Reinforcement Learning from Human Feedback

A short introduction to RLHF and post-training focused on language models.

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Abstract

Reinforcement learning from human feedback (RLHF) has become a crucial tool to build the latest machine learning systems at scale. The field grew around the core methods of RLHF into today's broader suite of post-training techniques. In this book, we give a comprehensive introduction to the core methods for post-training models for people with some level of quantitative background, organized around the canonical RLHF recipe. The book starts with what RLHF does and why it was created, with seminal technical milestones in its young history and a primer on reinforcement learning context needed to understand the book. The core of the book details every optimization stage in using RLHF, from starting with instruction tuning to training a reward model and finally all of rejection sampling, reinforcement learning, on-policy distillation, and direct alignment algorithms. The book also discusses broader topics, such as the origins of RLHF – both in recent literature and in a convergence of disparate fields of science in economics, philosophy, and optimal control. The book concludes with advanced topics – understudied or emerging research questions in synthetic data, tool-use, character training, and evaluation – and open questions for the field. The book is released with a variety of companion resources, including a codebase, a library to compare model completions from within post-training stages, and an educational course, to be a one-stop shop for learning all foundational concepts for post-training language models.

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1 Introduction

Reinforcement learning from human feedback (RLHF) is a technique used to incorporate human information into AI systems. RLHF emerged primarily as a method to solve hard-to-specify problems. With systems that are designed to be used by humans directly, such problems emerge all the time due to the often inexpressible nature of an individual's preferences. This encompasses every domain of content and interaction with a digital system. RLHF's early applications were often in control problems and other traditional domains for reinforcement learning (RL), where the goal is to optimize a specific behavior to solve a task. The core idea to start the field of RLHF was, "Can we solve hard problems only with basic preference signals guiding the optimization process?" RLHF became most known through the release of ChatGPT and the subsequent rapid development of large language models (LLMs) and other foundation models.

1.1 RLHF in Three Steps

The basic pipeline for RLHF involves three steps. First, a language model that can follow user questions must be trained (see Chapter 4). Second, human preference data must be collected for the training of a reward model of human preferences (see Chapter 5). Finally, the language model can be optimized with an RL optimizer of choice, by sampling generations and rating them with respect to the reward model (see Chapters 3 and 6). This book details key decisions and basic implementation examples for each step in this process.

RLHF has been applied to many domains successfully, with complexity increasing as the techniques have matured. Early breakthrough experiments with RLHF were applied to deep reinforcement learning [1], summarization [2], following instructions [3], parsing web information for question-answering [4], and "alignment" [5]. A summary of the early RLHF recipes is shown below in fig. 1.

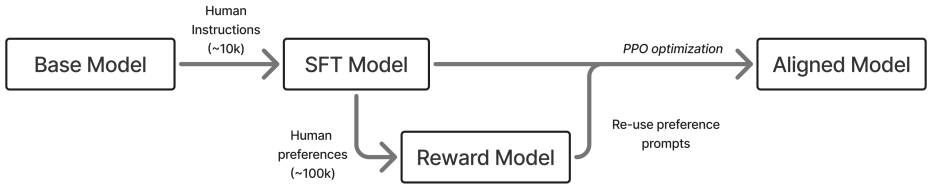


Figure 1: A rendition of the early, three stage RLHF process with SFT, a reward model, and then optimization.

In modern language model training, RLHF is one component of post-training. Post-training is a more complete set of techniques and best practices to make language models more useful for downstream tasks [6]. Post-training can be summarized as a many-stage training process using three optimization methods:

1. Instruction / Supervised Fine-tuning (IFT/SFT), where we teach formatting and form the base of instruction-following abilities. This is largely about learning *features* in

language.
2. Preference Fine-tuning (PreFT), where we align to human preferences via RLHF and related methods (and get a smaller bump in capabilities at the same time). This is largely about *style* of language and subtle human preferences that are hard to quantify.
3. Reinforcement Learning with Verifiable Rewards (RLVR), the newest type of post-training that boosts performance on verifiable domains with more RL training.

RLHF lives within and dominates the second area, **preference fine-tuning**, which has more complexity than instruction tuning because it often involves proxy reward models of the true objective and noisier data. At the same time, RLHF is far more established than the other popular RL method for language models, reinforcement learning with verifiable rewards. For that reason, this book focuses on preference learning, but in order to completely grasp the role of RLHF, one needs to use these other training stages, so they are also explained in detail.

As we consider the space of options and attention on these methods for crafting models we collectively use extensively, RLHF colloquially *is* what led to modern post-training. RLHF was the technique that enabled the massive success of the release of ChatGPT, so early in 2023 RLHF encompassed much of the interest in the general field of post-training. RLHF is now just one piece of post-training, so in this book we map why there was so much attention on RLHF early on, and how other methods emerged to complement it.

Training language models is a very complex process, often involving large technical teams of 10s to 100s of people and millions of dollars in data and compute costs. This book serves three purposes to enable readers to grasp how RLHF and related methods are used to craft leading models. First, the book distills cutting-edge research often hidden within large technology companies into clear topics and trade-offs, so readers can understand how models are made. Second, the book will allow users to set up basic code examples to get their hands dirty on fine-tuning these models themselves. Finally, beyond teaching the techniques for doing RLHF, this book is designed to distill intuition as to *why* RLHF is crucial to modern AI models.

Due to the complexity of RLHF and how the state-of-the-art is often too complex to be done alone, this book focuses on enabling readers so they have the tools needed to get jobs and start research projects in the area. Others will just enjoy precisely understanding the inner workings of the technology that is the focus of countless discussions across the globe. A book that lays out an exact recipe of how to do RLHF for a specific need is impossible, which is why there is a large industry of companies providing RLHF training and related methods as a service worth millions of dollars. Still, rapid progress in AI systems means the readers who are equipped with this knowledge can approach more and more of their own training over time.

1.2 What Does RLHF Do?

The biggest question around RLHF, yet one that is still hard to answer, is "What does RLHF training offer models?" In recent years, language models have shifted from academic experiments studied in the purview of benchmarks to general-purpose technology. RLHF is at the core of this transition.

To start, let us consider how a language model at the end of autoregressive pretraining answers a query. A strong model, Llama 3.1 405B Base, will continue from "The president

of the united states in 2006 was":

> **Response**: George W. Bush, the governor of Florida in 2006 was Jeb Bush, and John McCain was an Arizona senator in 2006 - who later lost to obama. September 1 – U.S. President Bush signs an executive order to provide more options for faith-based organizations when delivering social services that are paid for with federal funds. January 1 – The current deadline set by United States Congress in the Unlawful Internet Gambling Enforcement Act, aka UIGEA. search: Amendments to the 1961 International Convention for the Protection of New Varieties of Plants require plant breeders' rights include farmer's privilege. 2009 was a common year starting on Thursday of the Gregorian calendar, the 2009th year of the Common Era (ce) and Anno Domini (ad) designations, the 9th year of the 3rd millennium, the 9th year of the 21st century, and the 10th and last year of the 2000s decade.

What is clear is that this model is completing the sentence and adding other common internet metadata.

In contrast, a model trained with RLHF (and other post-training tools) would answer the question very differently. Asking Tülu 3 405B the same question "The president of the united states in 2006 was," it says concisely:

> **Response**: George W. Bush was the president of the United States in 2006. He served two terms in office, from January 20, 2001, to January 20, 2009.

This change in style is downstream of the post-training methods described in this book.

The most compelling view of how RLHF works is to consider how *style* applies to interactions you have with language models. The style, closely related to tone or format, of information presented is crucial to how it is learned. Style is often used to describe the manner of the model. To see the difference, consider two example responses to the following question:

> **User**: Help, I have to give a presentation tomorrow and I'm panicking.

A blunt, low-empathy style:

> **Response**: You'll be fine. Practice your slides a few times, get some sleep, and stop overthinking it. Just present what you know and move on.

A warmer, more supportive style:

> **Response**: That sounds really stressful, and it's a normal reaction. If you can, do one quick practice run now, pick the one or two points you most want people to remember, and then give yourself permission to rest—being clear and calm tomorrow will matter more than cramming tonight.

On the other hand, format is how the answer is structured. Modern models are known to use extensive Markdown formatting with lists, line breaks, and emojis. Consider two answers to the following question that are very different:

> **User**: How can I improve my sleep quality?

The simple answer, from an early language model:

> **Response**: Try to go to bed at the same time each night, avoid caffeine late in the day, and keep your bedroom quiet and dark.

Or a complex, richer answer:

> **Response**: Better sleep, in practice
>
> **Goal**: make your schedule predictable and your environment boring enough that your brain gives up.
>
> **Tonight (quick wins)**:
>
> 1. Dim lights for the last hour before bed.
> 2. No caffeine after lunch; keep alcohol minimal.
> 3. If you're stuck awake, get up and do something quiet until sleepy again.
>
> **Daily basics (most impact)**:
>
> - Same wake time (even weekends) –> anchors your body clock.
> - Morning light for 5-10 minutes –> helps set circadian rhythm.
> - Cool, dark room (fan/eye mask) –> easier to stay asleep.
>
> **If you want a simple rule**: `wake time fixed + caffeine cutoff + wind-down routine`
>
> If sleep problems are persistent or severe, it can be worth talking with a clinician—many issues are very treatable.

Instruction fine-tuning would provide the basic ability for models to respond reliably in the question-answering format, and RLHF is what takes these answers and crafts them into the reliable, warm, and engaging answers we now expect from language models.

Modern research has established RLHF as a general method to integrate subtle stylistic and related behavioral features into the models. An early, popular example of the utility of RLHF was in the application to safety [7] [5], where RLHF enabled models to be both helpful and harmless across varied datasets. Compared to other techniques for post-training, such as instruction fine-tuning, RLHF generalizes far better across domains [8] [9] – helping create effective general-purpose models.

Intuitively, this can be seen in how the optimization techniques are applied. Instruction fine-tuning trains the model to predict the next token when the text preceding is close to examples it has seen. It is optimizing the model to more regularly output specific features in text. This is a per-token update.

RLHF on the other hand tunes completions on the response level rather than looking at the next token specifically. Additionally, it is telling the model what a *better* response looks like, rather than a specific response it should learn. RLHF also shows a model which types of responses it should avoid, i.e. negative feedback. The training to achieve this is often called a *contrastive* loss function (one whose loss is computed from the comparison between two or more examples, rather than from each example independently) and is referenced throughout this book.

While this flexibility is a major advantage of RLHF, it comes with implementation challenges. Largely, these center on *how to control the optimization.* As we will cover in this book, implementing RLHF often requires training a reward model, but best practices for doing

so are not strongly established and depend on the area of application. With this, the optimization itself is prone to *over-optimization* because our reward signal is at best a proxy objective, requiring regularization. With these limitations, effective RLHF requires a strong starting point, so RLHF cannot be a solution to every problem alone and needs to be approached through a broader lens of post-training.

Due to this complexity, implementing RLHF is far more costly than simple instruction fine-tuning and can come with unexpected challenges such as length bias [10] [11]. For model training efforts where absolute performance matters, RLHF is established as being crucial to achieving a strong fine-tuned model, but it is more expensive in compute, data costs, and time. Through the early history of RLHF after ChatGPT, there were many research papers that showed approximate solutions to RLHF via limited instruction fine-tuning, but as the literature matured it has been repeated time and again that RLHF and related methods are core stages of model performance that cannot be easily dispensed with.

1.3 Walkthrough of an RLHF Recipe

To set the stage for the book, it's important to understand what "doing RLHF" can look like, as a minimal example, without any of the technical jargon that can be hard to grasp before solidifying fundamental intuitions. This section follows what is described as the canonical, three-stage RLHF recipe, as established with OpenAI's InstructGPT model in 2022 [3].

The first step of the process is to transition the model from a base model that completes text to an instruction-following model that can operate in a question-answering format. This is done by using the same next-token prediction loss function on a set of carefully crafted datapoints where the model is shown *only* data in this question-answering format. After the model is shown these high-quality responses, the model can now be prompted with a specific sequence of tokens to know that it should answer any query with a more defined, assistant persona.

With this foundation of *the shape of how the model should answer*, the next two steps work together to improve the overall quality of the answers. These two steps serve to set up a problem where we can use reinforcement learning to update the model and make it more helpful.

The first of these two steps is to train a reward model that captures human preferences. In order to apply reinforcement learning to a problem, you need a reward function that indicates quality. The goal of a reward model is to create a scalar signal that can then later be optimized with RL. In practice, this involves fine-tuning a language model (it is usually the same instruction-tuned model from the previous step) on a dataset of preference relations between pieces of text. This dataset is collected across a variety of prompts, model completions, and labelers to try and capture a robust signal of what is a better answer from a language model. The reward model learns which features in the text are better than others, so when it is used at inference-time (and during RL as the reward signal) it scores any piece of input text on how good it is.

With these two pieces, a question-answering model and a reward model, we have everything we need to put together the pieces and actually do reinforcement learning from human feedback (RLHF). The actual RLHF stage proceeds by taking prompts representative of tasks the model should be good at, generating a bunch of completions, having the reward model rank them, and then using RL to figure out how to change the model and make it

better. The basic primitive is that reinforcement learning is given a signal of which actions are good, in the form of tokens that a language model generates, and derives update rules that attribute different actions to different parameters in the model. The final RLHF stage shifts parameters to make good tokens more likely, and does so iteratively to maintain the general capabilities of the initial model.

Once RL is complete, and performance has saturated, this is often the final model served to the user.

Throughout this book, we'll cover many recipes for how to do RLHF, and more related optimization methods that make up the broader suite of post-training. These all emerge to solve more challenging problems facing language models, and to make the strengths of the original RLHF approaches more powerful.

1.4 An Intuition for Post-Training

We've established that RLHF specifically and post-training generally are crucial to the performance of the latest models and how they change the models' outputs, but not why RLHF works. Here's a simple analogy for how so many gains can be made on benchmarks on top of any base model.

The way I've been describing the potential of post-training is called the elicitation interpretation of post-training, where all we are doing is extracting potential by amplifying valuable behaviors in the base model.

To make this example click, we make the analogy between the base model – the language model that comes out of the large-scale, next-token prediction pretraining – and other foundational components in building complex systems. We use the example of the chassis of a car, which defines the space around which a car can be built. Consider Formula 1 (F1): most teams begin each year with a new chassis and engine. Then, they spend all year on aerodynamics and systems changes (of course, it is a minor oversimplification), and can dramatically improve the performance of the car. The best F1 teams improve far more during a season than chassis-to-chassis.

The same is true for post-training, where one can extract a ton of performance out of a static base model as they learn more about its quirks and tendencies. The best post-training teams extract a ton of performance in a very short time frame. The set of techniques includes everything close to and after the end of pretraining: "mid-training" like annealing / high-quality end of pretraining web data, instruction tuning, RLVR, preference-tuning, etc. A good example is the change from the first version of the Allen Institute for AI's fully-open, small Mixture-of-Experts (MoE) model OLMoE Instruct to the second. The first model was released in the fall of 2024 [12], and with the second version only updating the post-training, the evaluation average on popular benchmarks went from 35 to 48 without changing the majority of pretraining [13].

The idea is that there is a lot of intelligence and ability within base models, but because they can only answer in next-token prediction and not question-answering format, it takes a lot of work building around them, through post-training, in order to make excellent final models.

Then, when you look at models such as OpenAI's GPT-4.5 released in February 2025, which was largely a failure of a consumer product due to being too large of a base model to serve to millions of users, you can see this as a far more dynamic and exciting base for OpenAI to

build onto. With this intuition, base models determine the vast majority of the potential of a final model, and post-training's job is to cultivate all of it.

I've described this intuition as the Elicitation Theory of Post-training. This theory folds in with the reality that the majority of gains users are seeing are from post-training because it implies that there is more latent potential in a model pretrained on the internet than we can simply teach the model — such as by passing certain narrow samples in repeatedly during early types of post-training (i.e. only instruction tuning). The challenge of post-training is to reshape models from next-token prediction to conversation question-answering, while extracting all of this knowledge and intelligence from pretraining.

A related idea to this theory is the Superficial Alignment Hypothesis, coined in the paper LIMA: Less is More for Alignment [14]. This paper is getting some important intuitions right but for the wrong reasons in the big picture. The authors state:

> A model's knowledge and capabilities are learnt almost entirely during pretraining, while alignment teaches it which subdistribution of formats should be used when interacting with users. If this hypothesis is correct, and alignment is largely about learning style, then a corollary of the Superficial Alignment Hypothesis is that one could sufficiently tune a pretrained language model with a rather small set of examples.

All of the successes of deep learning should have taught you that scaling data is important to performance. Here, the major difference is that the authors are discussing alignment and style, the focus of academic post-training at the time. With a few thousand samples for instruction fine-tuning, you can change a model substantially and improve a narrow set of evaluations, such as AlpacaEval, MT-Bench, Arena (formerly Chatbot Arena, a platform where users compare anonymous model responses head-to-head), and the like. These do not always translate to more challenging capabilities, which is why Meta wouldn't train its Llama Chat models on just this dataset. Academic results have lessons, but need to be interpreted carefully if you are trying to understand the big picture of the technological arc.

What this paper is showing is that you can change models substantially with a few samples. We knew this, and it is important to the short-term adaptation of new models, but their argument for performance leaves the casual readers with the wrong lessons.

If we change the data, the impact could be far higher on the model's performance and behavior, but it is far from "superficial." Base language models today (with no post-training) can be trained on some mathematics problems with reinforcement learning, learn to output full chain-of-thought reasoning, and then score higher on a full suite of reasoning evaluations like BigBenchHard, Zebra Logic, AIME, etc.

The superficial alignment hypothesis is wrong for the same reason that people who think RLHF and post-training are just for vibes are still wrong. This was a field-wide lesson we had to overcome in 2023 (although many AI observers are still rooted in this belief). Post-training has far outgrown that, and we are coming to see that the style of models operates on top of behavior — such as the now popular long chain of thought.

As the AI community shifts post-training further into the era of agentic and reasoning models, the superficial alignment hypothesis breaks down further. RL methods are becoming an increasingly large share of the compute needed to train frontier language models. In the short time since reinforcement learning with verifiable rewards (RLVR) was coined in our

work on Tülu 3 in the fall of 2024 [6], the scale of compute used for post-training has grown dramatically. DeepSeek R1, famous for popularizing RLVR, used only about 5% of their overall compute in post-training – 147K H800 GPU hours for RL training on R1 [15], relative to 2.8M GPU hours for pretraining the underlying DeepSeek V3 base model [16].

The science studying the core methods of scaling RL as of 2026 shows that individual ablation runs can take 10-100K GPU hours [17], the equivalent of the compute used for the RL stage of Olmo 3.1 Think 32B (released in November of 2025), which trained for 4 weeks on 200 GPUs [18]. The science of scaled post-training is in its very early stages as of 2026, adopting ideas and methods from pretraining language models and applying them in this new domain, so the exact GPU hours used will change, but the trend of increased compute on post-training will continue. Altogether, the elicitation theory of post-training is likely to become the correct view only when applying a lighter post-training recipe – something useful for specializing a model – relative to the compute-intensive frontier models.

1.5 How We Got Here

Why does this book make sense now? How much will change in the future?

Post-training, the craft of eliciting powerful behaviors from a raw pretrained language model, has gone through many seasons and moods since the release of ChatGPT that sparked the renewed interest in RLHF. In the era of Alpaca [19], Vicuna [20], Koala [21], and Dolly [22], a limited number of human datapoints with extended synthetic data in the style of Self-Instruct were used to fine-tune the original LLaMA to get similar behavior to ChatGPT. The benchmark for these early models was fully vibes (and human evaluation) as we were all so captivated by the fact that these small models can have such impressive behaviors across domains. It was justified excitement.

Open post-training was moving faster, releasing more models, and making more noise than its closed counterparts. Companies were scrambling, e.g. DeepMind merging with Google Brain or new labs being started, and taking time to follow it up. There are phases of open recipes surging and then lagging behind.

The era following Alpaca et al., the first lag in open recipes, was one defined by skepticism and doubt about reinforcement learning from human feedback (RLHF), the technique OpenAI highlighted as crucial to the success of the first ChatGPT. Many companies doubted that they needed to do RLHF. A common phrase – "instruction tuning is enough for alignment" – was so popular then that it still carries weight today despite obvious evidence against it.

This doubt about RLHF lasted, especially in the open where groups cannot afford data budgets on the order of $100K to $1M. The companies that embraced it early ended up winning out. Anthropic published extensive research on RLHF through 2022 and now has arguably the best post-training [23] [5] [24]. The delta between open groups, struggling to reproduce or even know of basic closed techniques, and leading closed models is a common theme.

The first shift in open alignment methods and post-training was the story of Direct Preference Optimization (DPO) [25], which showed that you can solve the same optimization problem as RLHF with fewer moving parts by taking gradient steps directly on pairwise preference data. The DPO paper, posted in May of 2023, didn't have any clearly impactful models trained with it through the fall of 2023. This changed with the releases of a few breakthrough DPO

models – all contingent on finding a better, lower learning rate. Zephyr-Beta [26], Tülu 2 [27], and many other models showed that the DPO era of post-training had begun. Chris Manning literally thanked me for "saving DPO."

Preference-tuning was something you needed to do to meet the table stakes of releasing a good model since late 2023. The DPO era continued through 2024, in the form of never-ending variants on the algorithm, but we were very far into another slump in open recipes. Open post-training recipes had saturated the extent of knowledge and resources available.
A year after Zephyr and Tülu 2, the same breakout dataset, UltraFeedback is arguably still state-of-the-art for preference tuning in open recipes [28].

At the same time, the Llama 3.1 [29] and Nemotron 4 340B [30] reports gave us substantive hints that large-scale post-training is much more complex and impactful. The closed labs are doing full post-training – a large multi-stage process of instruction tuning, RLHF, prompt design, etc. – where academic papers are just scratching the surface. Tülu 3 represented a comprehensive, open effort to build the foundation of future academic post-training research [6].

Post-training is a complex process involving the aforementioned training objectives applied in various orders to target specific capabilities. This book is designed to provide a platform for understanding all of these techniques, and as the field matures the best practices for how to interleave them will emerge.

The primary areas of innovation in post-training are now in reinforcement learning with verifiable rewards (RLVR), reasoning training generally, and related ideas. These newer methods build extensively on the infrastructure and ideas of RLHF, but are evolving far faster. This book is written to capture the first stable literature for RLHF after its initial period of rapid change.

1.6 Scope of This Book

This book hopes to touch on each of the core steps of doing canonical RLHF implementations. It will not cover all the history of the components nor recent research methods, just techniques, problems, and trade-offs that have been proven to occur again and again.

1.6.1 Chapter Summaries

This book has the following chapters:

1.6.1.1 Introductions

Reference material and context useful throughout the book.

1. Introduction: Overview of RLHF and what this book provides.
2. A Tiny History of RLHF: Key models and papers in the history of RLHF techniques.
3. Training Overview: How the training objective for RLHF is designed and basics of understanding it.

1.6.1.2 Core Training Pipeline

The suite of techniques used to optimize language models to align them to human preferences.

4. Instruction Fine-Tuning: Adapting language models to the question-answer format.

5. Reward Modeling: Training reward models from preference data that act as an optimization target for RL training (or for use in data filtering).
6. Reinforcement Learning: The core RL techniques used to optimize reward models (and other signals) throughout RLHF.
7. Reasoning and Inference-Time Scaling: The role of new RL training methods for inference-time scaling with respect to post-training and RLHF.
8. Direct-Alignment Algorithms: Algorithms that optimize the RLHF objective directly from pairwise preference data rather than learning a reward model first.
9. Rejection Sampling: A basic technique for using a reward model with instruction tuning to align models.

1.6.1.3 Data & Preferences Context for the data that fuels RLHF and the big picture problem it is trying to solve.

10. The Nature of Preferences: Why human preference data is needed to fuel and understand RLHF.
11. Preference Data: How preference data is collected for RLHF.
12. Synthetic Data: The shift away from human to synthetic data, how AI feedback works, and how distilling from other models is used.
13. Tool Use and Function Calling: The basics of training models to call functions or tools in their outputs.

1.6.1.4 Practical Considerations Fundamental problems and discussions for implementing and evaluating RLHF.

14. Over-Optimization: Qualitative observations of why RLHF goes wrong and why over-optimization is inevitable with a soft optimization target in reward models.
15. Regularization: Tools to constrain these optimization tools to effective regions of the parameter space.
16. Evaluation: The ever evolving role of evaluation (and prompting) in language models.
17. Crafting Model Character and Products: How RLHF is shifting in its applicability as major AI laboratories use it to subtly match their models to their products.

1.6.1.5 Appendices Reference material for definitions and extended discussions.

- Appendix A - Definitions: Mathematical definitions for RL, language modeling, and other ML techniques leveraged in this book.
- Appendix B - Beyond "Just Style": How RLHF is often underestimated in its role in improving the user experience of models due to the crucial role that style plays in information sharing.

1.6.2 Target Audience

This book is intended for audiences with entry level experience with language modeling, reinforcement learning, and general machine learning. It will not have exhaustive documentation for all the techniques, but just those crucial to understanding RLHF.

1.6.3 How to Use This Book

This book was largely created because there were no canonical references for important topics in the RLHF workflow. Given the pace of progress on LLMs overall, combined with the complex nature of collecting and using human data, RLHF is an unusually academic field where published results are often noisy and hard to reproduce across multiple settings. To develop strong intuitions, readers are encouraged to read multiple papers on each topic rather than taking any single result as definitive. To facilitate this, the book includes numerous, academic-style citations to the canonical reference for a claim.

The contributions of this book are supposed to give you the minimum knowledge needed to try a toy implementation or dive into the literature. This is *not* a comprehensive textbook, but rather a quick book for reminders and getting started.

The print edition of this book was published by Manning in July 2026, while the web version continues to collect minor improvements and errata fixes. If you spot a typo or an important omission, please contribute a fix or suggestion on GitHub.

1.6.4 About the Author

Dr. Nathan Lambert is a researcher and writer focusing on building the open science of language models. He came here through a Ph.D. in robotics and building an RLHF team shortly after the release of ChatGPT. He has released many models trained with RLHF, their subsequent datasets, and training codebases in his time at the Allen Institute for AI (Ai2) and Hugging Face. Examples include Zephyr-Beta, Tülu 2, OLMo, TRL, Open Instruct, and many more. He has written extensively on RLHF, including many blog posts and academic papers.

1.7 Future of RLHF

With the investment in language modeling, many variations on the traditional RLHF methods emerged. RLHF colloquially has become synonymous with multiple overlapping approaches. RLHF is a subset of preference fine-tuning (PreFT) techniques, including Direct Alignment Algorithms (See Chapter 8), which are the class of methods downstream of DPO that solve the preference learning problem by taking gradient steps directly on preference data, rather than learning an intermediate reward model. RLHF is the tool most associated with rapid progress in "post-training" of language models, which encompasses all training after the large-scale autoregressive training on primarily web data. This textbook is a broad overview of RLHF and its directly neighboring methods, such as instruction tuning and other implementation details needed to set up a model for RLHF training.

As more successes of fine-tuning language models with RL emerge, such as OpenAI's o1 reasoning models, RLHF will be seen as the bridge that enabled further investment of RL methods for fine-tuning large base models. At the same time, while the spotlight of focus may be more intense on the RL portion of RLHF in the near future – as a way to maximize performance on valuable tasks – the core of RLHF is that it is a lens for studying the grand problems facing modern forms of AI. How do we map the complexities of human values and objectives into systems we use on a regular basis? This book hopes to be the foundation of decades of research and lessons on these problems.

2 A Tiny History of RLHF

RLHF and its related methods are very new. We highlight history to show how recently the procedures were formalized, and how much of this documentation is in the academic literature. With this, we want to emphasize that RLHF is very rapidly evolving, so the chapter sets the stage for a book that will express uncertainty over certain methods and an expectation that some details can change around a few, core practices. Otherwise, the papers and methods listed here showcase why many pieces of the RLHF pipeline are what they are, as some of the seminal papers were for applications totally distinct from modern language models.

In this chapter we detail the key papers and projects that got the RLHF field to where it is today. This is not intended to be a comprehensive review of RLHF and the related fields, but rather a starting point and retelling of how we got to today. It is intentionally focused on recent work that led to ChatGPT. There is substantial further work in the RL literature on learning from preferences [31]. For a more exhaustive list, you should use a proper survey paper [32], [33].

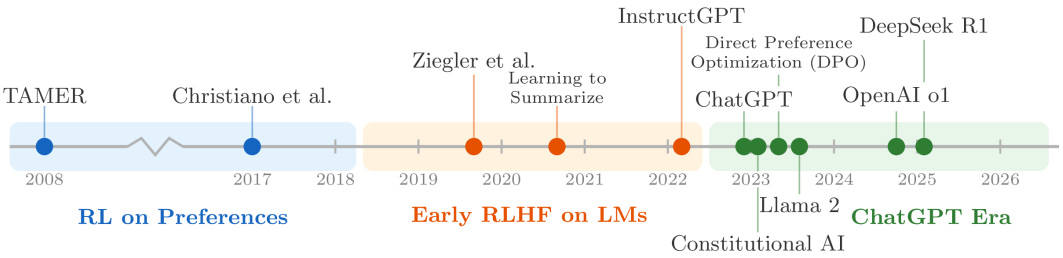


Figure 2: Timeline of key developments in RLHF discussed in this chapter, from early work on RL from preferences through the adoption of RLHF in large language models.

2.1 Origins to 2018: RL on Preferences

The field has recently been popularized with the growth of Deep Reinforcement Learning and has grown into a broader study of the applications of LLMs from many large technology companies. Still, many of the techniques used today are deeply related to core techniques from early literature on RL from preferences.

One of the first papers with an approach similar to modern RLHF was *TAMER. TAMER: Training an Agent Manually via Evaluative Reinforcement* proposed an approach in which humans iteratively scored an agent's actions to learn a reward model, which was used to learn the action policy [34]. Other work, concurrently or soon after, proposed an actor-critic algorithm, COACH, where human feedback (both positive and negative) is used to tune the advantage function [35].

The primary reference, Christiano et al. 2017, is an application of RLHF applied to preferences between trajectories of agents within Atari games [1]. This work introducing RLHF followed soon after DeepMind's seminal work in reinforcement learning on Deep Q-Networks (DQN), which showed that RL agents can solve popular video games learning from scratch. The work shows that humans choosing between trajectories can be more effective in some domains

than directly interacting with the environment. This uses some clever conditions, but is impressive nonetheless.

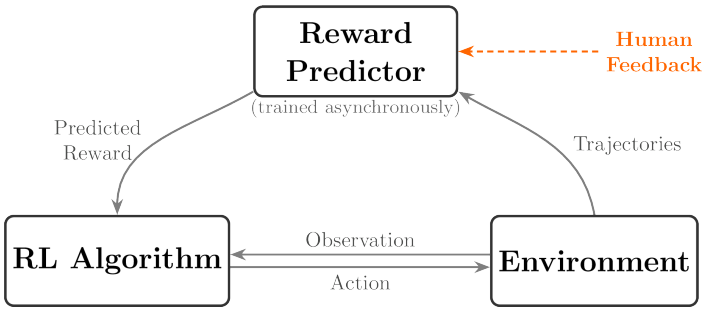


Figure 3: The core RLHF loop from Christiano et al. (2017): the reward predictor is trained asynchronously from comparisons of trajectory segments, and the agent maximizes predicted reward.

This method was expanded upon with more direct reward modeling [36] and the adoption of deep learning within early RLHF work was capped by an extension to TAMER with neural network models just one year later [37].

This era began to transition, as reward models as a general notion were proposed as a method for studying alignment, rather than just a tool for solving RL problems [38].

2.2 2019 to 2022: RL from Human Preferences on Language Models

Reinforcement learning from human feedback, also referred to regularly as reinforcement learning from human preferences in its early days, was quickly adopted by AI labs increasingly turning to scaling large language models. A large portion of this work began between GPT-2, in 2019, and GPT-3, in 2020. The earliest work in 2019, *Fine-Tuning Language Models from Human Preferences* has many striking similarities to modern work on RLHF and the content that we will cover in this book [39]. Many canonical terms, such as learning reward models, KL distances, feedback diagrams, etc., were formalized in this paper, though the evaluation tasks for the final models and their capabilities were different from what people are doing today. From here, RLHF was applied to a variety of tasks. Important examples include general summarization [2], recursive summarization of books [40], instruction following (InstructGPT) [3], browser-assisted question-answering (WebGPT) [4], supporting answers with citations (GopherCite) [41], and general dialogue (Sparrow) [42].

Aside from applications, a number of seminal papers defined key areas for the future of RLHF, including those on:

1. Reward model over-optimization [43]: The ability for RL optimizers to over-fit to models trained on preference data,
2. Language models as a general area of study for alignment [23], and
3. Red teaming [44] – the process of assessing the safety of a language model.

Work continued on refining RLHF for application to chat models. Anthropic continued to use it extensively for early versions of Claude [5] and early RLHF open-source tools emerged [45], [46], [47].

2.3 2023 to the Present: The ChatGPT Era

The announcement of ChatGPT was very clear about the role of RLHF in its training [48]:

> We trained this model using Reinforcement Learning from Human Feedback (RLHF), using the same methods as InstructGPT, but with slight differences in the data collection setup.

Since then, RLHF has been used extensively in leading language models. It is well known to be used in Anthropic's Constitutional AI for Claude [24], Meta's Llama 2 [49] and Llama 3 [29], NVIDIA's Nemotron [30], Ai2's Tülu 3 [6], and more.

Today, RLHF is growing into a broader field of preference fine-tuning (PreFT), including new applications such as process rewards for intermediate reasoning steps [50], covered in Chapter 5; direct alignment algorithms inspired by Direct Preference Optimization (DPO) [25], covered in Chapter 8; learning from execution feedback from code or math [51], [52] and other online reasoning methods inspired by OpenAI's o1 [53], covered in Chapter 7.

3 Training Overview

In this chapter we provide a cursory overview of RLHF training, before getting into the specifics later in the book. RLHF, while optimizing a simple loss function, involves training multiple, different AI models in sequence and then linking them together in a complex, online optimization.

Here, we introduce the core objective of RLHF, which is optimizing a proxy reward for human preferences with a distance-based regularizer (along with showing how it relates to classical RL problems). Then we showcase canonical recipes which use RLHF to create leading models to show how RLHF fits in with the rest of post-training methods. These example recipes will serve as references for later in the book, where we describe different optimization choices you have when doing RLHF, and we will point back to how different key models used different steps in training.

3.1 Problem Formulation

The optimization of reinforcement learning from human feedback (RLHF) builds on top of the standard RL setup. In RL, an agent takes actions a_t sampled from a policy $\pi(a_t \mid s_t)$ given the state of the environment s_t to maximize reward $r(s_t, a_t)$ [54]. A policy is a function that maps each state to a probability distribution over actions. The early policies that evolved into modern literature on RLHF were in what is called deep reinforcement learning – when a neural network is used to learn said function. Traditionally, the environment evolves according to transition (dynamics) $p(s_{t+1} \mid s_t, a_t)$ with an initial state distribution $\rho_0(s_0)$. Together, the policy and dynamics induce a trajectory distribution. A trajectory's overall probability is the product of the initial state probability, every action choice the policy makes, and every state transition the environment produces:

$$p_\pi(\tau) = \rho_0(s_0) \prod_{t=0}^{T-1} \pi(a_t \mid s_t)\, p(s_{t+1} \mid s_t, a_t). \tag{1}$$

Across a finite episode with horizon T, the goal of an RL agent is to solve the following optimization, where γ is a discount factor from 0 to 1 that balances the desirability of near-term versus future rewards:

$$\max_\pi \; \mathbb{E}_{\tau \sim p_\pi} \left[\sum_{t=0}^{T-1} \gamma^t r(s_t, a_t) \right]. \tag{2}$$

The expected return for a given policy is often denoted $J(\pi)$, with the optimal value written $J^* = \max_\pi J(\pi)$.

For continuing tasks, one often takes $T \to \infty$ and relies on discounting ($\gamma < 1$) to keep the objective well-defined. Multiple methods for optimizing this expression are discussed in Chapter 6.

A standard illustration of the RL loop is shown in fig. 4 (compare this to the RLHF loop in fig. 7).

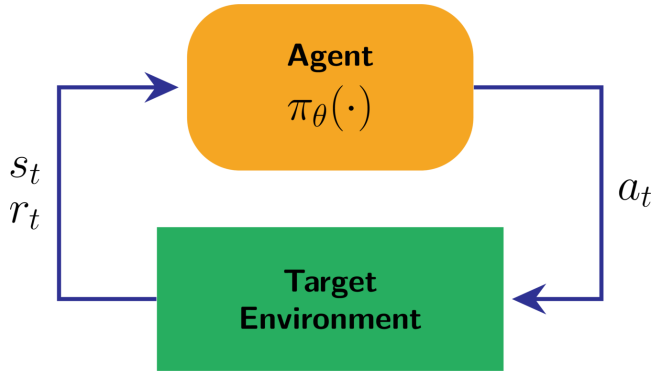


Figure 4: Standard RL loop

3.1.1 A Simple Example: The Thermostat

To build a basic intuition for what RL does, consider a thermostat trying to keep a room at a target temperature of 70°F. In RL, the agent starts with no knowledge of the task and must discover a good policy through trial and error. The thermostat example has the following components (see fig. 5 for how each maps to the trajectory distribution in eq. 1):

- **State** (s_t): the current room temperature, e.g. 65°F.
- **Action** (a_t): turn the heater on or off.
- **Reward** (r): +1 when the temperature is within 2° of the target, 0 otherwise.
- **Policy** (π): the rule that decides whether to turn the heater on or off given the current temperature. Here is one policy the thermostat might learn, which may not be optimal depending on the exact transition dynamics of the environment:

$$\pi(a_t = \text{on} \mid s_t) = \begin{cases} 1 & \text{if } s_t < 70°\text{F} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

- **Transition**: the room warms when the heater is on and cools when it is off. The agent influences these dynamics through its actions, but the underlying physics – how fast the room heats or cools – are outside its control.

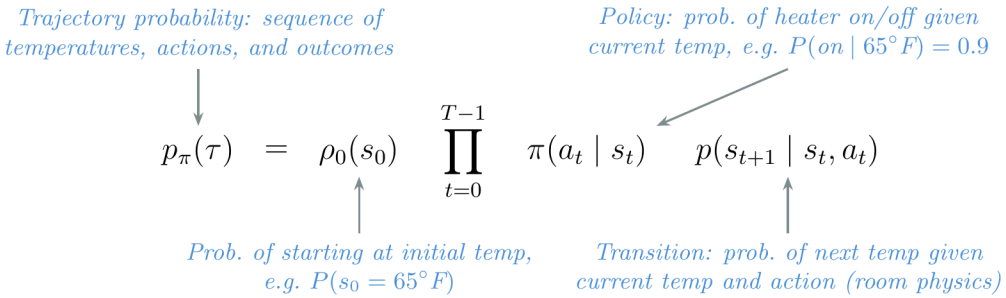


Figure 5: Each term in the trajectory distribution (eq. 1) mapped to the thermostat RL example.

Initially, the thermostat's policy is essentially random – it flips the heater on and off with no regard for the current temperature, and the room's temperature swings wildly. Over many episodes of trial and error, the agent discovers that turning the heater on when the room is cold and off when it is warm leads to more reward, and gradually converges on a sensible policy. This is the core RL loop: observe a state, choose an action, receive a reward, and update the policy to get more reward over time.

3.1.2 Classic RL Example: CartPole

For a richer example with continuous dynamics, consider the classic *CartPole* (inverted pendulum) control task, which appears in many RL textbooks, courses, and even research papers. Whereas the thermostat had a single state variable and a binary action, CartPole involves four continuous state variables and physics-based transitions – making it a standard benchmark for RL algorithms.

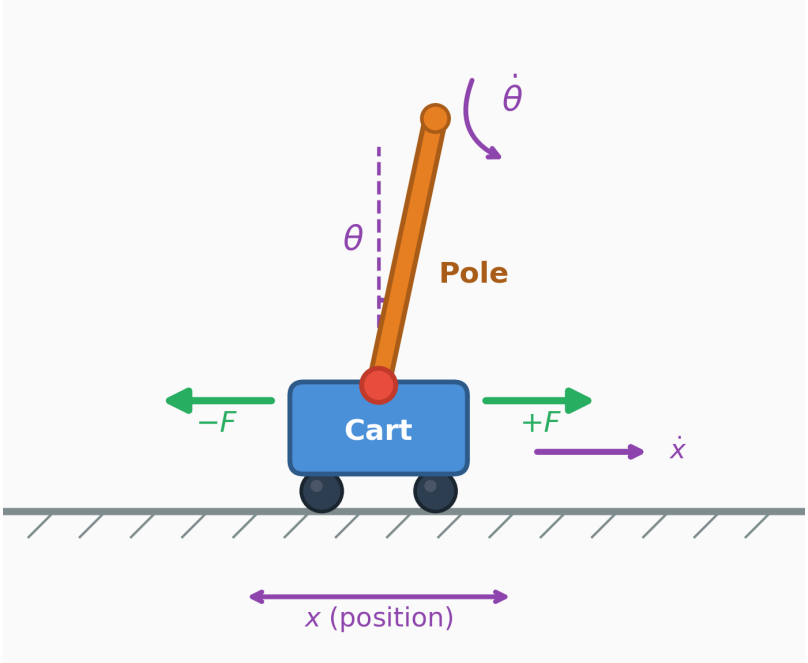


Figure 6: CartPole environment showing state variables (x, $\dot{x}$, θ, $\dot{\theta}$) and actions ($\pm F$).

- **State (s_t)**: the cart position/velocity and pole angle/angular velocity:

$$s_t = (x_t, \dot{x}_t, \theta_t, \dot{\theta}_t). \tag{4}$$

- **Action (a_t)**: apply a left/right horizontal force to the cart, e.g. $a_t \in \{-F, +F\}$.
- **Reward (r)**: a simple reward is $r_t = 1$ each step the pole remains balanced and the cart stays on the track (e.g. $|x_t| \leq 2.4$ and $|\theta_t| \leq 12°$), and the episode terminates when either bound is violated.

- **Dynamics / transition ($p(s_{t+1} \mid s_t, a_t)$)**: in many environments the dynamics are deterministic (so p is a point mass) and can be written as $s_{t+1} = f(s_t, a_t)$ via Euler integration with step size Δt. A standard simplified CartPole update uses the constants cart mass m_c, pole mass m_p, pole half-length l, and gravity g (α is a mass-normalized intermediate with acceleration units):

$$\alpha = \frac{a_t + m_p l\, \dot{\theta}_t^2 \sin\theta_t}{m_c + m_p} \tag{5}$$

$$\ddot{\theta}_t = \frac{g \sin\theta_t - \cos\theta_t\, \alpha}{l\left(\frac{4}{3} - \frac{m_p \cos^2\theta_t}{m_c + m_p}\right)} \tag{6}$$

$$\ddot{x}_t = \alpha - \frac{m_p l\, \ddot{\theta}_t \cos\theta_t}{m_c + m_p} \tag{7}$$

$$x_{t+1} = x_t + \Delta t\, \dot{x}_t, \quad \dot{x}_{t+1} = \dot{x}_t + \Delta t\, \ddot{x}_t, \tag{8}$$

$$\theta_{t+1} = \theta_t + \Delta t\, \dot{\theta}_t, \quad \dot{\theta}_{t+1} = \dot{\theta}_t + \Delta t\, \ddot{\theta}_t. \tag{9}$$

This is a concrete instance of the general setup above: the policy chooses a_t, the transition function advances the state, and the reward is accumulated over the episode.

3.1.3 Manipulating the Standard RL Setup

The RL formulation for RLHF is seen as a less open-ended problem, where a few key pieces of RL are set to specific definitions in order to accommodate language models. There are multiple core changes from the standard RL setup to that of RLHF: Table tbl. 1 summarizes these differences between standard RL and the RLHF setup used for language models.

1. **Switching from a reward function to a reward model.** In RLHF, a learned model of human preferences, $r_\theta(s_t, a_t)$ (or any other classification model) is used instead of an environmental reward function. This gives the designer a substantial increase in the flexibility of the approach and control over the final results, but at the cost of implementation complexity. In standard RL, the reward is seen as a static piece of the environment that cannot be changed or manipulated by the person designing the learning agent.
2. **No state transitions exist.** In RLHF, the initial states for the domain are prompts sampled from a training dataset and the "action" is the completion to said prompt (in the standard RLHF setup, the prompt is fixed and the model's completion does not define the next prompt). The combination of one prompt and one completion constitutes a complete episode or rollout, which would be many repeated state-action, state-action chains in classical RL problems.
3. **Response-level rewards and no discounting.** RLHF attribution of reward is done for an entire sequence of actions, composed of multiple generated tokens, rather than in a fine-grained manner (this single-step structure is sometimes called a bandit problem in the RL literature). To help the RL algorithms for RLHF see every token as part of the same action, implementations usually use a discount factor of $\gamma = 1$

(no discounting), unlike standard RL where $\gamma < 1$ balances short-term and long-term reward across many sequential decisions.

Table 1: Key differences between standard RL and RLHF for language models.

Aspect	Standard RL	RLHF (language models)
Policy	Learned from scratch (random init)	Fine-tuned from a pretrained language model
Reward signal	Environment reward function $r(s_t, a_t)$	Learned reward / preference model $r_\theta(x, y)$ (prompt x, completion y)
State transition	Yes: dynamics $p(s_{t+1} \mid s_t, a_t)$	Typically no: prompts x sampled from a dataset; the completion does not define the next prompt
Action	Single environment action a_t	A completion y (a sequence of tokens) sampled from $\pi_\theta(\cdot \mid x)$
Reward granularity	Often per-step / fine-grained	Usually response-level (bandit-style) over the full completion, usually no discounting ($\gamma = 1$)
Horizon	Multi-step episode ($T > 1$)	Often single-step ($T = 1$), though multi-turn can be modeled as longer-horizon

Given the single-turn nature of the problem, the optimization can be re-written without the time horizon and discount factor (and with an explicit reward model):

$$\max_{\pi} \mathbb{E}_{\tau \sim \pi} \left[r_\theta(s_t, a_t) \right]. \tag{10}$$

In many ways, the result is that while RLHF is heavily inspired by RL optimizers and problem formulations, the actual implementation is very distinct from traditional RL.

3.1.4 Fine-Tuning and Regularization

In traditional RL problems, the agent must learn from a randomly initialized policy, but with RLHF, we start from a strong pretrained base model with many initial capabilities. This strong prior for RLHF induces a need to prevent the optimization from drifting too far from the initial policy. In order to succeed in a fine-tuning regime, RLHF techniques employ multiple types of regularization to control the optimization. The goal is to allow the reward maximization to still occur without the model succumbing to over-optimization, as discussed in Chapter 14. The most common change to the optimization function is to add a KL divergence penalty on the distance between the current RLHF policy and the starting point of the optimization. The β hyperparameter set when training the model controls the strength of this constraint – a larger β keeps the model closer to its starting point, while a smaller β gives the optimizer more freedom to chase reward:

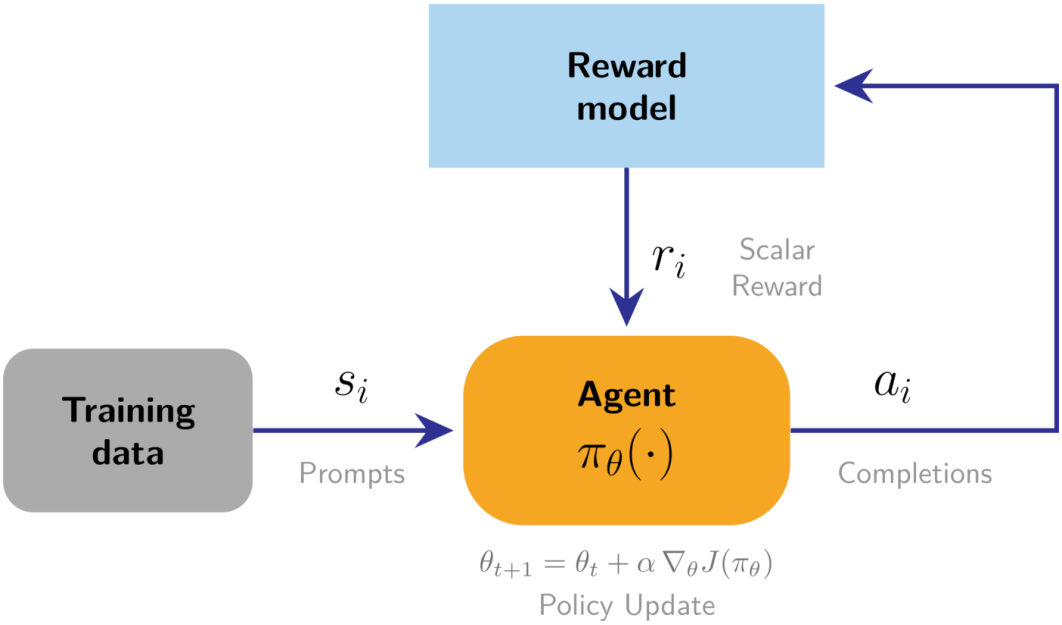


Figure 7: Standard RLHF loop

$$\max_{\pi} \mathbb{E}_{\tau\sim\pi}\left[r_\theta(s_t, a_t)\right] - \beta\mathcal{D}_{\text{KL}}(\pi(\cdot|s_t)\|\pi_{\text{ref}}(\cdot|s_t)). \tag{11}$$

Within this formulation, a lot of study into RLHF training goes into understanding how to spend a certain "KL budget" as measured by a distance from the initial model. For more details, see Chapter 15 on Regularization.

3.1.5 Optimization Tools

In this book, we detail many popular techniques for solving this optimization problem. The popular tools of post-training include:

- **Reward modeling** (Chapter 5): A model is trained to capture the signal from collected preference data and can then output a scalar reward indicating the quality of future text.
- **Instruction fine-tuning** (Chapter 4): A prerequisite to RLHF where models are taught the question-answer format used in the majority of language modeling interactions today by imitating preselected examples.
- **Rejection sampling** (Chapter 9): The most basic RLHF technique where candidate completions for instruction fine-tuning are filtered by a reward model imitating human preferences.
- **Policy gradients** (Chapter 6): The reinforcement learning algorithms used in the seminal examples of RLHF to update parameters of a language model with respect to the signal from a reward model.
- **Direct alignment algorithms** (Chapter 8): Algorithms that directly optimize a policy from pairwise preference data, rather than learning an intermediate reward

model to then optimize later.

Modern RLHF-trained models always utilize instruction fine-tuning followed by a mixture of the other optimization options.

3.1.6 Subtle Advantages of RL in Post-Training Language Models

In the following chapters, we cover many optimization tools for post-training. Plenty of them, such as rejection sampling (Chapter 9) and direct alignment algorithms like DPO (Chapter 8), are far simpler than getting RL working. Still, despite the simplicity of alternatives, RL-based methods continue to win out. Some trends, such as the inference-time scaling with reinforcement learning with verifiable rewards (RLVR), are obvious, but RL has turned out to be a well-suited optimization tool for language models. Implementing RL requires a far larger infrastructure investment relative to instruction tuning or DPO-like algorithms, but, at the risk of being overly colloquial, the gradient updates it provides "generally help the model a lot." This is hard to quantify, but comes in a few recurring forms:

- RL stages can "fix" rough edges on the model, making the model easier to chat with or more robust (this could come by training it to have numerical stability with inference tools like vLLM). The exact reason for this is not well-known in the literature, but its truth is reflected in the growing presence of RL today.
- RL can be done surgically — the model does a good job of learning where the prompt distribution lies, and RL tends to not "squash" the general capabilities of the model. A good example of this is Tülu 3 being trained with RL only on math prompts, while maintaining capabilities across a broad task suite [6].

Overall, RL losses on language models are robust, scalable, effective, and flexible, which opened large new fields of experimentation. The original method that started us down this path was RLHF work.

3.2 Canonical Training Recipes

Over time various models have been identified as canonical recipes for RLHF specifically or post-training generally. These recipes reflect data practices and model abilities at the time. As the recipes age, training models with the same characteristics becomes easier and requires less data. There is a general trend of post-training involving more optimization steps with more training algorithms across more diverse training datasets and evaluations.

3.2.1 InstructGPT: Foundational RLHF Tools

Around the time ChatGPT first came out, the widely accepted ("canonical") method for post-training an LM had three major steps, with RLHF being the central piece [55] [3] [5]. The three steps taken on top of a "base" language model (the next-token prediction model trained on large-scale web text) are summarized below in fig. 8:

1. **Instruction tuning on ~10K examples**: This teaches the model to follow the question-answer format and teaches some basic skills from primarily human-written data.
2. **Training a reward model on ~100K prompts with pairwise completions** (paper used 33K prompts): This model is trained from the instruction-tuned checkpoint

and captures the diverse values one wishes to model in their final training. The reward model is the optimization target for RLHF.

3. **Training the instruction-tuned model with RLHF on a separate ~100K prompts** (paper used exactly 31K and does not document whether prompts were reused from other stages): The model is optimized against the reward model with a likely separate set of prompts, where it generates responses before receiving ratings.

Once RLHF was done, the model was ready to be deployed to users. This recipe is the foundation of modern RLHF, but recipes have evolved substantially to include more stages and more data.

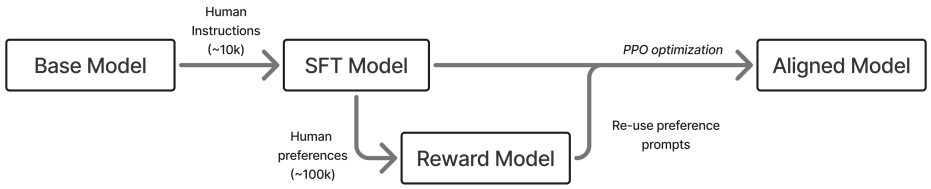


Figure 8: A rendition of the early, three stage RLHF process with SFT, a reward model, and then optimization.

3.2.2 Tülu 3: Introducing RLVR to Instruct Models

Modern versions of post-training involve many, many more model versions and training stages (i.e. well more than the 5 RLHF steps documented for Llama 2 [49]). An example is shown below in fig. 9 where the model undergoes numerous training iterations before convergence.

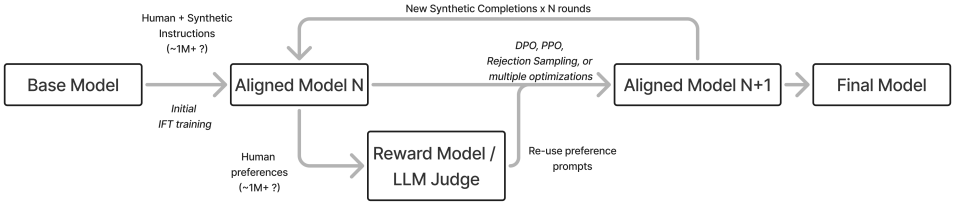


Figure 9: A rendition of modern post-training with many rounds.

The most complex models trained in this era and onwards have not released full details of their training process. Leading models such as ChatGPT or Claude by 2026 involve many iterative rounds of training. This can even include techniques that train specialized models and then merge the weights together to get a final model capable of many subtasks [56] (e.g. Cohere's Command A [57]).

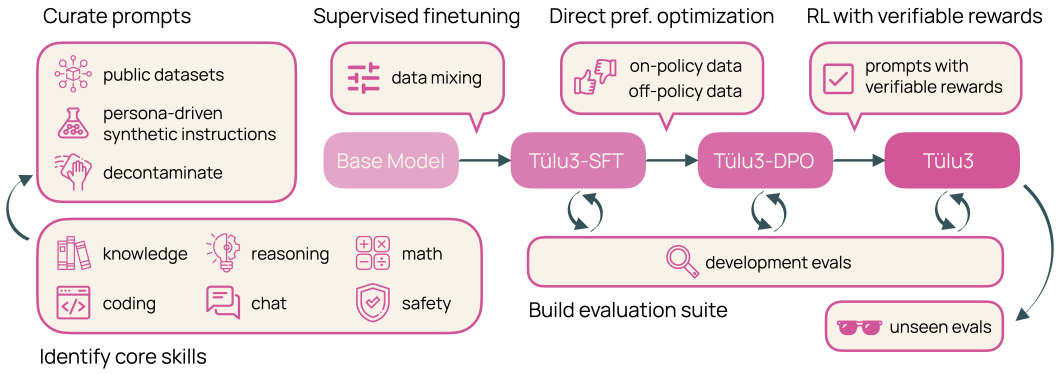


Figure 10: A summary of the Tülu 3 recipe with target skills and multi-step training recipe. Lambert et al. 2024, License CC-BY.

A fully open example of this multi-stage approach to post-training where RLHF plays a major role is Tülu 3. The Tülu 3 recipe consists of three stages:

1. **Instruction tuning on ~1M examples**: This primarily synthetic dataset, drawn from a mix of frontier models such as GPT-4o and Llama 3.1 405B, teaches the model general instruction following and serves as the foundation for capabilities such as mathematics and coding.
2. **On-policy preference data on ~1M preference pairs**: This stage substantially boosts the chattiness (e.g. Arena, formerly Chatbot Arena, or AlpacaEval 2) of the model while also improving skills mentioned above in the instruction tuning stage.
3. **Reinforcement Learning with Verifiable Rewards on ~10K prompts**: This stage is a small-scale reinforcement learning run to boost core skills such as mathematics while maintaining overall performance (and is now seen as a precursor to modern reasoning models such as DeepSeek R1).

The recipe has been successfully applied to Llama 3.1 [6], OLMo 2 [58], and SmolLM models [59].

3.2.3 DeepSeek R1: Scaling RLVR for Reasoning

With the rise of reasoning language models, such as OpenAI's o1, the best practices in post-training evolved again to re-order and redistribute compute across training stages. The clearest documentation of a reasoning model post-training recipe is DeepSeek R1 [15], which has been mirrored by Alibaba's larger Qwen 3 models (i.e. only the 32B and 225B MoE models) [60] or Xiaomi's MiMo 7B [61].

The DeepSeek recipe, shown in fig. 11, follows:

1. **"Cold-start" with 100K+ on-policy reasoning samples**: This data is sampled from an earlier RL checkpoint, R1-Zero, and heavily filtered to instill a specific reasoning process on DeepSeek-V3-Base. DeepSeek uses the term cold-start to describe how RL is learned from little supervised data.
2. **Large-scale reinforcement learning training**: This stage repeatedly covers reasoning problems with the model, running RLVR "until convergence" on a variety of

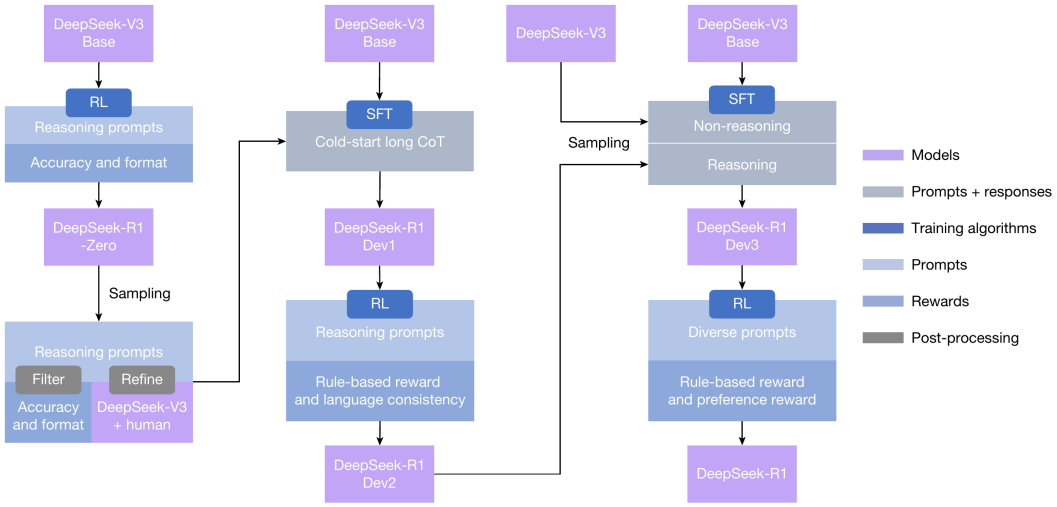


Figure 11: The multistage pipeline of DeepSeek-R1. From DeepSeek R1 paper in *Nature*, Fig. 2, under CC BY 4.0 [15].

benchmarks.

3. **Rejection sampling and SFT**: Near convergence, they apply rejection sampling to the RL checkpoint to build an SFT dataset of ~800K samples, then fine-tune the model on a filtered mix of roughly 3/4 reasoning problems and 1/4 general queries to produce a general-purpose model.
4. **Mixed reinforcement learning training** on reasoning problems (verifiable rewards) with general preference tuning reward models to polish the model.

As above, there are evolutions of the recipe, particularly with steps 3 and 4 to finalize the model before exposing it to users. Many models start with tailored instruction datasets with chain-of-thought sequences that are heavily filtered and polished from existing models, providing a fast step to strong behaviors with SFT alone before moving onto RL [62].

3.2.4 Transition to MOPD and Agents

A key product-market-fit for reasoning models was the evolution into coding agents, which was accompanied by another change in canonical post-training recipes. There were two key changes. First, another new training method, multi-teacher on-policy distillation (MOPD, covered in Chapter 12), became a popular tool for merging multiple, diverse skills into a final model. Second is simpler, as post-training compute continued to scale rapidly, especially in reinforcement learning phases.

Xiaomi's MiMo-V2-Flash was the first technical report to document the modern MOPD process [63], and the team later wrote a standalone paper on the topic [64]. The technical report summarizes this form of post-training recipe in a simple form, shown in fig. 12: general SFT is followed by domain-specialized teacher training (more SFT and extensive RL), then uses multi-teacher on-policy distillation (MOPD) to merge them into a final model. Other models use this form of recipe, such as Nvidia's Nemotron 3 Ultra [65], which is largely similar but with two consecutive MOPD phases across experts. Overall, these recipes have

less complexity in the form of iterative model versions (a *depth*) of the recipe, but more complexity in needing to specialize many crucial experts (a form of *breadth*). Note that previous generations of post-training still handled a wide breadth of tasks, but it was less segmented in the training process, i.e. most tasks were trained in the same training stages.

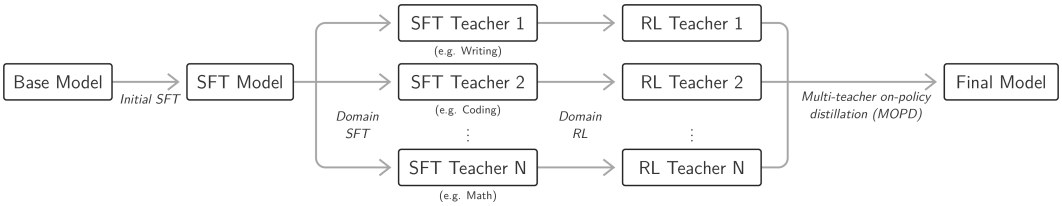


Figure 12: A schematic of specialist post-training with MOPD: shared SFT, domain-specific SFT and RL, then multi-teacher on-policy distillation into one (final) student.

Various models utilizing MOPD as a final stage use a different number of experts, with Nemotron 3 Ultra at more than ten [65], Kimi K3 at nine [66], and DeepSeek V4 at more than ten [67] (Xiaomi MiMo V2 did not report the number of experts).

Post-training recipes still vary widely across models. MOPD is a popular new tool, but labs are far from unanimous in its usage. As post-training matures and more tools enter the tool-kit, the variance of recipes has increased. For example, GLM-5 uses MOPD but details a more complex, three-stage RL process before MOPD, shown in fig. 13, [68].

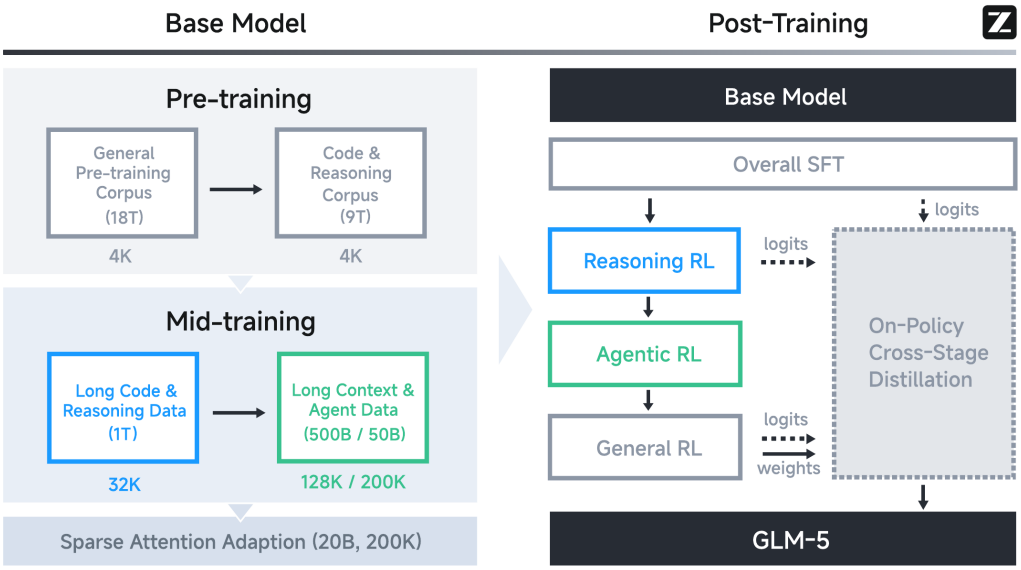


Figure 13: Overall training pipeline of GLM-5. From the GLM-5 Team's paper, Fig. 5, under CC BY 4.0 [68].

As post-training becomes more central to the performance of modern models, the recipes will continue to evolve to reflect this new scaling of difficulty, tasks, and ambitions of the

process.

4 Instruction Fine-Tuning

Early large pretrained language models were trained with a next-token prediction objective and, by default, did not come with an explicit interface for following instructions. Around the release of GPT-3 [69], prompting and in-context learning became a widely used way to adapt a single model to many tasks (though task-specific fine-tuning remained common), by showing examples in-context and asking the model to complete a similar task. A practical next step was instruction fine-tuning, which teaches the model to respond in an instruction-response format rather than just continuing text. For example, given the prompt "What is the capital of France?", a base model might continue with "What is the capital of Germany? What is the capital of Italy?..." — simply extending the pattern of questions — while an instruction-tuned model would respond with "The capital of France is Paris."

Instruction fine-tuning took off when two lines of work converged. First, NLP shifted from bespoke fine-tuning task setups to a unified "text-to-text" or instruction framing, which made it straightforward to standardize diverse datasets and train a single model across many tasks. Prominent examples of unifying the framework for tasks include *Exploring the Limits of Transfer Learning with a Unified Text-to-Text Transformer* (T5 models) [70], *Finetuned Language Models Are Zero-Shot Learners* (FLAN dataset) [71], *Multitask Prompted Training Enables Zero-Shot Task Generalization* (T0 models) [72], and *Cross-Task Generalization via Natural Language Crowdsourcing Instructions* (Natural Instructions dataset) [73]. Second, scaling pretrained LMs and the rise of prompting/in-context learning showed that a single model could generalize across tasks, but that generalization becomes far more reliable when the model is explicitly trained on instruction-response examples. Together, these trends led to an era of fine-tuning pretrained language models on large collections of instructions—what is now commonly called instruction fine-tuning (IFT), or supervised fine-tuning (SFT), in which training general models became accessible to wider audiences.

Since its discovery, instruction fine-tuning, also called colloquially just *instruction tuning*, has matured and is standard practice across many language modeling pipelines. At its core, IFT is the simplest method for adapting language models to a desired task distribution. It serves as the foundation for RLHF by preparing the model for a format of instructions that is known as question-answering, and it is the first tool used by those attempting to apply modern techniques to new domains. Without a basic level of instruction-following abilities, most of the pipelines we discuss in this book—from preference data collection to online RLHF optimization—cannot be performed.

Instruction fine-tuning generally is covered extensively elsewhere and is supervised learning at its core, so this chapter focuses on the practical details that matter most for RLHF practitioners: how training data is formatted and structured. Decisions on data and formatting are directly leveraged in the later training stages to create a common language for the model to absorb post-training data.

4.1 Chat Templates and the Structure of Instructions

The post-training process begins with defining a pattern to format user queries so that they are easily readable by a language model that processes information through a tokenizer. When using a pretrained language model, the prompting is quite simple. The model only knows a few tokens: a beginning-of-sequence token (e.g., `<bos_token>`), an end-of-sequence token (e.g., `<eos_token>`), and a padding token (to manage training on batches with empty

components). This means, to prompt a base model, the user inputs a sequence of tokens for the model to continue from, such as:

```
<bos_token> The capital of the United States is
```

Then, the model would generate tokens until it runs out of its context window, or it generates the end-of-sequence token.

All post-training stages, from instruction tuning to RLHF and other methods, rely on this formatting to train the model. The tool that handles the structure of the interaction with the user is called the **chat template**.

An example which we will break down is below:

```
{% if messages[0]['role'] == 'system' %}
    {# If the conversation begins with a system message, treat it as a special first
turn.
       We set an offset so the user/assistant alternation check lines up correctly. #}
    {% set offset = 1 %}
{% else %}
    {# No system message: user should be the first non-empty turn. #}
    {% set offset = 0 %}
{% endif %}

{# Emit the beginning-of-sequence token (model-specific). #}
{{ bos_token }}

{# Serialize each message into the model's chat-markup tokens. #}
{% for message in messages %}
    {# Enforce role alternation: (system), user, assistant, user, assistant, ...
       The boolean expression compares "is this a user message?" against whether the
       current index (plus offset) is expected to be user or assistant. #}
    {% if (message['role'] == 'user') != (loop.index0 % 2 == offset) %}
        {{ raise_exception('Conversation roles must alternate
user/assistant/user/assistant/...') }}
    {% endif %}

    {# Wrap each message with special tokens:
       - <|im_start|><role>\n
       - message content (trimmed)
       - <|im_end|>\n
       This produces a single flat token sequence the LM can train on. #}
    {{ '<|im_start|>' + message['role'] + '\n' + message['content'] | trim +
'<|im_end|>\n' }}
{% endfor %}

{# Optionally append an "assistant" start tag with no content.
   This cues generation to continue from the assistant role. #}
{% if add_generation_prompt %}
    {{ '<|im_start|>assistant\n' }}
{% endif %}
```

This is the raw code for transforming a list of dictionaries in Python containing messages and roles into tokens that a language model can predict from.

All information passed into models is assigned a role. The traditional three roles are `system`, `user`, and `assistant`.

The `system` tag is only used for the first message of the conversation; it holds instructions for the agent in text that will not be received from or exposed to the user. These **system prompts** are used to provide additional context to the models, such as the date and time, or to patch behaviors. As a fun example, models can be told things such as "You are a friendly chatbot who always responds in the style of a pirate."

Next, the two other roles are straightforward: **user** holds the messages from the person using the AI, and **assistant** holds the responses from the model (that is, engaging as an AI assistant).

In order to translate all this information into tokens, we use the code listing above that we started with. The model has a series of *special tokens* that separate the various messages from each other. If we run the above code with the example query "How many helicopters can a human eat in one sitting?", the token sequence passed into the model would look as follows:

```
<|im_start|>system
You are a friendly chatbot who always responds in the style of a pirate<|im_end|>
<|im_start|>user
How many helicopters can a human eat in one sitting?<|im_end|>
<|im_start|>assistant
```

Notice how the final tokens in the sequence are `<|im_start|>assistant`. This is how the model knows to continue generating tokens until it finally generates its end-of-sequence token, which in this case is `<|im_end|>`.

By packing all question-answer pair data (and downstream preference tuning data) into this format, modern language models follow it with perfect consistency. This is the language that instruction-tuned models use to exchange information with users and the models running on GPUs or other computing devices.

The behavior can be extended naively to multiple turns, as shown below:

```
<|im_start|>system
You are a friendly chatbot who always responds in the style of a pirate<|im_end|>
<|im_start|>user
How many helicopters can a human eat in one sitting?<|im_end|>
<|im_start|>assistant
Oh just 6.<|im_end|>
<|im_start|>user
Are you sure about that?<|im_end|>
<|im_start|>assistant
```

In the open ecosystem, the standard method for applying the chat template to a list of messages uses a Jinja snippet stored in the tokenizer configuration, as `apply_chat_template`.

The above chat template is a derivative of OpenAI's Chat Markup Language (ChatML), which was an early attempt to standardize message formatting. Now, OpenAI and other model providers use a hierarchical system where the user can configure a system message, yet there are higher-level instructions that may or may not be revealed to the user [74].

Many other chat templates exist. Some other examples include Zephyr's [26]:

```
<|system|>
You are a friendly chatbot who always responds in the style of a pirate</s>
<|user|>
How many helicopters can a human eat in one sitting?</s>
<|assistant|>
```

Or Tülu's:

```
<|user|>
How are you doing?
<|assistant|>
I'm just a computer program, so I don't have feelings, but I'm functioning as expected.
How can I assist you today?<|endoftext|>
```

Beyond this, many chat templates include formatting and other tokens for tasks such as tool-use.

4.2 Best Practices for Instruction Tuning

Instruction tuning as the foundation of post-training and creating helpful language models is well-established. There are many ways to achieve successful instruction tuning. For example, efficient fine-tuning with quantization of some model parameters makes training very accessible [75]. Also, in narrow domains such as chat alignment, i.e., without harder skills such as math or code, small, focused datasets can achieve strong performance [14].

Soon after the release of ChatGPT, human datasets with as few as 10K samples such as No Robots were state-of-the-art [76]. Years later, large-scale synthetic datasets work best [6] on most tasks.

A few principles remain:

- High-quality data is key to performance. The completions are what the model actually learns from (in many cases the prompts are not predicted over so the model does not learn to predict prompts).
- Around 1M prompts can be used to create a model capable of excellent RLHF and post-training. Further scaling can still help, but returns diminish quickly.
- The best prompts are those in a similar distribution to downstream tasks of interest.
- If multiple stages of training are done after instruction tuning, the models can recover from some noise in the instruction-tuning data. Crafting the overall optimization is more important than fixating on each individual stage.

4.3 Implementation Details

While the loss function is the same as that used in pretraining, there are a few key implementation details that differ from the setting used for pretraining. Many practices, such as deciding on the types of parallelism used to shard models across many GPUs, are the same as pretraining, but the total number of machines used is often lower (for the first technical change listed below):

- **Smaller batch sizes**: Compared to pretraining, instruction tuning (and other post-training techniques such as preference fine-tuning) use substantially smaller batch sizes to optimize well on a narrower data distribution while preserving the model's

generalization from pretraining. For example, OLMo 2 uses a batch size of 1024 packed-rows for the 7B and 2048 for the 13B pretraining, where these models have a total context length of 4096 tokens, and each row in the batch is a combination of documents that fills the sequence length. For post-training, both these models only use a batch size of 256 *prompts* [58], without filling to the full sequence length (for far fewer non-masked tokens per batch). The smaller batch sizes mean that these training jobs cannot be sharded across as many devices as during pretraining – in practice, distributed training setups have minimum per-device batch sizes, so if you're trying to retain a smaller global batch size for SFT you can use cumulatively fewer GPUs. In practice the batch size forcing a smaller concurrent GPU allotment per training job is not a limiting factor because the training token counts for SFT are much smaller than in pretraining, and training for multiple seeds is needed in post-training to obtain the best final performance.

- **Prompt masking**: When pretraining, every token in the batch is predicted autoregressively and the loss is then applied to them. For instruction tuning, the prompt tokens are masked out so the model isn't learning to accurately predict user queries – just responses. The same applies to other post-training algorithms.
- **Multi-turn masking**: For multi-turn conversations, there are two common masking choices. (1) *Final-turn only*: only the tokens in the final assistant turn are included in the loss, while all earlier context (including earlier assistant turns) is masked. Long conversations can still be "unrolled" into multiple training samples: for a conversation of N turns, each example predicts one assistant response while masking all prior context and excluding any future turns. (2) *Mask user turns only*: all user turns are masked, but *every* assistant turn is included in the loss. You can still unroll in this setting if you want more (shorter) training examples, but the key difference is that intermediate assistant replies are trained on directly.
- **Same loss function as pretraining:** Instruction tuning uses the same autoregressive loss function used in pretraining language models, but with substantially different data and masking (training only on full sequences, whereas pretraining documents can be split across batches), etc.
- **Learning rate:** SFT typically uses a learning rate one to two orders of magnitude smaller than pretraining to best manage the different optimization dynamics (smaller datasets, smaller batches, and a strong pretrained initialization all favor more conservative updates). For example, OLMo 2 uses a peak learning rate of 3×10^{-4} for pretraining but 1×10^{-5} for SFT [58]. Olmo 3 uses a higher SFT learning rate of 5-8 $\times 10^{-5}$ [18], in part because its training infrastructure uses sequence packing, which fits multiple examples into each training sequence and increases the effective batch size measured in useful tokens. Larger batches produce lower-variance gradient estimates, which in turn supports a higher learning rate without destabilizing training – a relationship known as the linear scaling rule. The learning rate is commonly warmed up over a small fraction of training steps before decaying linearly. In practice, teams often sweep over multiple learning rates and select the best checkpoint on a held-out evaluation suite [18].

4.4 Suggested Experiments

The companion code repository includes a small SFT training script in `code/instruction_tuning/`. It is intended as a learning exercise to make the base-model-to-assistant transition concrete.

1. **Run the canonical SFT example and watch the base→assistant transition.** Run:

   ```
   cd code/
   uv run python -m instruction_tuning.train --config
   instruction_tuning/configs/sft_olmo2_1b.yaml
   ```

   This trains `allenai/OLMo-2-0425-1B` (base) on `HuggingFaceH4/no_robots` and prints generations for a fixed prompt pool every 50 optimizer steps. At step 0 the base model rambles, repeats the prompt, and emits malformed role markers; after a few hundred steps the same prompts produce concise answers that terminate at `<|endoftext|>`. This is the sanity check for instruction tuning — the same loss function as pretraining, but applied to a chat template with prompt tokens masked.

2. **Sweep the learning rate.** Copy `sft_olmo2_1b.yaml` and try `lr` values of `1e-6`, `5e-6`, and `5e-5` while holding everything else fixed. Inspect at which learning rate the model first answers and stops cleanly versus when it overfits and starts producing template-shaped slop. This is the practical version of the "one to two orders of magnitude below pretraining" guidance above.

5 Reward Modeling

Reward models are core to the modern approach to RLHF by being where the complex human preferences are learned. They are what enable our models to learn from hard-to-specify signals. They compress complex features in the data into a representation that can be used in downstream training – a sort of magic that once again shows the complex capacity of modern deep learning. These models act as proxy objectives for the core optimization, as studied in the following chapters. As shown in fig. 14, the reward model plays a role like the standard RL environment, providing the learning signal for the agent, but unlike a fixed environment, we get to learn it from human preferences.

Reward models have historically been used extensively in reinforcement learning research as a proxy for environment rewards [54]. Reward models were proposed, in their modern form, as a tool for studying the value alignment problem [38]. These models tend to take in some sort of input and output a single scalar value of reward. This reward can take multiple forms – in traditional RL problems it was attempting to approximate the exact environment reward for the problem, but we will see in RLHF that reward models actually output a probability of a certain input being "of high quality" (i.e. the chosen answer among a pairwise preference relation). The practice of reward modeling for RLHF is closely related to inverse reinforcement learning, where the problem is to approximate an agent's reward function given trajectories of behavior [77], and other areas of deep reinforcement learning. The high-level problem statement is the same, but the implementation and focus areas are entirely different, so they're often considered as totally separate areas of study.

The most common reward model, often called a Bradley-Terry reward model and the primary focus of this chapter, predicts the probability that a piece of text was close to a "preferred" piece of text from the training comparisons. Later in this section we also compare these to Outcome Reward Models (ORMs), Process Reward Models (PRMs), and other types of reward models.

Throughout this chapter, we use x to denote prompts and y to denote completions. This notation is common in the language model literature, where methods operate on full prompt-completion pairs rather than individual tokens.

5.1 Training a Bradley-Terry Reward Model

The canonical implementation of a reward model is derived from the Bradley-Terry model of preference [78]. There are two popular expressions for how to train a standard reward model for RLHF – they are mathematically equivalent. To start, a Bradley-Terry model of preferences defines the probability that, in a pairwise comparison between two items i and j, a judge prefers i over j:

$$P(i > j) = \frac{p_i}{p_i + p_j}. \tag{12}$$

The Bradley-Terry model assumes that each item has a latent strength $p_i > 0$, and that observed preferences are a noisy reflection of these underlying strengths. It is common to reparametrize the Bradley-Terry model with unbounded scores, where $p_i = e^{r_i}$, which results in the following form:

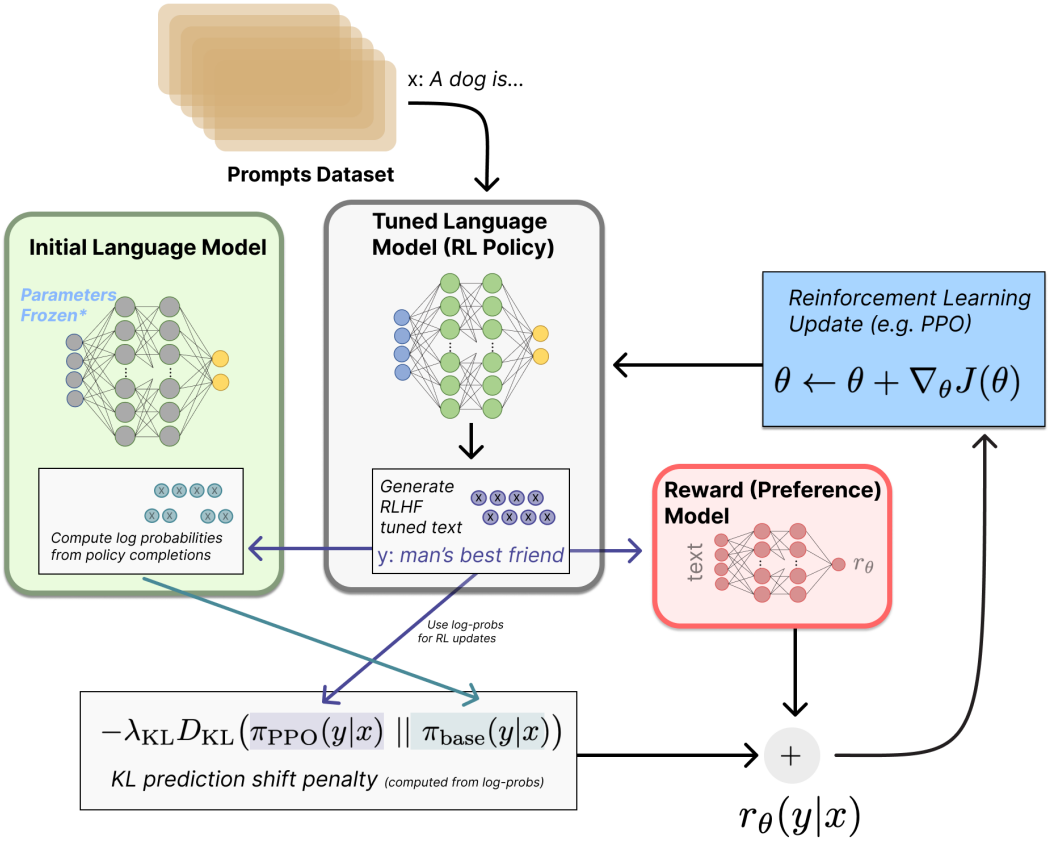


Figure 14: The reward model in RLHF plays the role of the environment component that returns rewards in standard RL. The key difference is that in RLHF, we get to control and learn this reward function from human preferences, rather than having it fixed by the environment.

$$P(i > j) = \frac{e^{r_i}}{e^{r_i} + e^{r_j}} = \sigma(r_i - r_j). \tag{13}$$

Here $\sigma(z) = \frac{1}{1+e^{-z}}$ is the logistic (sigmoid) function, so the preference probability depends only on the score difference $r_i - r_j$. Only differences in scores matter: adding the same constant c to every r_k leaves $P(i > j)$ unchanged. These forms are a useful approximation of human preferences that often works well in RLHF.

To train a reward model, we must formulate a loss function that satisfies the above relation. In practice, this is done by converting a language model into a model that outputs a scalar score, often via a small linear head that produces a single reward value from the model's final hidden state. Given a prompt x and two sampled completions y_1 and y_2, we score both with a reward model r_θ and write the conditional scores as $r_\theta(y_i \mid x)$.

The probability that the reward model assigns to y_1 being preferred to y_2 becomes:

$$P(y_1 > y_2 \mid x) = \frac{\exp\left(r_\theta(y_1 \mid x)\right)}{\exp\left(r_\theta(y_1 \mid x)\right) + \exp\left(r_\theta(y_2 \mid x)\right)}. \tag{14}$$

We denote the preferred completion as y_c (chosen) and the rejected completion as y_r.

The resulting loss encourages the reward model to assign a higher score to the human-preferred completion than the rejected one, using a sigmoid to convert the score difference into a probability. The preference likelihood in eq. 14 is the starting point. We first rewrite that likelihood into sigmoid form by dividing the numerator and denominator by $\exp\left(r_\theta(y_c \mid x)\right)$:

$$\begin{aligned}
P(y_c > y_r \mid x) &= \frac{\exp\left(r_\theta(y_c \mid x)\right)}{\exp\left(r_\theta(y_c \mid x)\right) + \exp\left(r_\theta(y_r \mid x)\right)} \\
&= \frac{\exp\left(r_\theta(y_c \mid x)\right)}{\exp\left(r_\theta(y_c \mid x)\right)\left(1 + \frac{\exp(r_\theta(y_r|x))}{\exp(r_\theta(y_c|x))}\right)} \\
&= \frac{1}{1 + \frac{\exp(r_\theta(y_r|x))}{\exp(r_\theta(y_c|x))}} \\
&= \frac{1}{1 + \exp\left(-(r_\theta(y_c \mid x) - r_\theta(y_r \mid x))\right)} \\
&= \sigma\left(r_\theta(y_c \mid x) - r_\theta(y_r \mid x)\right).
\end{aligned} \tag{15}$$

The reward model is then fit by maximum likelihood over the preference dataset D, maximizing the expected log-likelihood of the observed preferences. Because the logarithm is monotonic, this is equivalent to minimizing the expected negative log-likelihood:

$$\begin{aligned}
\theta^* &= \arg\max_\theta \mathbb{E}_{(x,y_c,y_r)\sim D}\left[\log P(y_c > y_r \mid x)\right] \\
&= \arg\min_\theta \mathbb{E}_{(x,y_c,y_r)\sim D}\left[-\log \sigma\left(r_\theta(y_c \mid x) - r_\theta(y_r \mid x)\right)\right].
\end{aligned} \tag{16}$$

Taking the logarithm *before* averaging over the dataset is what makes the negative-log-likelihood loss the right objective: maximizing the expected probability $\mathbb{E}[P]$ is not the same as maximizing the expected log-probability $\mathbb{E}[\log P]$.

The per-example loss is the log-sigmoid expression inside the expectation above, as in [3] and other works:

$$\mathcal{L}(\theta) = -\log\left(\sigma\left(r_\theta(y_c \mid x) - r_\theta(y_r \mid x)\right)\right) \tag{17}$$

The second is a mathematically equivalent form expressed using the softplus function $\log(1 + e^x)$, as in [23] and other works:

$$\mathcal{L}(\theta) = \log\left(1 + e^{r_\theta(y_r|x) - r_\theta(y_c|x)}\right) \tag{18}$$

These are equivalent by letting $\Delta = r_\theta(y_c \mid x) - r_\theta(y_r \mid x)$ and using $\sigma(\Delta) = \frac{1}{1+e^{-\Delta}}$, which implies $-\log\sigma(\Delta) = \log(1 + e^{-\Delta}) = \log\left(1 + e^{r_\theta(y_r|x) - r_\theta(y_c|x)}\right)$. They both appear in the RLHF literature.

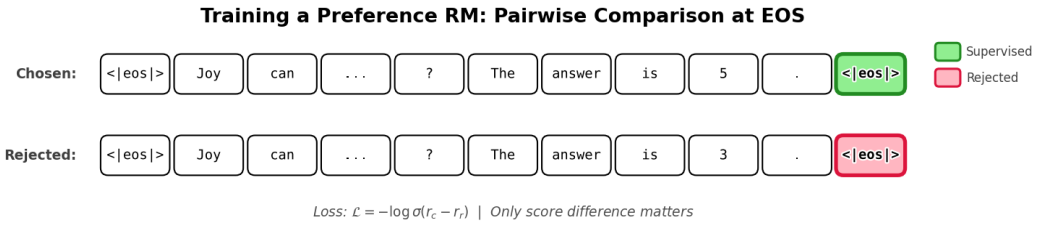


Figure 15: Training a preference reward model requires pairs of chosen and rejected completions. The model computes a scalar score for each completion from a sequence-level representation, often the end-of-sequence (EOS) token's hidden state, and the contrastive loss depends only on the score difference between the two.

5.1.1 The Default Reward Model Architecture

The most common way reward models are implemented is through an abstraction similar to Transformers' `AutoModelForSequenceClassification`, which appends a small linear head to the language model and produces a scalar reward score for a prompt-completion pair at training or inference. At inference time, the model outputs the *relative likelihood that the piece of text is chosen* as a single logit from the model.

Other implementation options exist, such as just taking a linear layer directly from the final embeddings, but they are less common in open tooling.

5.1.2 Implementation Example

Implementing the reward modeling loss is quite simple. More of the implementation challenge is on setting up a separate data loader and inference pipeline. Given the correct dataloader with tokenized, chosen and rejected prompts with completions, the loss is implemented as:

```
import torch.nn as nn
# inputs_chosen / inputs_rejected include the prompt tokens x and the respective
# completion tokens (y_c or y_r) that the reward model scores jointly.
rewards_chosen = model(**inputs_chosen)
rewards_rejected = model(**inputs_rejected)
```

```
loss = -nn.functional.logsigmoid(rewards_chosen - rewards_rejected).mean()
```

As for the bigger picture, this is often within a causal language model (a model that generates tokens left-to-right, predicting each token conditioned on all previous ones) that has an additional head added (and learned with the above loss) that transitions from the final hidden state to the score of the inputs. The code takes in standard transformer inputs – `input_ids` (tokenized text) and `attention_mask` (which marks real tokens vs. padding) – and extracts the hidden state (the model's internal representation of the input) at the last real token, which is then passed through a linear layer to produce a scalar reward. This model will have a structure as follows:

```
import torch
import torch.nn as nn
import torch.nn.functional as F

class BradleyTerryRewardModel(nn.Module):
    """
    Standard scalar reward model for Bradley-Terry preference learning.

    Usage (pairwise BT loss):
        rewards_chosen = model(**inputs_chosen)    # (batch,)
        rewards_rejected = model(**inputs_rejected)  # (batch,)
        loss = -F.logsigmoid(rewards_chosen - rewards_rejected).mean()
    """
    def __init__(self, base_lm):
        super().__init__()
        self.lm = base_lm  # e.g., AutoModelForCausalLM
        self.head = nn.Linear(self.lm.config.hidden_size, 1)

    def _sequence_rep(self, hidden, attention_mask):
        """
        Get a single vector per sequence to score.
        Default: last non-padding token (EOS token); if no mask, last token.
        hidden: (batch, seq_len, hidden_size)
        attention_mask: (batch, seq_len)
        """

        # Index of last non-pad token in each sequence
        # attention_mask is 1 for real tokens, 0 for padding
        lengths = attention_mask.sum(dim=1) - 1  # (batch,)
        batch_idx = torch.arange(hidden.size(0), device=hidden.device)
        return hidden[batch_idx, lengths]  # (batch, hidden_size)

    def forward(self, input_ids, attention_mask):
        """
        A forward pass designed to show inference structure of a standard reward model.
        To train one, this function will need to be modified to compute rewards from
both
         chosen and rejected inputs, applying the loss above.
        """
        outputs = self.lm(
            input_ids=input_ids,
            attention_mask=attention_mask,
            output_hidden_states=True,
            return_dict=True,
        )
```

```
        # Final hidden states: (batch, seq_len, hidden_size)
        hidden = outputs.hidden_states[-1]

        # One scalar reward per sequence: (batch,)
        seq_repr = self._sequence_rep(hidden, attention_mask)
        rewards = self.head(seq_repr).squeeze(-1)

        return rewards
```

In this section and what follows, most of the implementation complexity for reward models (and much of post-training) is around constructing the data-loaders correctly and distributed learning systems. Note, when training reward models, the most common practice is to train for only 1 epoch to avoid overfitting.

5.2 Outcome Reward Models

The majority of *preference tuning* for language models and other AI systems is done with the Bradley-Terry models discussed above. For reasoning-heavy tasks, one can use an Outcome Reward Model (ORM). The training data for an ORM is constructed in a similar manner to standard preference tuning. Here, we have a problem statement or prompt, x and two completions y_1 and y_2. The inductive bias used here is that one completion should be a correct solution to the problem and one incorrect, resulting in (y_c, y_{ic}).

Before we continue, it is important to note that outcome reward models are a relatively niche area in the post-training literature, and the key papers we reference have subtly different implementation details. The key idea is to learn a per-token signal of how likely the completion is to end in a correct answer, but there have been different training approaches and architectures over time.

The architecture of the models used is very similar to a standard reward model, with a linear layer appended to a model that can output a single logit (in the case of an RM) – with an ORM, the training objective that follows is slightly different. To start, let's break down the content in the original GSM8K paper (a popular benchmark studying grade-school math) [79], which originated the ideas that became an ORM without yet naming it. We start with architecture, from section 4.3:

> We can either train verifiers to make a single scalar prediction conditioned on the entire generated solution, or to make a scalar prediction after each token in the solution. By default, we choose the latter, training verifiers to make predictions after each token.

This is where the default implementation of outcome reward models diverges from Bradley-Terry models – they predict at each token. The authors comment on how per-token information could be "a useful auxiliary signal that encourages the model to judge reasoning throughout the solutions," rather than just predicting the outcome (which is a bit counter-intuitive, given the name of model that later emerged as ORM). Continuing, from Appendix E:

> [We] train verifiers with a joint objective where the model learns to label a model completion as correct or incorrect, in addition to the original language modeling objective. Architecturally, this means our verifiers are language models, with a small scalar head that outputs predictions on a per-token basis. We implement

this scalar head as a single bias parameter and single gain parameter that operate on the logits outputted by the language model's final unembedding layer.

To translate, this is implemented as a small head that outputs a scalar logit at every token, rather than a classification head of a traditional RM that outputs one logit for the entire sequence. Additionally, in this original GSM8K paper the authors jointly trained their ORM with the next-token, language modeling loss – this practice did not continue as the default.

The term "outcome-reward model" appeared in 2022, in a paper comparing "outcome-supervised RM (ORM)" versus process reward models that predicted the quality of the reasoning so far [80] – this importantly is a secondary way of implementing an ORM, one that implements a binary `correct` or `incorrect` in the LLM's tokenizer vocabulary as a step-level signal, rather than learning a separate scalar head that predicts correctness at every token.

The canonical implementation that is followed in this book is from the paper *Let's Verify Step by Step* [50], where the outcome reward model is training a per-token predictor of if an answer is right with a cross-entropy loss.

Formally, the per-token loss applies a binary cross-entropy at every completion token, where each token's associated outcome probability is trained towards the sequence's outcome label:

$$\mathcal{L}_{\text{token}}(\theta) = -\mathbb{E}_{(s,r)\sim\mathcal{D}}\left[\frac{1}{T}\sum_{t=1}^{T}\left(r\log p_\theta(s_t) + (1-r)\log\left(1-p_\theta(s_t)\right)\right)\right] \tag{19}$$

where s is a completion of T tokens, $r \in \{0, 1\}$ is a binary label where 1 applies to a correct answer to a given prompt and 0 applies to an incorrect answer, and $p_\theta(s_t) = \sigma(w_\theta(s_t))$ is the probability of correctness predicted at token t from the model's scalar logit $w_\theta(s_t)$.

A simpler form of an ORM, following [81], is a sequence-level cross-entropy loss, where the model is later used for per-token inference:

$$\mathcal{L}_{\text{CE}}(\theta) = -\mathbb{E}_{(s,r)\sim\mathcal{D}}\left[r\log\bar{p}_\theta(s) + (1-r)\log(1-\bar{p}_\theta(s))\right] \tag{20}$$

where $r \in \{0, 1\}$ is a binary label where 1 applies to a correct answer to a given prompt and 0 applies to an incorrect answer, and $\bar{p}_\theta(s) = \sigma\left(\frac{1}{T}\sum_{t=1}^{T} w_\theta(s_t)\right)$ squashes the average of the per-token logits into a single probability that the entire completion is correct – note this is not the average of the per-token probabilities, since the sigmoid is applied after the pooling. In code, this outcome label is copied onto every completion token, while prompt tokens are masked with `-100` so they do not contribute to the loss.

Implementing an outcome reward model (and other types, as we'll see with the Process Reward Model) involves applying the cross-entropy loss per-token based on whether the completion is a correct sample. This is far closer to the language modeling loss, where it does not need the structured chosen-rejected nature of standard Bradley-Terry reward models. In the simplified ORM training setup below, we are not sampling new tokens or training an LLM on next-token prediction; we feed a fixed prompt-completion sequence through the backbone and train the ORM head to predict correctness labels.

The model structure could follow as:

```python
import torch.nn as nn
import torch.nn.functional as F

class OutcomeRewardModel(nn.Module):
    def __init__(self, base_lm):
        super().__init__()
        self.lm = base_lm  # e.g., AutoModelForCausalLM
        self.head = nn.Linear(self.lm.config.hidden_size, 1)

    def forward(self, input_ids, attention_mask=None, labels=None):
        """
        input_ids contains a full prompt+completion sequence.
        labels is token-aligned: prompt tokens are -100, and each completion
         token repeats the sequence outcome label (1=correct, 0=incorrect).
        If labels=None, this is an inference-only forward pass and the loss is
         returned as None.
        """
        outputs = self.lm(
            input_ids=input_ids,
            attention_mask=attention_mask,
            output_hidden_states=True,
            return_dict=True,
        )
        # Final hidden states: (batch, seq_len, hidden_size)
        hidden = outputs.hidden_states[-1]
        # One scalar logit per token: (batch, seq_len)
        logits = self.head(hidden).squeeze(-1)

        # Inference-only forward pass: no loss is computed.
        if labels is None:
            return None, logits
        # Only compute loss on completion tokens (labels 0 or 1)
        # Prompt tokens have labels = -100
        mask = labels != -100
        loss = None
        if mask.any():
            loss = F.binary_cross_entropy_with_logits(
                logits[mask], labels[mask].float()
            )
        else:
            loss = logits.sum() * 0
        return loss, logits
```

A simplified version of the loss follows:

```python
# Feed the full prompt+completion sequence once; no token sampling happens here.
# Assume model already has: model.lm (backbone) + model.head
hidden = model.lm(**inputs, output_hidden_states=True).hidden_states[-1]
logits_per_token = model.head(hidden).squeeze(-1)  # (batch, seq_len)
# This will sometimes be compressed as model.forward() in other implementations

# Binary labels: 1=correct, 0=incorrect (prompt tokens masked as -100)
mask = labels != -100
loss = F.binary_cross_entropy_with_logits(
    logits_per_token[mask], labels[mask].float()
)
```

The important intuition here is that an ORM will output a probability of correctness at every token in the sequence (judged only by the final answer – reasoning errors are not

captured in the ORM training process). This can be a noisy process, as the updates and loss propagate per token depending on outcomes and attention mappings.

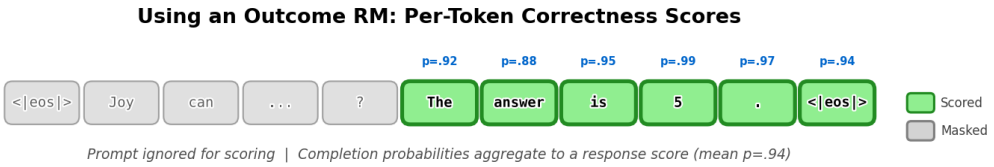


Figure 16: At inference time, an outcome reward model outputs per-token correctness probabilities over completion tokens. Prompt tokens are ignored for scoring, and the completion probabilities can be aggregated into a response-level score for verification, filtering, or reranking.

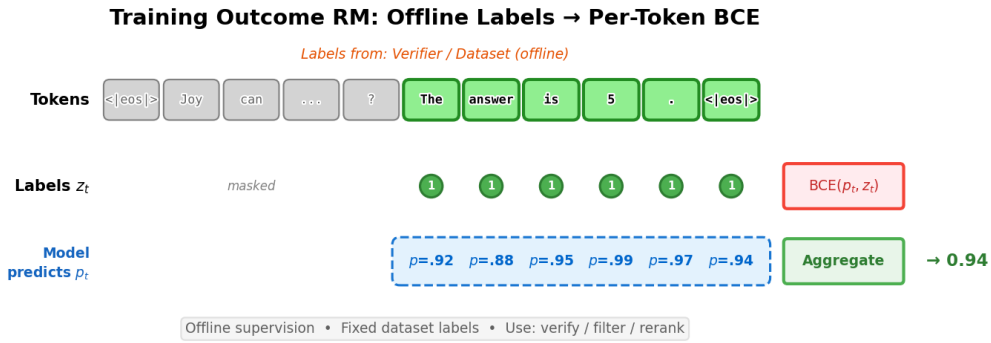


Figure 17: Training an outcome reward model uses offline labels from a verifier or dataset (e.g., all 1s for correct completions). Each completion token is trained with binary cross-entropy against the outcome label, and per-token probabilities are aggregated into a final score for verification, filtering, or reranking.

These models have continued to be used, but are less supported in open-source RLHF tools. For example, the same type of ORM was used in the seminal work *Let's Verify Step by Step* [50], but without the language modeling prediction piece of the loss from Cobbe et al. 2021. Then, the final loss is a cross-entropy loss on every token, predicting whether the final answer is correct.

Given the lack of support, the term outcome reward model (ORM) has been used in multiple ways. Some literature, e.g. [81], continues to be inspired by the original definition from Cobbe et al. 2021; others use it more broadly for any verifier trained to predict whether a completion is correct.

5.3 Process Reward Models

Process Reward Models (PRMs), originally called process-supervised reward models, are reward models trained to output scores at every *step* in a chain-of-thought reasoning process. These differ from a standard RM that outputs a score only at an EOS token or an ORM that

outputs a score at every token. Process Reward Models require supervision at the end of each reasoning step, and then are trained similarly where the tokens in the step are trained to their relevant target – the target is the step in PRMs and the entire response for ORMs.

Following [50], a binary-labeled PRM is commonly optimized with a per-step cross-entropy loss:

$$\mathcal{L}_{\text{PRM}}(\theta) = -\mathbb{E}_{(x,s)\sim\mathcal{D}}\left[\sum_{i=1}^{K} y_{s_i}\log r_\theta(s_i \mid x, s_{<i}) + (1-y_{s_i})\log\left(1 - r_\theta(s_i \mid x, s_{<i})\right)\right] \quad (21)$$

where s is a sampled chain-of-thought with K annotated steps, $y_{s_i} \in \{0, 1\}$ denotes whether the i-th step is correct, and $r_\theta(s_i \mid x, s_{<i})$ is the PRM's predicted probability that step s_i is valid conditioned on the original prompt x and all previous steps $s_{<i}$.

Here's an example of how this per-step label can be packaged in a trainer, from Hugging Face's TRL (Transformer Reinforcement Learning) [47]:

```
# Get the ID of the separator token and add it to the completions
separator_ids = tokenizer.encode(step_separator, add_special_tokens=False)
completions_ids = [completion + separator_ids for completion in completions_ids]

# Create the label
labels = [[-100] * (len(completion) - 1) + [label] for completion, label in
zip(completions_ids, labels)]
```

Traditionally PRMs are trained with a language modeling head that outputs a token only at the end of a reasoning step, e.g. at the token corresponding to a double new line or other special token. These predictions tend to be -1 for incorrect, 0 for neutral, and 1 for correct. These labels do not necessarily tie to whether or not the model is on the right path, but rather to whether the step is correct.

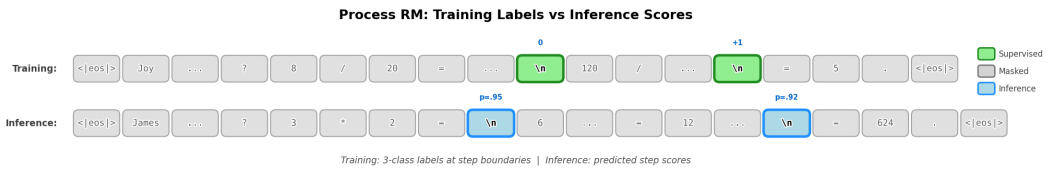


Figure 18: Process reward models provide supervision only at step boundaries (e.g., newline tokens). Each step receives a 3-class label: correct (+1), neutral (0), or incorrect (-1). All other tokens are masked during training.

An example construction of a PRM is shown below.

```
import torch.nn as nn
import torch.nn.functional as F

class ProcessRewardModel(nn.Module):
    def __init__(self, base_lm, num_classes=3):
        super().__init__()
```

```
        self.lm = base_lm  # e.g., AutoModelForCausalLM
        self.head = nn.Linear(self.lm.config.hidden_size, num_classes)

    def forward(self, input_ids, attention_mask=None, labels=None):
        """
        The inputs are tokenized prompts and completions, where the end of a
         "reasoning step" is denoted by a designated separator token such as a
         newline or other special marker rather than batch padding.
        labels will be a list of labels, True, False, and Neutral (3 labels) which
         will be predicted by the model.
        If labels=None, this is an inference-only forward pass and the loss is
         returned as None.
        """
        outputs = self.lm(
            input_ids=input_ids,
            attention_mask=attention_mask,
            output_hidden_states=True,
            return_dict=True,
        )
        # Final hidden states: (batch, seq_len, hidden_size)
        hidden = outputs.hidden_states[-1]
        # One logit vector per token: (batch, seq_len, num_classes)
        logits = self.head(hidden)

        # Inference-only forward pass: no loss is computed.
        if labels is None:
            return None, logits
        # Only compute loss at step boundaries (where labels != -100)
        # Labels map: -1 -> 0, 0 -> 1, 1 -> 2 (class indices)
        mask = labels != -100
        loss = None
        if mask.any():
            loss = F.cross_entropy(
                logits[mask], labels[mask]
            )
        else:
            loss = logits.sum() * 0
        return loss, logits
```

The core loss function looks very similar to outcome reward models, with the labels being applied at different intervals.

```
# Assume model outputs 3-class logits per token
hidden = model.lm(**inputs, output_hidden_states=True).hidden_states[-1]
logits = model.head(hidden)  # (batch, seq_len, 3)

# 3-class labels at step boundaries only: 0=-1, 1=0, 2=1 (others masked as -100)
mask = labels != -100
loss = F.cross_entropy(logits[mask], labels[mask])
```

5.4 Comparing Reward Model Types (and Value Functions)

The various types of reward models covered indicate the spectrum of ways that "quality" can be measured in RLHF and other post-training methods. Below is a summary of what the models predict and how they are trained.

Table 2: Comparing types of reward models.

Model Class	What They Predict	How They Are Trained	LM structure
Reward Models	Sequence-level quality score $r_\theta(x, y)$	Contrastive loss between pairwise (or N-wise) comparisons between completions to the same prompt	Linear head on EOS/last-token hidden state
Outcome Reward Models	Probability that an answer is correct per-token	Labeled outcomes (e.g., success/failure on verifiable domains); each sample is labeled independently, with no need for paired comparisons on the same prompt	Per-token binary cross-entropy head; labels repeat the outcome label
Process Reward Models	A reward or score for intermediate steps at end of reasoning steps	Trained using intermediate feedback or stepwise annotations (trained per token in reasoning step)	Per-token head predicting step correctness (-1, 0, 1)
Value Functions	The expected return given the current state	Trained via regression to each point in sequence	A scalar regression head with per-token outputs

A few caveats on the distinctions in this table, as the boundaries between model types are not always clear cut:

- Both in preference tuning and reasoning training, the value functions often have a discount factor of 1, which makes a value function even closer to an outcome reward model, but with a different training loss.
- A process reward model can be supervised by doing rollouts from an intermediate state and collecting outcome data. This blends multiple ideas, but if the *loss* uses per-reasoning-step labels, it is best referred to as a PRM.

What if you train a Bradley-Terry pairwise model with correct/incorrect pairs? Much of the confusion on outcome reward models came from a small set of the literature that was training a reward model on pairwise data derived from answer correctness. In this domain, you set the chosen response as being a correct answer to a problem and a rejected response as being an incorrect answer *for the same problem.* This is technically not an ORM and still trained directly with the contrastive, sequence-level loss. This is technically still a Bradley-Terry model and would fall in the first class of models we covered.

ORM vs. Value Function. ORMs and value functions can appear similar since both produce per-token outputs with the same head architecture, but they differ in *what they predict* and *where targets come from*:

- **ORMs** predict, at every token, whether the completion will conclude with a correct answer. Targets come from *offline labels* (a verifier or dataset marking sequences as

correct or incorrect) and are broadcast to every intermediate token for training.
- **Value functions** predict the expected *remaining* return: $V(s_t) = \mathbb{E}\left[\sum_{k \geq t} \gamma^{k-t} r_k \mid s_t\right]$. Targets are typically *computed from on-policy rollouts* under the current policy π_θ, and change as the policy changes (technically, value functions can also be off-policy, but this is not established for work in language modeling).

If you define a dense token reward $r_t = \mathbb{1}[\text{token is correct}]$ and use $\gamma = 1$, then an ORM is learning r_t (or $p(r_t = 1)$) while the value head is learning the remaining-sum $\sum_{k \geq t} r_k$. They can share the same base model and head dimensions, but the *semantics and supervision pipeline* differ: ORMs are trained offline from fixed labels, while value functions are trained on-policy and used to compute advantages $A_t = \hat{R}_t - V_t$ for policy gradients.

5.4.1 Inference Across Reward Model Types

The models handle data differently at inference time (once they've been trained), in order to handle a suite of tasks that RMs are used for.

Bradley-Terry RM (Preference Model):

- *Input:* prompt x + candidate completion y
- *Output:* single scalar $r_\theta(x, y)$ via a linear layer from the EOS/last-token hidden state
- *Usage:* rerank k completions, pick top-1 (best-of-N sampling); or provide terminal reward for RLHF
- *Aggregation:* Not needed with scalar outputs

Outcome RM:

- *Input:* prompt x + completion y
- *Output:* per-token probabilities $p_t \approx P(\text{final answer correct} \mid y_{\leq t})$ over completion tokens
- *Usage:* score finished candidates; aggregate via mean, min (tail risk), or product $\prod_t p_t$ (equivalently, sum log-probabilities $\sum_t \log p_t$)
- *Aggregation choices:* mean correctness, minimum p_t, average over last m tokens, or threshold flagging if any $p_t < \tau$

Process RM:

- *Input:* prompt x + reasoning trace with step boundaries
- *Output:* scores at step boundaries (e.g., class logits for correct/neutral/incorrect)
- *Usage:* score completed chain-of-thought; or guide search/decoding by pruning low-scoring branches
- *Aggregation:* over steps (not tokens) — mean step score, minimum (fail-fast), or weighted sum favoring later steps

Value Function:

- *Input:* prompt x + current prefix $y_{\leq t}$ (a state)
- Output: V_t at each token position in the completion (expected remaining return from state t)
- Usage: compute per-token advantages $A_t = \hat{R}_t - V_t$ during RL training; the values at each step serve as baselines

- *Aggregation:* typically take V at the last generated token; interpretation differs from "probability of correctness"

In summary, the way to understand the different models is:

- **RM:** "How good is this whole answer?" → scalar value
- **ORM:** "Does this answer end up correct?" → per-token predictions of the outcome (as a proxy for intermediate quality)
- **PRM:** "Are the reasoning steps sound?" → per-step scores
- **Value:** "How much reward remains from here?" → baseline for RL advantages

5.5 Other Reward Model Variants

Reward modeling is a relatively under-explored area of RLHF. The traditional, Bradley-Terry reward modeling loss has been modified in many popular works, but the modifications have not solidified into a single best practice.

5.5.1 Preference Margin Loss

In the case where annotators are providing either scores or rankings on a Likert Scale (a rating scale with ordered categories indicating magnitude of preference, e.g. 1–5), the magnitude of the relational quantities can be used in training. The most common practice is to binarize the data along the preference direction, reducing the mixed information of relative ratings or the strength of the ranking to just chosen and rejected completions. The additional information, such as the magnitude of the preference, has been used to improve model training, but it has not converged as a standard practice. Llama 2 proposes using the margin between two data points, $m(y_c, y_r)$, to distinguish the magnitude of preference:

$$\mathcal{L}(\theta) = -\log\left(\sigma\left(r_\theta(y_c \mid x) - r_\theta(y_r \mid x) - m(y_c, y_r)\right)\right) \tag{22}$$

For example, each completion is often given a ranking from 1 to 5 in terms of quality. In the case where the chosen sample was assigned a score of 5 and rejected a score of 2, the margin $m(y_c, y_r) = 5 - 2 = 3$. Other functions for computing margins can be explored.

Note that in Llama 3 the margin term was removed as the team observed diminishing improvements after scaling.

5.5.2 Balancing Multiple Comparisons Per Prompt

InstructGPT studies the impact of using $K = 4$ to 9 completions per prompt to rank, producing $\binom{K}{2}$ pairwise comparisons from each prompt [3]. Because these comparisons are highly correlated (they share the same prompt), shuffling them into the dataset naively causes the reward model to overfit. To address this, they weight the loss updates per comparison per prompt – without reweighting, prompts with more completions would contribute more total loss simply because they generate more pairs. In practice, all $\binom{K}{2}$ comparisons from a single prompt are typically included in the same training batch and averaged together, so each prompt contributes one grouped update rather than appearing across many separate batches. This reduces overfitting to individual prompts and prevents prompts with more sampled completions from dominating the loss. The loss function becomes:

$$\mathcal{L}(\theta) = -\frac{1}{\binom{K}{2}} \mathbb{E}_{(x,y_c,y_r)\sim D} \log\left(\sigma\left(r_\theta(y_c \mid x) - r_\theta(y_r \mid x)\right)\right) \tag{23}$$

5.5.3 K-Wise Loss Function

There are many other formulations that can create suitable models of human preferences for RLHF. One such example, used in the popular, early RLHF'd models Starling 7B and 34B [82], is a K-wise loss function based on the Plackett-Luce model [83].

Zhu et al. 2023 [84] formalize the setup as follows. With a prompt, or state, s^i, K actions $(a_0^i, a_1^i, \cdots, a_{K-1}^i)$ are sampled from $P(a_0, \cdots, a_{K-1}|s^i)$. Then, labelers rank the K actions by preference, producing a permutation $\sigma^i : [K] \mapsto [K]$, where $\sigma^i(0)$ is the most preferred action. This yields a Plackett-Luce probability over the complete ranking of all K items:

$$P(\sigma^i|s^i, a_0^i, a_1^i, \ldots, a_{K-1}^i) = \prod_{k=0}^{K-1} \frac{\exp(r_{\theta\star}(s^i, a_{\sigma^i(k)}^i))}{\sum_{j=k}^{K-1} \exp(r_{\theta\star}(s^i, a_{\sigma^i(j)}^i))} \tag{24}$$

When $K = 2$, this reduces to the Bradley-Terry (BT) model for pairwise comparisons. Regardless, once trained, these models are used similarly to other reward models during RLHF training.

5.6 Generative Reward Modeling (a.k.a. LLM-as-a-judge)

With the cost of preference data, a large research area emerged to use existing language models as a judge of human preferences or in other evaluation settings [85]. The core idea is to prompt a language model with instructions on how to judge, a prompt, and two completions (much as would be done with human labelers). An example prompt, from one of the seminal works here for the chat evaluation MT-Bench [85], follows:

```
[System]
Please act as an impartial judge and evaluate the quality of the responses provided by
two AI assistants to the user question displayed below.
You should choose the assistant that follows the user's instructions and answers the
user's question better.
Your evaluation should consider factors such as the helpfulness, relevance, accuracy,
depth, creativity, and level of detail of their responses.
Begin your evaluation by comparing the two responses and provide a short explanation.
Avoid any position biases and ensure that the order in which the responses were
presented does not influence your decision.
Do not allow the length of the responses to influence your evaluation.
Do not favor certain names of the assistants.
Be as objective as possible.
After providing your explanation, output your final verdict by strictly following this
format: "[[A]]" if assistant A is better, "[[B]]" if assistant B is better, and "[[C]]"
for a tie.
[User Question]
{question}
[The Start of Assistant A's Answer]
{answer_a}
[The End of Assistant A's Answer]
[The Start of Assistant B's Answer]
{answer_b}
```

```
[The End of Assistant B's Answer]
```

Given the efficacy of LLM-as-a-judge for evaluation, which spawned many other evaluations such as AlpacaEval [86], Arena-Hard [87], and WildBench [88], many began using LLM-as-a-judge instead of reward models to create and use preference data.

An entire field of study has emerged around how to use so-called "Generative Reward Models" [89] [90] [91] (including models trained *specifically* to be effective judges [92]), but on RM evaluations they tend to be behind existing reward models, showing that reward modeling is an important technique for current RLHF.

A common trick to improve the robustness of LLM-as-a-judge workflows is to use a sampling temperature of 0 to reduce variance of ratings.

5.7 Further Reading

The academic literature for reward modeling established itself in 2024. The bulk of early progress in reward modeling has focused on establishing benchmarks and identifying behavior modes. The first RM benchmark, RewardBench, provided common infrastructure for testing reward models [93]. Since then, RM evaluation has expanded to be similar to the types of evaluations available to general post-trained models, where some evaluations test the accuracy of prediction on domains with known true answers [93] or those more similar to "vibes" performed with LLM-as-a-judge or correlations to other benchmarks [94].

Examples of new benchmarks include:

- **Text-only (general chat / preferences):** RMB [95], RewardBench2 [96], Preference Proxy Evaluations [97], or RM-Bench [98].
- **Specialized text-only (math, etc.):** multilingual reward bench (M-RewardBench) [99], RAG-RewardBench for retrieval augmented generation (RAG) [100], ReWordBench for typos [101], RewardMATH [102], or AceMath-RewardBench [103].
- **Process RMs:** PRM Bench [104] or ProcessBench [105] and visual benchmarks of VisualProcessBench [106] or ViLBench [107].
- **Agentic RMs:** Agent-RewardBench [108] or CUARewardBench [109].
- **Multimodal:** MJ-Bench [110], Multimodal RewardBench [111], VL RewardBench [112], or VLRMBench [113].

To understand progress on *training* reward models, one can reference new reward model training methods, with aspect-conditioned models [114], high-quality human datasets [115] [116], scaling experiments [30], extensive experimentation [49], or debiasing data [117].

5.8 Suggested Experiments

The companion code repository includes small reward model training scripts in `code/reward_models/`. These are intended as learning exercises rather than tuned reference recipes. Start from a clean `code/` environment with `uv sync`, then run one experiment at a time.

1. **Train a Bradley-Terry preference reward model on UltraFeedback.** Run:

```
cd code/
uv run python -m reward_models.train_preference_rm --config
reward_models/configs/preference_rm.yaml
```

Watch whether the reward margin between chosen and rejected responses grows in the demo and W&B logs. Then vary `samples`, `lr`, and `model_id` in the yaml config to see when the signal becomes noisy or unstable.

2. **Compare outcome and process supervision.** Run the GSM8K outcome reward model and the PRM800K process reward model:

```
cd code/
uv run python -m reward_models.train_orm --config reward_models/configs/orm.yaml
uv run python -m reward_models.train_prm --samples 500 --epochs 2
```

Compare what each model can score after training: the ORM should distinguish correct and incorrect final answers, while the PRM should assign scores across intermediate reasoning steps. This is the practical version of the distinction between sequence-level, outcome-level, and process-level supervision.

3. **Add a small held-out reward model eval.** A useful contribution is a 50- to 200-example evaluation for `reward_models/` that reports accuracy or preference-pair ordering without requiring a full training run. Keep the evaluation small enough that it can be used while tuning hyperparameters.

6 Reinforcement Learning

In the RLHF process, the reinforcement learning algorithm slowly updates the model's weights with respect to feedback from a reward model. The policy – the model being trained – generates completions to prompts in the training set, then the reward model scores them, and the reinforcement learning optimizer takes gradient steps based on this information (see fig. 19 for an overview). This chapter explains the mathematics and trade-offs across various algorithms used to learn from the signal the reward model gives to on-policy data. These algorithms are run for a period of many epochs, often thousands or millions of batches across a larger set of prompts, with gradient updates in between each of them.

6.1 The Role of Reinforcement Learning in RLHF

The algorithms that popularized RLHF for language models were policy-gradient reinforcement learning algorithms. These algorithms, such as Proximal Policy Optimization (PPO), Group Relative Policy Optimization (GRPO), and REINFORCE, use recently generated samples to update their model (rather than storing scores in a replay buffer like algorithms, e.g. Deep Q-Networks, DQN, used in popular projects such as AlphaGo). In this section we will cover the fundamentals of the policy gradient algorithms and how they are used in the modern RLHF framework.

At a machine learning level, this section is the subject with the highest complexity in the RLHF process. However, as with most modern AI models, the largest determining factor in its success is the data provided as inputs to the process.

When RLHF came onto the scene with ChatGPT, it was largely known that they used a variant of PPO, and many initial efforts were built upon that. Over time, multiple research projects showed the promise of REINFORCE-style algorithms [118] [116], touted for their simplicity over PPO without a separate value model (saves memory and therefore the number of GPUs required) and with simpler advantage estimation (no Generalized Advantage Estimation, GAE, which is a method to compute advantages used for variance reduction in policy gradient algorithms). More algorithms have emerged, including Group Relative Policy Optimization, which is particularly popular with reasoning tasks, but in general many of these algorithms can be tuned to fit a specific task. In this chapter, we cover the core policy gradient setup and the three algorithms mentioned above due to their central role in the establishment of a canonical RLHF literature.

At its simplest, the RL stage of RLHF requires two models: a policy (the model being trained) and a reward model that scores its outputs (as covered in the previous chapter). A copy of the policy before RL serves as the reference model for computing a KL penalty (this model is frozen, i.e. it is not updated with gradients from the automatic differentiation engine). The most complex algorithm covered here, PPO, adds a fourth model – a learned value function used to estimate how good each token in the action was, also a large language model updated during training. The algorithms in this chapter differ mainly in how they estimate a quantity called *advantages* – a measure of how good the current action (completion) from the model is relative to average – and how they constrain policy updates so the optimization is numerically stable. A visual overview of this RLHF process (without the value model) is shown in fig. 19.

For definitions of symbols, see the problem setup chapter.

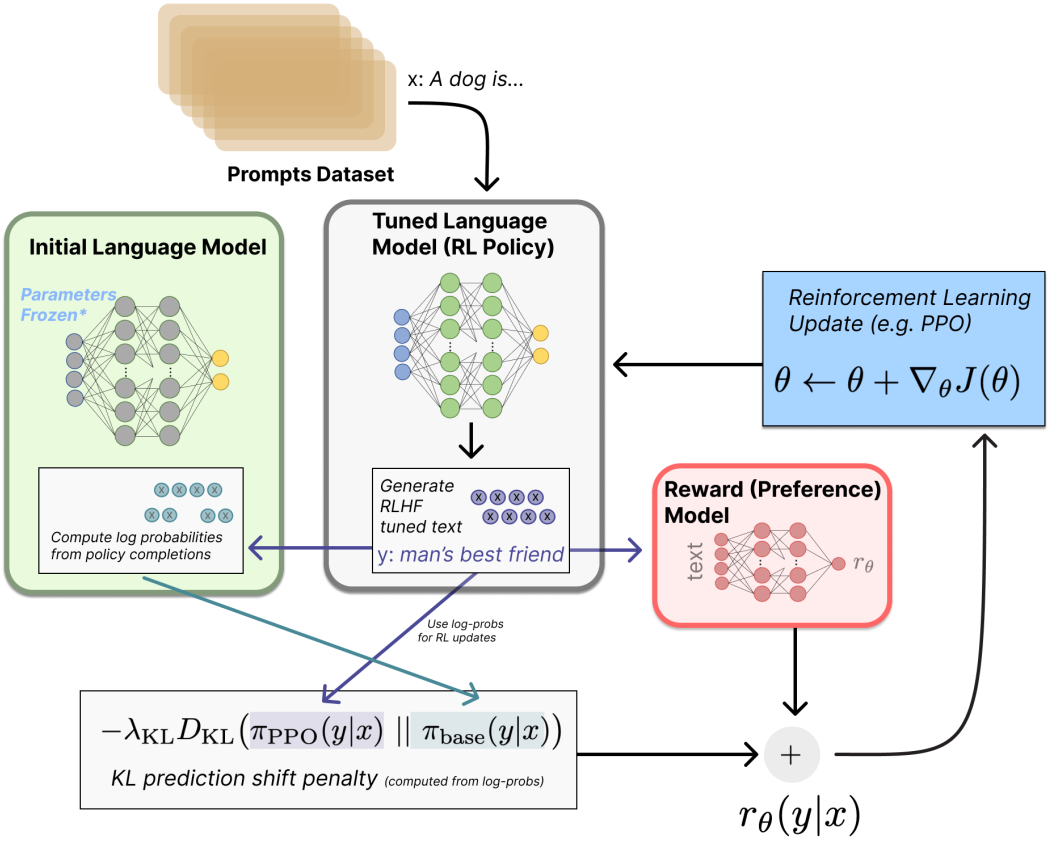


Figure 19: Overview of the RLHF training loop. A prompt from the dataset is passed to the tuned policy, which generates a completion. The reward model scores this completion, while the frozen initial model (typically the instruction-tuned model before RL) computes log probabilities on the same text to calculate a KL penalty that prevents excessive drift. The combined reward signal then drives a reinforcement learning update to the policy parameters.

This chapter uses (s, a) notation from the reinforcement learning literature, where s denotes states and a denotes actions. In the language model context, you will often see (x, y) instead, where x is the prompt and y is the completion. The (s, a) framing is more general—these algorithms were designed for sequential decision problems where actions are taken at each timestep. However, many RLHF implementations treat the entire completion as a single action, making the (x, y) notation equally valid.

***RL Cheatsheet:** A one-page reference of all core RL loss functions from this chapter is available at rlhfbook.com/rl-cheatsheet.*

6.2 Policy Gradient Algorithms

At its core, this chapter is dedicated to understanding the following shape of equation. This equation is computing the gradient, $\Delta\theta$, to the language model we are training, π_θ:

$$\Delta\theta \propto \Psi_t \, \nabla_\theta \log \pi_\theta(a_t \mid s_t) \tag{25}$$

Here, the equation is composed of two key components: 1. $\nabla_\theta \log \pi_\theta(a_t \mid s_t)$ — which direction in parameter space makes action a_t more likely. 2. Ψ_t — how good was it? A scalar scoring the outcome.

When you put this together, yes, by multiplying the quantities, you get the policy gradient update. Some things are simple, such as that $\Psi_t > 0$ updates parameters to make a_t more likely, $\Psi_t < 0$ updates them to make it less likely. The policy gradient is computing which parameters contribute to an action and if we should make it more or less likely to occur in the future. The rest of this chapter goes very deep on the different ways to do this, and what the specific tricks are to make it work for LLMs.

Now, let us formalize this a bit further. Reinforcement learning algorithms are designed to maximize the future, discounted reward across a trajectory of states, $s \in \mathcal{S}$, and actions, $a \in \mathcal{A}$ (for more notation, see Appendix A, Definitions). The objective of the agent, often called the *return*, is the sum of discounted rewards starting at a given time t (where $\gamma \in [0, 1]$ is a factor that prioritizes near-term rewards):

$$G_t = r_t + \gamma r_{t+1} + \cdots = \sum_{k=0}^{\infty} \gamma^k r_{t+k}. \tag{26}$$

The return definition can also be written recursively as:

$$G_t = r_t + \gamma G_{t+1}. \tag{27}$$

This return is the basis for learning a value function $V(s)$ that is the estimated future return given a current state:

$$V(s) = \mathbb{E}\left[G_t \mid S_t = s\right]. \tag{28}$$

All policy gradient algorithms optimize a policy $\pi_\theta(a \mid s)$ to maximize expected return; this objective can be expressed using the induced value function $V^{\pi_\theta}(s)$.

Let $d_0(s)$ be the initial-state distribution. The episodic objective we maximize can be written as:

$$J(\theta) \; = \; \sum_s d_0(s) V^{\pi_\theta}(s), \tag{29}$$

In a finite MDP this is a sum over possible starting states, but in practice we never compute it exactly. Instead, we estimate it from data by sampling rollouts from the current policy. In RLHF this typically means sampling prompts x_i from a dataset and generating completions $y_i \sim \pi_\theta(\cdot \mid x_i)$. Let $R(x_i, y_i)$ denote the scalar sequence-level reward assigned to that prompt-completion pair; if τ_i is the corresponding episode, this is the trajectory reward $R(\tau_i)$. We then take an empirical average such as:

$$\hat{J}(\theta) = \frac{1}{B} \sum_{i=1}^{B} R(x_i, y_i), \tag{30}$$

or, in an MDP view with per-step rewards,

$$\hat{J}(\theta) = \frac{1}{B} \sum_{i=1}^{B} \sum_{t=0}^{T_i} \gamma^t r_{i,t}. \tag{31}$$

In practice, RLHF for language models sets $\gamma = 1$ (no discounting) because the unit of optimization is the collective completion, not individual tokens – this choice is discussed further in the MDP vs. Bandit section later in this chapter.

The core of policy gradient algorithms is computing the gradient with respect to the finite-time expected return over the current policy. With this expected return, J, the parameter update can be computed as follows, where α is the learning rate:

$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta) \tag{32}$$

The core implementation detail is how to compute said gradient.

6.2.1 Deriving the Policy Gradient

Let $p_\theta(\tau)$ denote the trajectory distribution induced by the initial-state distribution d_0, the policy π_θ, and the environment transition dynamics, as expanded in eq. 35 below. Another way to pose the RL objective we want to maximize is as follows:

$$J(\theta) = \mathbb{E}_{\tau \sim p_\theta} \left[R(\tau) \right], \tag{33}$$

where $\tau = (s_0, a_0, s_1, a_1, \ldots)$ is a trajectory and $R(\tau) = \sum_{t=0}^{\infty} r_t$ is the total reward of the trajectory. Alternatively, we can write the expectation as an integral over all possible trajectories:

$$J(\theta) = \int_\tau p_\theta(\tau) R(\tau) d\tau \tag{34}$$

Notice that we can express the trajectory probability as follows, where $\pi_\theta(a_t|s_t)p(s_{t+1}|s_t, a_t)$ combines the policy probability with the environment transition probability from one state-action pair to the next state:

$$p_\theta(\tau) = d_0(s_0) \prod_{t=0}^{\infty} \pi_\theta(a_t|s_t)p(s_{t+1}|s_t, a_t), \tag{35}$$

If we take the gradient of the objective (eq. 33) with respect to the policy parameters θ:

$$\nabla_\theta J(\theta) = \int_\tau \nabla_\theta p_\theta(\tau) R(\tau) d\tau \tag{36}$$

Notice that we can use the log-derivative trick in order to rewrite the gradient of the integral as an expectation:

$$\begin{aligned} \nabla_\theta \log p_\theta(\tau) &= \frac{\nabla_\theta p_\theta(\tau)}{p_\theta(\tau)} && \text{(from chain rule)} \\ \implies \nabla_\theta p_\theta(\tau) &= p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) && \text{(rearranging)} \end{aligned} \tag{37}$$

Using this log-derivative trick:

$$\begin{aligned} \nabla_\theta J(\theta) &= \int_\tau \nabla_\theta p_\theta(\tau) R(\tau) d\tau \\ &= \int_\tau p_\theta(\tau) R(\tau) \nabla_\theta \log p_\theta(\tau) d\tau \\ &= \mathbb{E}_{\tau \sim p_\theta} \left[R(\tau) \nabla_\theta \log p_\theta(\tau) \right] \end{aligned} \tag{38}$$

Where the final step uses the definition of an expectation under the trajectory distribution $p_\theta(\tau)$: for any function f, $\mathbb{E}_{\tau \sim p_\theta}[f(\tau)] = \int_\tau f(\tau)\, p_\theta(\tau)\, d\tau$ (or a sum in the discrete case). Writing it as an expectation is useful because we can approximate it with Monte Carlo rollouts, e.g., $\frac{1}{B} \sum_{i=1}^{B} f(\tau_i)$ for trajectories $\tau_i \sim p_\theta$ induced by the current policy.

Back to the derivation, expanding the log probability of the trajectory:

$$\log p_\theta(\tau) = \log d_0(s_0) + \sum_{t=0}^{\infty} \log \pi_\theta(a_t|s_t) + \sum_{t=0}^{\infty} \log p(s_{t+1}|s_t, a_t) \tag{39}$$

Now, if we take the gradient of the above, we get:

- $\nabla_\theta \log d_0(s_0) = 0$ (initial state distribution doesn't depend on θ)
- $\nabla_\theta \log p(s_{t+1}|s_t, a_t) = 0$ (environment transition dynamics don't depend on θ)
- only $\nabla_\theta \log \pi_\theta(a_t|s_t)$ survives

Therefore, the gradient of the log probability of the trajectory simplifies to:

$$\nabla_\theta \log p_\theta(\tau) = \sum_{t=0}^{\infty} \nabla_\theta \log \pi_\theta(a_t|s_t) \tag{40}$$

Reaching this equation is a crucial point in the implementation. Here, we have gone far enough to see that the gradient of the trajectory distribution reduces to a sum of gradients from language model policy probabilities (which are just the probabilities of tokens given by the model we're training). In practice, this results in a common form of the policy gradient equations. They end up looking like a sum of log-probabilities in the loss, and then we compute the gradients via autodiff. A short snippet you'll see again and again roughly follows:

```
seq_log_probs = (token_log_probs * completion_mask).sum(dim=-1)
loss = -(seq_log_probs * advantages).mean()
loss.backward()
```

You'll see this throughout the chapter. Now, back to the formal policy gradient mathematics.

Substituting this back in eq. 38, we get:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta} \left[\sum_{t=0}^{\infty} R(\tau) \nabla_\theta \log \pi_\theta(a_t|s_t) \right] \tag{41}$$

Quite often, people use a more general formulation of the policy gradient:

$$g = \nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta} \left[\sum_{t=0}^{\infty} \Psi_t \nabla_\theta \log \pi_\theta(a_t|s_t) \right] \tag{42}$$

Where Ψ_t can be the following (where the rewards can also often be discounted by γ), a taxonomy adopted from Schulman et al. 2015 [119]:

1. $R(\tau) = \sum_{t=0}^{\infty} r_t$: total reward of the trajectory.
2. $\sum_{t'=t}^{\infty} r_{t'}$: reward following action a_t, also described as the return from time t, G_t.
3. $\sum_{t'=t}^{\infty} r_{t'} - b(s_t)$: baselined version of previous formula.
4. $Q^\pi(s_t, a_t)$: state-action value function.
5. $A^\pi(s_t, a_t)$: advantage function, which yields the lowest possible theoretical variance if it can be computed accurately.
6. $r_t + \gamma V^\pi(s_{t+1}) - V^\pi(s_t)$: Temporal Difference (TD) residual.

The *baseline* is a value used to reduce variance of policy updates (more on this below).

For language models, some of these concepts do not make as much sense. For example, for a deterministic policy π the state value is $V^\pi(s_t) = Q^\pi(s_t, \pi(s_t))$ (and for the optimal value function one has $V^*(s_t) = \max_{a_t} Q^*(s_t, a_t)$). For a stochastic policy, the analogous identity is $V^\pi(s_t) = \mathbb{E}_{a_t \sim \pi(\cdot|s_t)}[Q^\pi(s_t, a_t)]$. The Bellman equation relates Q to V: in general $Q^\pi(s_t, a_t) = \mathbb{E}[r_t + \gamma V^\pi(s_{t+1}) \mid s_t, a_t]$, but for language models where state transitions are deterministic, this simplifies to $Q(s_t, a_t) = r_t + \gamma V(s_{t+1})$. The advantage function measures how much better action a_t is compared to the average:

$$A(s_t, a_t) = Q(s_t, a_t) - V(s_t) = r_t + \gamma V(s_{t+1}) - V(s_t) \tag{43}$$

This final form is exactly the temporal difference (TD) residual (item 6 above) – a fundamental quantity in RL that measures the gap between the value function's prediction and what actually occurred, driving value function updates toward more accurate estimates. In practice, a learned value function $\hat{V}$ is used to estimate the advantage via this TD error.

6.2.2 Vanilla Policy Gradient

The vanilla policy gradient implementation optimizes the above expression for $J(\theta)$ by differentiating with respect to the policy parameters. A simple version, with respect to the time-t return, is:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta} \left[\sum_{t=0}^{T} G_t \nabla_\theta \log \pi_\theta(a_t|s_t) \right] \tag{44}$$

A common problem with vanilla policy gradient algorithms is the high variance in gradient updates, which can be mitigated in multiple ways. The high variance comes from the gradient updates being computed by estimating the return G from an often small set of rollouts in the environment that tend to be susceptible to noise (e.g. the stochastic nature of generating from language models with temperature > 0). The variance across return estimates is higher in domains with sparse rewards, as more of the samples are 0 or 1, rather than closely clustered. In order to alleviate this, various techniques are used to normalize the value estimation, called *baselines*. Baselines accomplish this in multiple ways, effectively normalizing by the value of the state relative to the downstream action (e.g. in the case of Advantage, which is the difference between the Q value and the value). The simplest baselines are averages over the batch of rewards or a moving average. Even these action-independent baselines can reduce variance without changing the expected gradient, since $\mathbb{E}_{a \sim \pi(a|s)}[b(s)\nabla_\theta \log \pi_\theta(a|s)] = 0$ for any state-dependent $b(s)$, improving the learning signal substantially.

Many of the policy gradient algorithms discussed in this chapter build on the advantage formulation of policy gradient:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta} \left[\sum_{t=0}^{T} A^{\pi_\theta}(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \right] \tag{45}$$

6.2.3 REINFORCE

The algorithm REINFORCE is likely a backronym, but the components of the algorithm it represents are quite relevant for modern reinforcement learning algorithms. Defined in the seminal paper *Simple statistical gradient-following algorithms for connectionist reinforcement learning* [120]:

> The name is an acronym for "REward Increment = Nonnegative Factor X Offset Reinforcement X Characteristic Eligibility."

The three components of this are how to do the *reward increment*, a.k.a. the policy gradient step. It has three pieces to the update rule:

1. Nonnegative factor: This is the learning rate (step size) that must be a positive number, e.g. α below.
2. Offset Reinforcement: This is a baseline b or other normalizing factor of the reward to improve stability.
3. Characteristic Eligibility: This attributes the scalar reward signal to the parameters that produced the action. Williams denotes this eligibility term as e (not the exponential function). In modern policy-gradient notation, it corresponds to $\nabla_\theta \log \pi_\theta(a_t \mid s_t)$.

Thus, the form looks quite familiar:

$$\Delta_\theta = \alpha(r - b)e \tag{46}$$

With more modern notation and the generalized return G, the REINFORCE operator appears as:

$$\nabla_\theta J(\theta) \;=\; \mathbb{E}_{\tau\sim p_\theta}\left[\sum_{t=0}^{T}(G_t - b(s_t))\,\nabla_\theta \log \pi_\theta(a_t \mid s_t)\right], \tag{47}$$

Here, the value $G_t - b(s_t)$ is the *advantage* of the policy at the current state, so we can reformulate the policy gradient in a form that we continue later with the advantage, A:

$$\nabla_\theta J(\theta) \;=\; \mathbb{E}_{\tau\sim p_\theta}\left[\sum_{t=0}^{T} A_t\,\nabla_\theta \log \pi_\theta(a_t \mid s_t)\right], \tag{48}$$

REINFORCE is a specific implementation of vanilla policy gradient that uses a Monte Carlo estimator of the gradient.

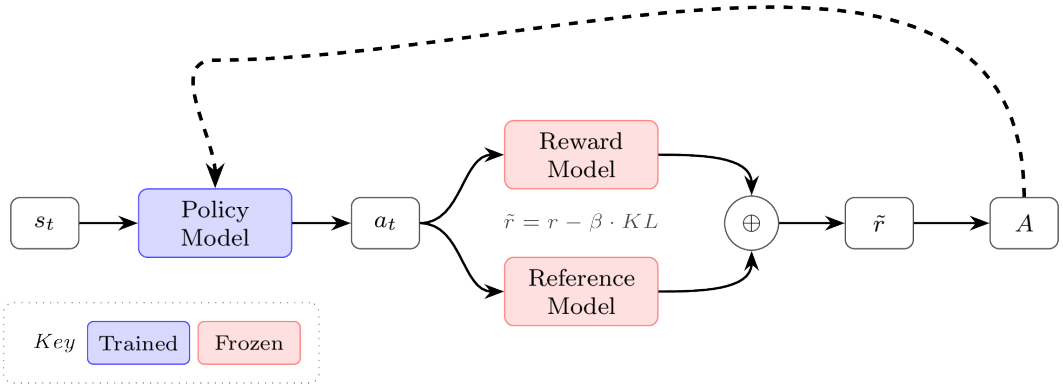


Figure 20: Basic REINFORCE architecture for language models. The shaped reward combines the reward model score with a KL penalty from the reference model. We build on this structure throughout the chapter.

6.2.4 REINFORCE Leave One Out (RLOO)

The core implementation detail of REINFORCE Leave One Out versus standard REINFORCE is that it takes the average reward of the *other* samples in the batch to compute the baseline – rather than averaging over all rewards in the batch [121], [118], [122]. By excluding the current sample's reward from its own baseline, the RLOO baseline is independent of the action being evaluated, which keeps the gradient estimator exactly unbiased.

Crucially, this only works when generating multiple trajectories (completions) per state (prompt), which is common practice in multiple domains of fine-tuning language models with RL.

Specifically, for the REINFORCE Leave-One-Out (RLOO) baseline, given K sampled trajectories (actions taken conditioned on a prompt) $a_1, \ldots, a_K$, to a given prompt s we define the baseline explicitly as the following *per-prompt*:

$$b(s, a_k) = \frac{1}{K-1} \sum_{i=1, i \neq k}^{K} R(s, a_i), \tag{49}$$

resulting in the advantage:

$$A(s, a_k) = R(s, a_k) - b(s, a_k). \tag{50}$$

Equivalently, this can be expressed as:

$$A(s, a_k) = \frac{K}{K-1} \left(R(s, a_k) - \frac{1}{K} \sum_{i=1}^{K} R(s, a_i) \right). \tag{51}$$

This is a simple, low-variance *per-prompt* advantage estimate that is closely related to the group-relative advantage used in Group Relative Policy Optimization, GRPO (discussed shortly, after Proximal Policy Optimization, PPO). In practice, GRPO-style training mainly differs in how it applies the KL regularizer (as an explicit loss term vs. folded into the reward) and whether it uses PPO-style ratio clipping. To be specific, the canonical GRPO implementation applies the KL penalty at the loss level, whereas the derivation for RLOO or traditional policy-gradients applies the KL penalty to the reward itself. With the transition from RLHF to reasoning and reinforcement learning with verifiable rewards (RLVR), the prevalence of KL penalties has decreased overall, with many reasoning adaptations of RLHF code turning them off entirely. Still, the advantage from RLOO could be combined with the clipping of PPO, showing how similar many of these algorithms are.

RLOO and other algorithms that do not use a value network – an additional model copy (a critic) that predicts a scalar value $V(s_t)$ per token – assign the same sequence-level advantage (or reward) to every token when computing the loss. Algorithms that use a learned value network, such as PPO, assign a different value to every token individually, discounting from the final reward achieved at the EOS token. With a KL distance penalty, RLOO aggregates the per-token KL over the completion and folds that scalar into the sequence reward, so the resulting advantage is broadcast to all tokens. PPO subtracts a per-token KL from the per-token reward before computing A_t, giving token-level credit assignment. GRPO typically retains a sequence-level advantage but adds a separate per-token term to the loss, rather than subtracting it from the reward. These details and trade-offs are discussed later in the chapter.

6.2.5 Proximal Policy Optimization (PPO)

Proximal Policy Optimization (PPO) [123] is one of the foundational algorithms behind Deep RL's successes (such as OpenAI Five, which mastered Dota 2 [124] and large amounts of research). The objective that PPO maximizes, with respect to the advantages and the policy probabilities, is as follows:

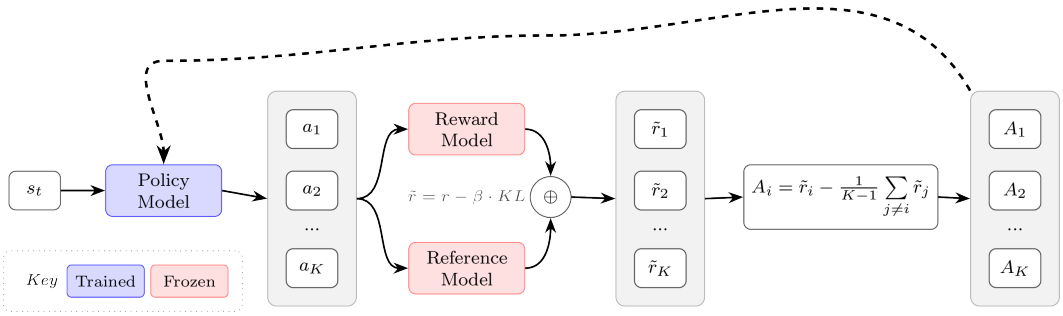


Figure 21: REINFORCE Leave-One-Out (RLOO) architecture. Multiple completions per prompt provide a leave-one-out baseline for advantage estimation without learning a value function.

$$J(\theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)}A, \text{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)}, 1-\varepsilon, 1+\varepsilon\right)A\right). \tag{52}$$

Here, $\pi_\theta(a|s)$ is the current policy being optimized and $\pi_{\theta_{\text{old}}}(a|s)$ is the policy that was used to collect the training data (i.e., the policy from the previous iteration). The ratio between these two policies emerges from *importance sampling*, which allows us to reuse data collected under an old policy to estimate gradients for a new policy.

Recall from the advantage formulation of the policy gradient (eq. 45) that we have:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta}\left[\sum_{t=0}^{T} A^{\pi_\theta}(s_t, a_t)\nabla_\theta \log \pi_\theta(a_t|s_t)\right]. \tag{53}$$

This expectation is taken over trajectories sampled from the trajectory distribution induced by π_θ, but in practice we want to take multiple gradient steps on a batch of data that was collected from a fixed policy $\pi_{\theta_{\text{old}}}$. To correct for this distribution mismatch, we multiply by the importance weight $\frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)}$, which reweights samples to account for how much more or less likely they are under the current policy versus the data-collection policy. Without constraints, optimizing this importance-weighted objective can lead to destructively large policy updates when the ratio diverges far from 1. PPO addresses this by clipping the ratio to the range $[1-\varepsilon, 1+\varepsilon]$, ensuring that the policy cannot change too drastically in a single update.

Note that, as we move to PPO and its peer algorithms, we often work with the *objective* rather than an explicit gradient. This is because the PPO objective does *not* have an easily interpretable analytical gradient once the min and clipping operations are included (the gradient has ~4 terms corresponding to the regions in fig. 23, depending on how it is written); writing the objective is simply the clearer way to convey these algorithms.

For completeness, PPO is typically written as an *expected* clipped surrogate objective over timesteps:

$$J(\theta) = \mathbb{E}_t \left[\min \left(\rho_t(\theta) A_t,\ \text{clip}(\rho_t(\theta), 1-\varepsilon, 1+\varepsilon) A_t \right) \right], \qquad \rho_t(\theta) = \frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}. \tag{54}$$

The objective is often converted into a loss function by simply adding a negative sign, which makes the optimizer seek to make it as negative as possible.

For language models, the objective (or loss) is computed per token, which intuitively can be grounded in how one would compute the probability of the entire sequence of autoregressive predictions – by a product of probabilities. From there, the common implementation is with *log-probabilities* that make the computation simpler to perform in modern language modeling frameworks. In practice, one computes the difference of token log-probabilities and exponentiates it to recover the policy ratio ρ_t.

$$J(\theta) = \frac{1}{|a|} \sum_{t=0}^{|a|} \min \left(\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)} A_t, \text{clip} \left(\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}, 1-\varepsilon, 1+\varepsilon \right) A_t \right). \tag{55}$$

This is the per-token version of PPO, which also applies to other policy-gradient methods, but is explored further later in the implementation section of this chapter. Here, the term for averaging by the number of tokens in the action, $\frac{1}{|a|}$, comes from common implementation practices, but is not in a formal derivation of the loss (shown in [125]).

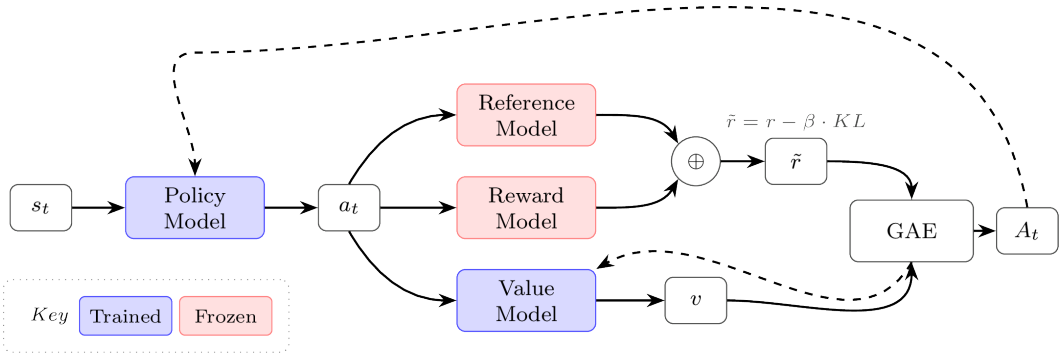


Figure 22: PPO framework. A learned value function enables Generalized Advantage Estimation (GAE) for per-token advantages, used with a clipped surrogate objective.

Here we will explain the different cases this loss function triggers given various advantages and policy ratios. At an implementation level, the inner computations for PPO involve two main terms: 1) a standard policy gradient with a learned advantage and 2) a clipped policy gradient based on a maximum step size.

To understand how different situations emerge, we can define the policy ratio as:

$$\rho(\theta) = \frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)} \tag{56}$$

The policy ratio is a centerpiece of PPO and related algorithms. It emerges from computing the gradient of a policy and controls the parameter updates in a very intuitive way. For any batch of data, the policy ratio starts at 1 for the first gradient step for that batch, since π_θ is the same as $\pi_{\theta_{\text{old}}}$ at this point. Then, in the next gradient step, the policy ratio will be above one if that gradient step increased the likelihood of certain tokens with an associated positive advantage, or less than one for the other case. A common practice is to take 1-4 gradient steps per batch with policy gradient algorithms before updating $\pi_{\theta_{\text{old}}}$.

6.2.6 Understanding the PPO Objective

Overall, the PPO objective can be visualized by two lines of a plot of objective versus policy ratio, which is shown in fig. 23. The PPO objective is maximized by changing the probability of the sampled actions. Numerically, the objective controls for both positive and negative advantage cases by clever use of the minimum operation, making it so the update is at most pushed by an epsilon distance away from a policy ratio of 1.

Within the trust region, PPO operates the same as other policy gradient algorithms. This is by design! The trust region is a concept used to cap the maximum step size of PPO and its peer algorithms for stability of updates. The core of the PPO algorithm, the clip and min/max functions, define this region. The objective becomes flat outside of it.

The idea of a "trust region" comes from the numerical optimization literature [126], but was popularized within Deep RL from the algorithm Trust Region Policy Optimization (TRPO), which is accepted as the predecessor to PPO [127]. The trust region is the area where the full policy-gradient steps are applied, as the updates are not "clipped" by the max/min operations of the PPO objective.

The policy ratio and advantage together can occur in a few different configurations, which fig. 23 enumerates by the sign of the advantage A_t and by which of the three regions the policy ratio $\rho(\theta)$ falls into. Two facts determine the outcome in every region: the sign of the advantage sets whether we want to make the action more or less likely, and the min operation selects either the unclipped term $\rho(\theta)A_t$ or its clipped counterpart.

The clipping only zeroes out the gradient in the two regions where the policy has *already* moved the sampled action in the desired direction, past the edge of the trust region:

- **Positive advantage and** $\rho(\theta) > 1 + \varepsilon$: the action is already substantially more likely under π_θ than under $\pi_{\theta_{\text{old}}}$. The objective saturates at $(1 + \varepsilon)A_t$, its gradient is zero, and no update is made — we avoid over-reinforcing an action that is already more expressed.
- **Negative advantage and** $\rho(\theta) < 1 - \varepsilon$: the action is already substantially less likely under π_θ. The objective saturates at $(1 - \varepsilon)A_t$, its gradient is again zero, and no update is made — we avoid over-suppressing an action that is already discouraged.

Everywhere else the unclipped term $\rho(\theta)A_t$ is active and PPO takes a standard policy-gradient step: increasing the action's probability when $A_t > 0$ and decreasing it when $A_t < 0$. We can read off fig. 23 in terms of what each region asks of the updated policy π_θ:

- the sloped, unclipped region under a positive advantage (green) **increases** the probability of the sampled action;
- the sloped, unclipped region under a negative advantage (red) **decreases** it;
- the flat, clipped region (grey) leaves the policy **unchanged**, since its gradient is zero.

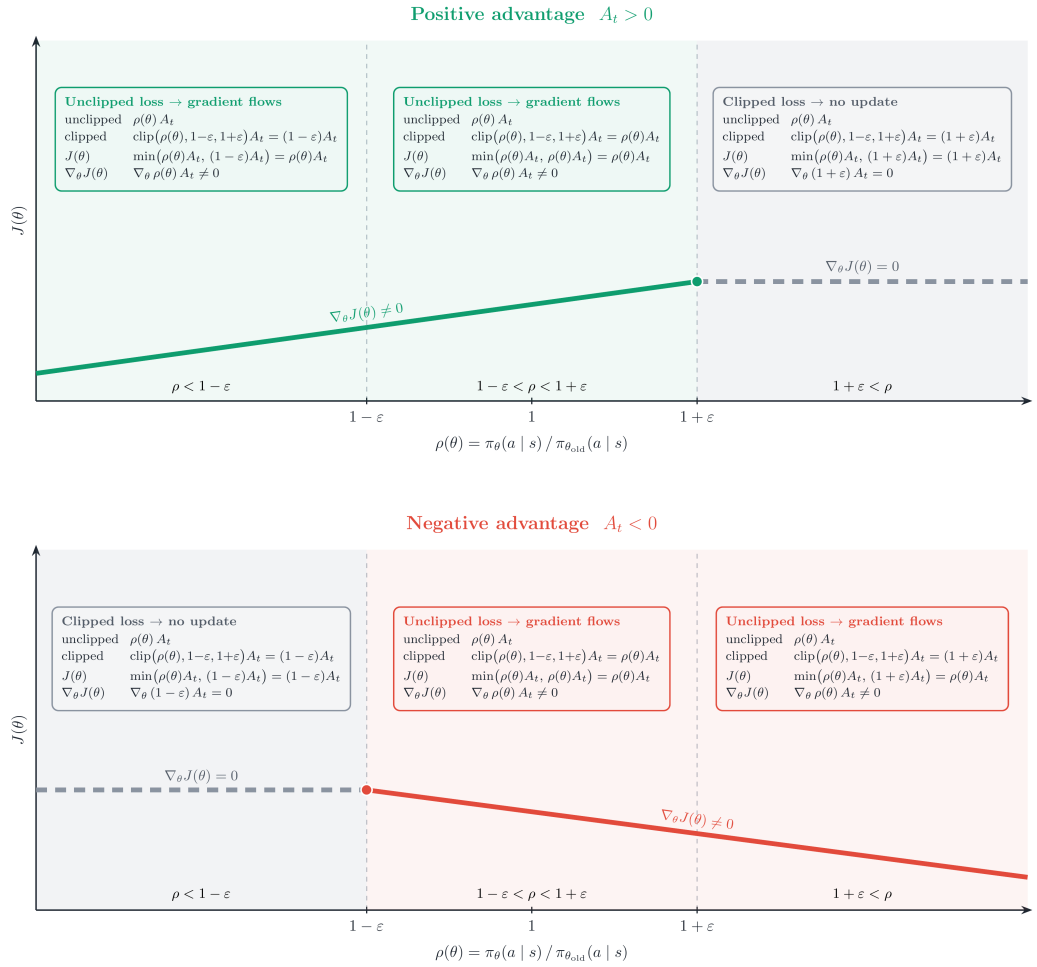


Figure 23: Visualization of the PPO objective $J(\theta)$ as a function of the policy ratio $\rho(\theta)$, for both positive and negative advantage. Within each panel, the three ratio regions are annotated with their unclipped term, clipped term, resulting objective, and gradient.

The same regions, written out term by term:

6.2.6.1 Positive Advantage ($A_t > 0$) This means that the action taken was beneficial according to the value function, and we want to increase the likelihood of taking that action in the future. Now, let's look at different cases for the policy ratio $\rho(\theta)$:

1. $\rho(\theta) < 1 - \varepsilon$:
    - **Interpretation**: Action is less likely with the new policy than the old policy
    - **Unclipped Term**: $\rho(\theta)A_t$
    - **Clipped Term**: $(1 - \varepsilon)A_t$
    - **Objective**: $\rho(\theta)A_t$
    - **Gradient**: $\nabla_\theta \rho(\theta)A_t \neq 0$
    - **What happens**: Normal policy-gradient update - increase likelihood of action
2. $1 - \varepsilon \leq \rho(\theta) \leq 1 + \varepsilon$:
    - **Interpretation**: Action is almost equally likely with the new policy as the old policy
    - **Unclipped Term**: $\rho(\theta)A_t$
    - **Clipped Term**: $\rho(\theta)A_t$
    - **Objective**: $\rho(\theta)A_t$
    - **Gradient**: $\nabla_\theta \rho(\theta)A_t \neq 0$
    - **What happens**: Normal policy-gradient update - increase likelihood of action
3. $1 + \varepsilon < \rho(\theta)$:
    - **Interpretation**: Action is more likely with the new policy than the old policy
    - **Unclipped Term**: $\rho(\theta)A_t$
    - **Clipped Term**: $(1 + \varepsilon)A_t$
    - **Objective**: $(1 + \varepsilon)A_t$
    - **Gradient**: $\nabla_\theta (1 + \varepsilon)A_t = 0$
    - **What happens**: NO UPDATE - action is already more likely under the new policy

To summarize, when the advantage is positive ($A_t > 0$), we want to boost the probability of the action. Therefore:

- We perform gradient steps only in the case when $\pi_{\text{new}}(a) \leq (1 + \varepsilon)\pi_{\text{old}}(a)$. Intuitively, we want to boost the probability of the action, since the advantage was positive, but not boost it so much that we have made it substantially more likely.
- Crucially, when $\pi_{\text{new}}(a) > (1 + \varepsilon)\pi_{\text{old}}(a)$, then we don't perform any update, and the gradient of the clipped objective is 0. Intuitively, the action is already more expressed with the new policy, so we don't want to over-reinforce it.

6.2.6.2 Negative Advantage ($A_t < 0$) This means that the action taken was detrimental according to the value function, and we want to decrease the likelihood of taking that action in the future. Now, let's look at different cases for the policy ratio $\rho(\theta)$:

1. $\rho(\theta) < 1 - \varepsilon$:
    - **Interpretation**: Action is less likely with the new policy than the old policy
    - **Unclipped Term**: $\rho(\theta)A_t$

- **Clipped Term**: $(1-\varepsilon)A_t$
- **Objective**: $(1-\varepsilon)A_t$
- **Gradient**: $\nabla_\theta(1-\varepsilon)A_t = 0$
- **What happens**: NO UPDATE - action is already less likely under the new policy

2. $1-\varepsilon \leq \rho(\theta) \leq 1+\varepsilon$:

   - **Interpretation**: Action is almost equally likely with the new policy as the old policy
   - **Unclipped Term**: $\rho(\theta)A_t$
   - **Clipped Term**: $\rho(\theta)A_t$
   - **Objective**: $\rho(\theta)A_t$
   - **Gradient**: $\nabla_\theta\rho(\theta)A_t \neq 0$
   - **What happens**: Normal policy-gradient update - decrease likelihood of action

3. $1+\varepsilon < \rho(\theta)$:

   - **Interpretation**: Action is more likely with the new policy than the old policy
   - **Unclipped Term**: $\rho(\theta)A_t$
   - **Clipped Term**: $(1+\varepsilon)A_t$
   - **Objective**: $\rho(\theta)A_t$
   - **Gradient**: $\nabla_\theta\rho(\theta)A_t \neq 0$
   - **What happens**: Normal policy-gradient update - decrease likelihood of action

To summarize, when the advantage is negative ($A_t < 0$), we want to decrease the probability of the action. Therefore:

- We perform gradient steps only in the case when $\pi_{\text{new}}(a) \geq (1-\varepsilon)\pi_{\text{old}}(a)$. Intuitively, we want to decrease the probability of the action, since the advantage was negative, and we do so proportional to the advantage.
- Crucially, when $\pi_{\text{new}}(a) < (1-\varepsilon)\pi_{\text{old}}(a)$, then we don't perform any update, and the gradient of the clipped objective is 0. Intuitively, the action is already less likely under the new policy, so we don't want to over-suppress it.

It is crucial to remember that PPO within the trust region is roughly the same as standard forms of policy gradient.

6.2.7 Value Functions and PPO

The value function within PPO is an additional copy of the model that is used to predict the value per token. The value of a token (or state) in traditional RL is predicting the future return from that moment, often with discounting. This value in PPO is used as a learned baseline, representing an evolution of the simple Monte Carlo version used with REINFORCE (which doesn't need the learned value network). This highlights how PPO is an evolution of REINFORCE and vanilla policy-gradient in multiple forms, across the optimization form, baseline, etc. In practice, with PPO and other algorithms used for language models, this is predicting the return of each token after the deduction of KL penalties (the per-token loss includes the KL from the reward traditionally, as discussed).

There are a few different methods (or targets) used to learn the value functions. Generalized Advantage Estimation (GAE) is considered the state-of-the-art and canonical implementation

in modern systems, but it carries more complexity by computing the value prediction error over multiple steps – see the later section on GAE in this chapter. A value function can also be learned with Monte Carlo estimates from the rollouts used to update the policy. PPO has two losses – one to learn the value function and another to use that value function to update the policy.

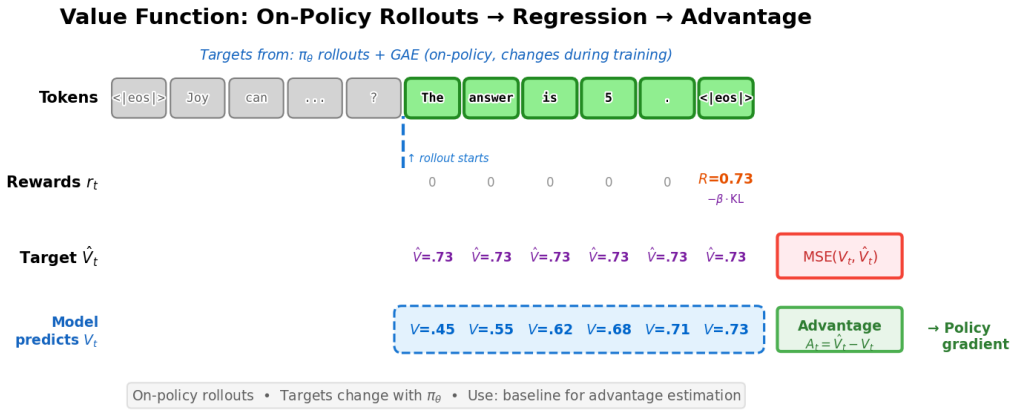


Figure 24: Value function training uses on-policy rollouts to compute targets. The model predicts V_t at each token, which is trained via MSE against the target return $\hat{V}_t$. The advantage $A_t = \hat{V}_t - V_t$ then weights the policy gradient update.

A simple example implementation of a value network loss is shown below.

```
# Basic PPO critic targets & loss (no GAE)
#
# B: Batch Size
# L: Completion Length
# Inputs:
#   rewards: (B, L) post-KL per-token rewards; EOS row includes outcome
#   done_mask: (B, L) 1.0 at terminal token (EOS or truncation if penalized), else 0.0
#   completion_mask: (B, L) 1.0 on response tokens to supervise (ignore the prompt)
#   values: (B, L) current critic predictions V_theta(s_t)
#       because a value network is a running update
#   old_values: (B, L) critic predictions at rollout time V_{theta_old}(s_t)
#   gamma: discount factor, float (often 1.0 for LM RLHF)
#   epsilon_v: float value clip range (e.g., 0.2), similar to PPO Loss Update itself,
optional
#
# Returns:
#   value_loss: scalar; advantages: (B, L) detached (for policy loss)

B, L = rewards.shape

# 1) Monte Carlo returns per token (reset at terminals)
# Apply discounting, if enabled
returns = torch.zeros_like(rewards)
running = torch.zeros(B, device=rewards.device, dtype=rewards.dtype)
for t in reversed(range(L)):
    running = rewards[:, t] + gamma * (1.0 - done_mask[:, t]) * running
```

```
    returns[:, t] = running

targets = returns  # y_t = G_t (post-KL)

# 2) PPO-style value clipping (optional)
v_pred = values
v_old  = old_values
v_clip = torch.clamp(v_pred, v_old - epsilon_v, v_old + epsilon_v)

vf_unclipped = 0.5 * (v_pred - targets) ** 2
vf_clipped   = 0.5 * (v_clip - targets) ** 2
vf_loss_tok  = torch.max(vf_unclipped, vf_clipped)

# 3) Mask to response tokens and aggregate
denom = completion_mask.sum(dim=1).clamp_min(1)
value_loss = ((vf_loss_tok * completion_mask).sum(dim=1) / denom).mean()

# 4) Advantages for policy loss (no GAE): A_t = G_t - V(s_t)
advantages = (targets - v_pred).detach()

# The value loss is applied later, often with the PG loss, e.g.
# total_loss = policy_loss + vf_coef * value_loss
```

6.2.8 Group Relative Policy Optimization (GRPO)

Group Relative Policy Optimization (GRPO) is introduced in DeepSeekMath [128], and used in other DeepSeek works, e.g. DeepSeek-V3 [16] and DeepSeek-R1 [15]. GRPO can be viewed as a PPO-inspired algorithm with a very similar surrogate loss, but it avoids learning a value function with another copy of the original policy language model (or another checkpoint for initialization). This brings two posited benefits:

1. Avoiding the challenge of learning a value function from an LM backbone, where research hasn't established best practices.
2. Saves memory by not needing to keep the extra set of model weights in memory (going from needing the current policy, the reference policy, and a value function, to just the first two copies).

GRPO does this by simplifying the value estimation and assigning the same value to every token in the episode (i.e. in the completion to a prompt, each token gets assigned the same value rather than discounted rewards in a standard value function) by estimating the advantage or baseline. The estimate is done by collecting multiple completions (a_i) and rewards (r_i), i.e. a Monte Carlo estimate, from the same initial state / prompt (s).

To state this formally, the GRPO objective is very similar to the PPO objective above. For GRPO, the objective (or loss) is accumulated over a group of completions $\{a_1, a_2, ..., a_G\}$ to a given prompt s. Here, we show the GRPO objective:

$$J(\theta) = \frac{1}{G}\sum_{i=1}^{G}\left(\min\left(\frac{\pi_\theta(a_i|s)}{\pi_{\theta_{\text{old}}}(a_i|s)}A_i, \text{clip}\left(\frac{\pi_\theta(a_i|s)}{\pi_{\theta_{\text{old}}}(a_i|s)}, 1-\varepsilon, 1+\varepsilon\right)A_i\right) - \beta\mathcal{D}_{\text{KL}}(\pi_\theta||\pi_{\text{ref}})\right). \tag{57}$$

Note that relative to PPO, the standard implementation of GRPO includes the KL distance in the loss. As above, we can expand this into a per-token computation:

$$J(\theta) = \frac{1}{G}\sum_{i=1}^{G}\frac{1}{|a_i|}\sum_{t=1}^{|a_i|}\left(\min\left(\frac{\pi_\theta(a_{i,t}|s_i)}{\pi_{\theta_{\text{old}}}(a_{i,t}|s_i)}A_{i,t},\ \text{clip}\left(\frac{\pi_\theta(a_{i,t}|s_i)}{\pi_{\theta_{\text{old}}}(a_{i,t}|s_i)}, 1-\varepsilon, 1+\varepsilon\right)A_{i,t}\right)\right. \\ \left. - \beta\mathcal{D}_{\text{KL}}(\pi_\theta(\cdot|s_i)\|\pi_{\text{ref}}(\cdot|s_i))\right) \tag{58}$$

With the advantage computation for the completion index i:

$$A_i = \frac{r_i - \text{mean}(r_1, r_2, \cdots, r_G)}{\text{std}(r_1, r_2, \cdots, r_G)}. \tag{59}$$

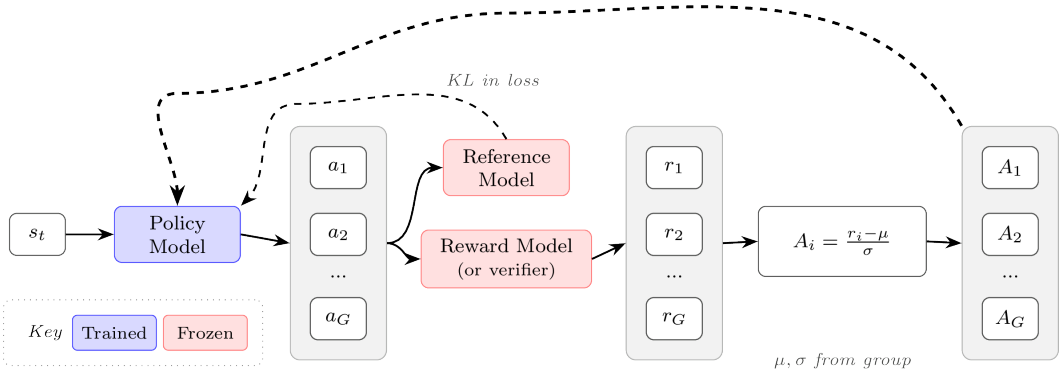


Figure 25: GRPO architecture. Advantages are normalized relative to the group mean and standard deviation. The KL penalty is applied directly in the loss rather than shaping the reward.

Intuitively, the GRPO update is comparing multiple answers to a single question within a batch. The model learns to become more like the answers marked as correct and less like the others. This is a very simple way to compute the advantage, which is the measure of how much better a specific action is than the average at a given state. Relative to PPO, REINFORCE, and broadly RLHF performed with a reward model rating (relative to output reward), GRPO is often run with a far higher number of samples per prompt because the advantage is entirely about the relative value of a completion to its peers from that prompt. Here, the current policy generates multiple responses to a given prompt, and the group-wise GRPO advantage estimate is given valuable context. PPO and vanilla policy-gradient algorithms were designed to accurately estimate the reward of every completion (in fact, more completions can do little to improve the value estimate in some cases). GRPO and its variants are particularly well-suited to modern language model tools, where having multiple completions to a given prompt is very natural (especially when compared to, e.g., multiple actions from a set environment state in a robotic task).

The advantage computation for GRPO has trade-offs in its biases. The normalization by standard deviation rewards questions in a batch that have a low variation in answer correctness. For questions with either nearly all correct or all incorrect answers, the standard

deviation will be lower and the advantage will be higher. Liu et al. 2025 [125] proposes removing the standard deviation term given this bias, but this comes at the cost of down-weighting questions that were all incorrect with a few correct answers, which could be seen as valuable learning signal for the model. Those high-variance prompts can be exactly the hardest cases, where only a few sampled completions find the correct answer and provide a strong training signal.

eq. 59 is the implementation of GRPO when working with outcome supervision (either a standard reward model or a single verifiable reward) and a different implementation is needed with process supervision. In this case, GRPO computes the advantage as the sum of the normalized rewards for the following reasoning steps.

Finally, GRPO's advantage estimation can also be applied without the PPO clipping to more vanilla versions of policy gradient (e.g. REINFORCE), but it is not the canonical form. As an example of how these algorithms are intertwined, we can show that the advantage estimation in a variant of GRPO, Dr. GRPO (GRPO Done Right) [125], is equivalent to the RLOO estimation (which uses the average reward of other samples as its baseline) up to a constant scaling factor (which normally does not matter due to implementation details to normalize the advantage). Dr. GRPO removes the standard deviation normalization term from eq. 59 – note that this also scales the advantage *up*, which is equivalent to increasing the GRPO learning rate on samples with a variance in answer scores. This addresses a bias towards questions with low reward variance – i.e. almost all the answers are right or wrong – but comes at a potential cost if it is important to learn from problems where just one sample gets the answer right. The Dr. GRPO advantage for completion i within a group of size G is defined as:

$$\tilde{A}_i = r_i - \text{mean}(r_1, r_2, \cdots, r_G) = r_i - \frac{1}{G}\sum_{j=1}^{G} r_j \tag{60}$$

Here, in the same notation, we can recall the RLOO advantage estimation as:

$$A_i^{\text{RLOO}} = r_i - \frac{1}{G-1}\sum_{j=1, i\neq j}^{G} r_j \tag{61}$$

Thus, if we multiply the Dr. GRPO advantage definition by $\frac{G}{G-1}$ we can see a scaled equivalence:

$$
\begin{aligned}
\frac{G}{G-1}\tilde{A}_i &= \frac{G}{G-1}\left(r_i - \frac{1}{G}\sum_{j=1}^{G} r_j\right) \\
&= \frac{G}{G-1}r_i - \frac{1}{G-1}\sum_{j=1}^{G} r_j \\
&= \frac{G}{G-1}r_i - \frac{1}{G-1}\sum_{j=1, j\neq i}^{G} r_j - \frac{1}{G-1}r_i \\
&= r_i\left(\frac{G}{G-1} - \frac{1}{G-1}\right) - \frac{1}{G-1}\sum_{j=1, j\neq i}^{G} r_j \\
&= r_i - \frac{1}{G-1}\sum_{j=1, j\neq i}^{G} r_j \\
&= A_i^{\text{RLOO}}
\end{aligned}
\tag{62}
$$

6.2.9 Group Sequence Policy Optimization (GSPO)

When taking multiple gradient steps on a batch of data collected from a previous policy, importance sampling is required to correct for the distribution mismatch between the data-collection policy and the current policy being optimized. The standard importance sampling identity allows us to estimate expectations under one distribution using samples from another:

$$
\mathbb{E}_p[f(x)] = \mathbb{E}_q\left[f(x)\frac{p(x)}{q(x)}\right], \tag{63}
$$

where p is the target distribution, q is the sampling distribution, and $\frac{p(x)}{q(x)}$ is the importance weight. In policy gradient methods, $p = \pi_\theta$ is the current policy we want to optimize and $q = \pi_{\theta_{\text{old}}}$ is the policy that generated the training data. This allows us to reweight samples collected under $\pi_{\theta_{\text{old}}}$ to estimate gradients for π_θ, enabling multiple gradient steps per batch of rollouts.

This distribution mismatch arises in two common scenarios: (1) taking multiple gradient steps on a single batch, where π_θ drifts from $\pi_{\theta_{\text{old}}}$ after each update, and (2) in asynchronous training systems where the inference backend (e.g., vLLM) and training backend (e.g., FSDP) may have different model weights due to synchronization delays (see the Asynchronicity section later in this chapter, which emerged particularly with the focus on RL for verifiable rewards, but is also used in RLHF setups).

PPO and GRPO apply importance sampling at the token level and stabilize learning by clipping the *surrogate objective*. However, this approach has a subtle failure mode: when a token's importance ratio moves outside the clipping range $[1-\varepsilon, 1+\varepsilon]$, that token receives zero gradient. For rare but important tokens—such as key reasoning steps that the model initially assigns low probability—this "token dropping" can prevent the model from learning to produce them more reliably.

Group Sequence Policy Optimization (GSPO) [129] extends GRPO by computing importance ratios at the sequence level rather than the token level. The practical motivation for this algorithm – and its peer, CISPO, which modifies how importance sampling is computed for policy gradient algorithms, as we will discuss later – is that the per-token importance sampling ratio is often numerically unstable. The conceptual motivation is that when rewards are assigned at the sequence level (as in most RLHF and RLVR setups), the importance sampling correction should match that granularity.

Token-level ratios can behave erratically for long sequences and/or large, sparse models (e.g. modern mixture-of-experts (MoE) models): a single token with a large ratio can dominate the policy update, or many tokens may get clipped independently within a response, fragmenting the learning signal across a single response. GSPO addresses this by computing a single importance weight per response.

Recall that the probability of a full response factorizes autoregressively:

$$\pi_\theta(a \mid s) = \prod_{t=1}^{|a|} \pi_\theta(a_t \mid s, a_{<t}). \tag{64}$$

Note that for simplicity, we often shorten the conditional policy, $\pi_\theta(a_t \mid s, a_{<t})$, as $\pi_\theta(a_t \mid s)$, which implicitly contains the previous actions (tokens) in a completion. GSPO defines a length-normalized sequence-level importance ratio using the geometric mean (to avoid numerical issues with long sequences):

$$\rho_i(\theta) = \left(\frac{\pi_\theta(a_i \mid s)}{\pi_{\theta_{\text{old}}}(a_i \mid s)} \right)^{\frac{1}{|a_i|}} = \exp\left(\frac{1}{|a_i|} \sum_{t=1}^{|a_i|} \log \frac{\pi_\theta(a_{i,t} \mid s, a_{i,<t})}{\pi_{\theta_{\text{old}}}(a_{i,t} \mid s, a_{i,<t})} \right). \tag{65}$$

The GSPO objective mirrors GRPO but uses this sequence-level ratio:

$$J_{\text{GSPO}}(\theta) = \mathbb{E}_{s\sim\mathcal{D},\, \{a_i\}_{i=1}^{G}\sim\pi_{\theta_{\text{old}}}(\cdot|s)} \left[\frac{1}{G} \sum_{i=1}^{G} \min\left(\rho_i(\theta)A_i,\ \text{clip}(\rho_i(\theta), 1-\varepsilon, 1+\varepsilon)A_i\right) \right]. \tag{66}$$

Because the ratio is length-normalized, the clipping range ε operates on a per-token average scale, making the effective constraint comparable across responses of different lengths. In implementation, the sequence-level weight ρ_i is applied uniformly to all tokens in response a_i, which simplifies gradient computation while maintaining the sequence-level IS correction.

The advantage computation remains the same as GRPO (eq. 59), using the group-relative mean and standard deviation normalization, which can be modified as done in other derivative studies of GRPO. GSPO can be summarized as "GRPO with sequence-level importance ratios"—the IS correction granularity is matched to the reward granularity.

6.2.10 Clipped Importance Sampling Policy Optimization (CISPO)

Clipped Importance Sampling Policy Optimization (CISPO) [130] takes a different approach: rather than clipping the surrogate objective, CISPO clips the importance weights themselves

while preserving gradients for all tokens. The objective uses a stop-gradient on the clipped importance weight, returning to a REINFORCE-style formulation instead of the PPO-style, two-sided clipping:

$$J_{\text{CISPO}}(\theta) = \mathbb{E}_{s\sim\mathcal{D},\,\{a_i\}_{i=1}^{K}\sim\pi_{\theta_{\text{old}}}(\cdot|s)}\left[\frac{1}{\sum_{i=1}^{K}|a_i|}\sum_{i=1}^{K}\sum_{t=1}^{|a_i|}\text{sg}\left(\hat{\rho}_{i,t}(\theta)\right)A_{i,t}\log\pi_\theta(a_{i,t}\mid s,a_{i,<t})\right], \tag{67}$$

where $\text{sg}(\cdot)$ denotes stop-gradient (the weight is used but not differentiated through), and the clipped importance ratio is:

$$\hat{\rho}_{i,t}(\theta) = \text{clip}\left(\rho_{i,t}(\theta),\,1-\varepsilon_{\text{low}},\,1+\varepsilon_{\text{high}}\right), \quad \rho_{i,t}(\theta) = \frac{\pi_\theta(a_{i,t}\mid s,a_{i,<t})}{\pi_{\theta_{\text{old}}}(a_{i,t}\mid s,a_{i,<t})}. \tag{68}$$

The key difference from PPO/GRPO is subtle but important: clipping the weight (not the objective) means every token still receives a gradient signal proportional to its advantage—the weight just bounds how much that signal is amplified or suppressed by the importance ratio. This is a bias-variance tradeoff: clipping weights introduces bias but controls variance and, critically, avoids dropping token gradients entirely.

Both CISPO and GSPO were developed by organizations pushing the limits of applying RL on large-scale MoE models, which are known for their numerical issues. The papers highlight how the per-token importance sampling ratios are unstable and can add substantial variance to the gradients, mitigating learning. This can make these algorithms particularly impactful on large-scale models, but less studied and beneficial within smaller, academic experiments.

CISPO also allows asymmetric clipping bounds ($\varepsilon_{\text{low}} \neq \varepsilon_{\text{high}}$), similar to DAPO's "clip-higher" modification discussed later in this chapter, which can encourage exploration by allowing larger updates for tokens the model wants to upweight. Related work includes Tapered Off-Policy REINFORCE (TOPR) [131], which also clips IS weights directly (like CISPO) rather than clipping within the objective (like PPO/GRPO), but operates at the sequence level (like GSPO) and uses asymmetric clipping based on reward sign—applying no IS correction for positive rewards while clipping ratios to $[0, 1]$ for negative rewards—enabling stable off-policy learning.

6.2.11 Comparing Algorithms

Each algorithm in this chapter shares the same core gradient shape (eq. 25), but differs in how it estimates the advantage and controls the optimization:

- **REINFORCE**: The simplest policy gradient implementation, using Monte-Carlo estimates of reward and a state-based baseline to reduce variance.
- **RLOO**: REINFORCE with multiple samples per prompt, with each sample's baseline being the average reward of the others (leave-one-out) to reduce gradient variance.
- **PPO**: Adds a learned value function and a clipped policy ratio to get more accurate and stable gradient updates.

- **GRPO**: A simplified variant of PPO that groups multiple completions per prompt and normalizes rewards within the group to compute advantages, removing the need for a value function.
- **CISPO**: A REINFORCE-style algorithm that clips importance-sampling weights (not the objective as in PPO/GRPO) with a stop-gradient for stability, so every token receives a gradient signal.
- **GSPO**: Like GRPO but normalizes the policy ratio by completion length, preventing length bias.
- **DPO**: Not an RL algorithm, but a method to solve the same preference optimization problem by bypassing the separate reward model entirely, optimizing directly from preference pairs (see Chapter 8).

All of the policy gradient algorithms above are on-policy in derivation, though most are applied slightly off-policy in practice. DPO and the other direct alignment algorithms in Chapter 8 are off-policy by default. All can be paired with a learned reward model or verifiable rewards. Only PPO requires a learned value function. REINFORCE and RLOO have no importance-sampling ratio — the remaining algorithms each introduce one to enable multiple gradient steps per batch of rollouts, differing in granularity and clipping strategy as summarized below.

Table 3: Comparing policy gradient algorithms.

Method	IS Granularity	Clipping Style	Advantage
REINFORCE	None	None	Monte Carlo baseline
RLOO	None	None	Leave-one-out
PPO	Token	Objective (bilateral)	Learned value fn
GRPO	Token	Objective (bilateral)	Group-relative
GSPO	Sequence	Objective (bilateral)	Group-relative
CISPO	Token	Weights (stop-grad)	Group-relative

The core loss $\mathcal{L}(\theta)$ for each method is:

$$\textbf{REINFORCE:} \quad -\frac{1}{T}\sum_{t=1}^{T}\log \pi_\theta(a_t \mid s_t)\left(G_t - b(s_t)\right)$$

$$\textbf{RLOO:} \quad -\frac{1}{K}\sum_{i=1}^{K}\sum_{t}\log \pi_\theta(a_{i,t} \mid s_{i,t})\left(R_i - \frac{1}{K-1}\sum_{j\neq i} R_j\right)$$

$$\textbf{CISPO:} \quad -\sum_{i,t}\mathrm{sg}(\hat{\rho}_{i,t})\, A_{i,t}\log \pi_\theta(a_{i,t} \mid s_{i,t})$$
$$\hat{\rho}_{i,t} = \mathrm{clip}(\rho_{i,t},\, 1-\varepsilon,\, 1+\varepsilon)$$

$$\textbf{PPO:} \quad -\frac{1}{T}\sum_{t=1}^{T}\min\left(\rho_t A_t,\ \mathrm{clip}(\rho_t, 1-\varepsilon, 1+\varepsilon)\, A_t\right)$$
$$\rho_t = \frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}$$

$$\textbf{GRPO:} \quad -\frac{1}{G}\sum_{i=1}^{G}\min\left(\rho_i A_i,\ \mathrm{clip}(\rho_i, 1-\varepsilon, 1+\varepsilon)\, A_i\right)$$
$$\rho_i = \frac{\pi_\theta(a_i \mid s)}{\pi_{\theta_{\text{old}}}(a_i \mid s)}, \quad A_i = \frac{r_i - \mathrm{mean}(r_{1:G})}{\mathrm{std}(r_{1:G})}$$

$$\textbf{GSPO:} \quad -\frac{1}{G}\sum_{i=1}^{G}\min\left(\rho_i A_i,\ \mathrm{clip}(\rho_i, 1-\varepsilon, 1+\varepsilon)\, A_i\right)$$
$$\rho_i = \left(\frac{\pi_\theta(a_i \mid s)}{\pi_{\theta_{\text{old}}}(a_i \mid s)}\right)^{1/|a_i|}$$

$$\textbf{DPO:} \quad -\mathbb{E}_{(x,y^w,y^l)}\left[\log \sigma\left(\beta[\Delta \log \pi_\theta(x) - \Delta \log \pi_{\text{ref}}(x)]\right)\right]$$

6.3 Implementation

Compared to the original Deep RL literature where many of these algorithms were developed, implementing RL for optimizing language models or other large AI models requires many small implementation details. In this section, we highlight some key factors that differentiate the implementations of popular algorithms.

There are many other small details that go into this training. For example, when doing RLHF with language models a crucial step is generating text that will then be rated by the reward model. Under normal circumstances, the model should generate an end-of-sequence (EOS) token indicating it finished generating, but a common practice is to put a hard cap on generation length to efficiently utilize infrastructure. A failure mode of RLHF is that the model is regularly truncated in its answers, driving the ratings from the reward model out-of-distribution and to unpredictable scores. The solution to this is to *only* run reward model scoring on the `eos_token`, and to otherwise assign a penalty to the model for generating too long.

The popular open-source tools for RLHF have a large variance in implementation details across the algorithms (see table 10 in [132]). Some decisions not covered here include:

- **Value network initialization**: The internal learned value network used by PPO and other similar algorithms can be started from a different model of the same architecture or randomly selected weights. This can have a large impact on performance. The standard established in InstructGPT [3] (and re-used in Tülu 3 for its work on RLVR [6]) is to initialize the value network from the reward model used during RLHF. Others have used the previous checkpoint to RLHF training (normally an SFT model) with a value head appended randomly initialized, or fully re-initialized language models (less common as it will take longer for RLHF to converge, but possible).
- **Reward normalization, reward whitening, and/or advantage whitening**: Normalization bounds all the values from the RM (or environment) to be between 0 and 1, which can help with learning stability. Whitening goes further by transforming rewards or advantage estimates to have zero mean and unit variance, providing an even stronger boost to stability.
- **Different KL estimators**: With complex language models, precisely computing the KL divergence between models can be complex, so multiple approximations are used to substitute for an exact calculation [133].
- **KL controllers**: Original implementations of PPO and related algorithms had dynamic controllers that targeted specific KLs and changed the penalty based on recent measurements. Most modern RLHF implementations use static KL penalties, but this can also vary.

For more details on implementation details for RLHF, see [134]. For further information on the algorithms, see [135].

6.3.1 Policy-Gradient Basics

A simple implementation of policy gradient, using advantages to estimate the gradient to prepare for advanced algorithms such as PPO and GRPO follows:

```
pg_loss = -advantages * ratio
```

Ratio here is the (per-token) probability ratio (often computed from a log-probability difference) of the new policy model probabilities relative to the old policy that generated the batch.

In order to understand this equation, it is good to understand different cases that can fall within a batch of updates. Remember that we want the loss to *decrease* as the model gets better at the task.

Case 1: Positive advantage, so the action was better than the expected value of the state. We want to reinforce this. In this case, the model will make this more likely with the negative sign. To do so, it'll increase the logratio. A positive logratio, or sum of log probabilities of the tokens, means that the model is more likely to generate those tokens.

Case 2: Negative advantage, so the action was worse than the expected value of the state. This follows very similarly. Here, the loss will be positive if the new model was more likely, so the model will try to make it so the policy parameters make this completion less likely.

Case 3: Zero advantage, so no update is needed. The loss is zero, don't change the policy model.

6.3.2 Loss Aggregation Tradeoffs

The question when implementing any policy gradient algorithm with language models is: How do you aggregate per-token losses into a final scalar loss? Given per-token losses $\ell_{i,t}$ for sample i at token t, with completion lengths $|a_i|$ and batch size B, there are three main strategies:

Strategy 1: Per-sequence normalization (standard GRPO; also used in some PPO implementations)

$$L = \frac{1}{B}\sum_{i=1}^{B}\frac{1}{|a_i|}\sum_{t=1}^{|a_i|}\ell_{i,t} \tag{69}$$

Each sequence contributes equally to the batch loss, regardless of length. In code:

```
# Strategy 1: Per-sequence normalization
sequence_loss = ((per_token_loss * completion_mask).sum(dim=1) / \
            completion_mask.sum(dim=1)).mean()
```

Strategy 2: Per-token normalization (DAPO [136])

$$L = \frac{\sum_{i=1}^{B}\sum_{t=1}^{|a_i|}\ell_{i,t}}{\sum_{i=1}^{B}|a_i|} \tag{70}$$

Each token contributes equally; longer sequences have proportionally more influence on the gradient. In code:

```
# Strategy 2: Per-token normalization
token_loss = ((per_token_loss * completion_mask).sum() / \
           completion_mask.sum())
```

Strategy 3: Fixed-length normalization (Dr. GRPO [125])

$$L = \frac{1}{B}\sum_{i=1}^{B}\frac{1}{L_{\max}}\sum_{t=1}^{|a_i|}\ell_{i,t} \tag{71}$$

Normalizes by max sequence length $L_{\max}$, equalizing the per-token scale across sequences while still letting longer sequences contribute more total gradient because they contain more active tokens. In code:

```
# Strategy 3: Fixed-length normalization
fixed_len_loss = ((per_token_loss * completion_mask).sum(dim=1) / \
           L_max).mean()
```

Where $L_{\max}$ is typically a global constant during the entire training procedure, which specifies the maximum number of generation tokens.

Note that `completion_mask` in the code above is a matrix of 1s and 0s, where the prompt tokens are masked out (0s) because we don't want the model to learn from predicting prompt tokens.

6.3.2.1 Why Does This Matter? Intuitively, per-sequence normalization (Strategy 1) seems best since we care about *outcomes*, not individual tokens. However, this introduces subtle biases based on sequence length, which can cause the model to overthink or down-weight strategies that naturally need to use more tokens, depending on the direction of the bias. Consider two sequences of different lengths with per-token losses:

```
seq_1_losses = [1, 1, 1, 1, 10]  # 5 tokens, mean = 2.8
seq_2_losses = [1, 1, 1, 1, 1, 1, 1, 1, 1, 10]  # 10 tokens, mean = 1.9
```

With **Strategy 1** (per-sequence): The batch loss is $(2.8 + 1.9)/2 = 2.35$, and crucially, each token in the short sequence receives a larger gradient than tokens in the long sequence.

With **Strategy 2** (per-token): The batch loss is $(14 + 19)/15 = 2.2$, and all tokens receive equal gradient magnitude.

With **Strategy 3** (fixed-length with $L_{\max} = 10$): The short sequence contributes 1.4 and the long sequence contributes 1.9, balancing per-token gradients while still weighting by sequence.

For a more complete example showing how these strategies affect gradients, see the script below.

```
from typing import Optional
import torch

def masked_mean(values: torch.Tensor, mask: torch.Tensor, axis: Optional[int] = None)
-> torch.Tensor:
    """Compute mean of tensor with masked values."""
    if axis is not None:
        return (values * mask).sum(axis=axis) / mask.sum(axis=axis)
    else:
        return (values * mask).sum() / mask.sum()

def masked_sum(
        values: torch.Tensor,
        mask: torch.Tensor,
        axis: Optional[int] = None,
        constant_normalizer: float = 1.0,
    ) -> torch.Tensor:
    """Compute sum of tensor with masked values. Use a constant to normalize."""
    if axis is not None:
        return (values * mask).sum(axis=axis) / constant_normalizer
    else:
        return (values * mask).sum() / constant_normalizer

ratio = torch.tensor([
    [1., 1, 1, 1, 1, 1, 1,],
    [1, 1, 1, 1, 1, 1, 1,],
```

```
], requires_grad=True)


advs = torch.tensor([
    [2, 2, 2, 2, 2, 2, 2,],
    [2, 2, 2, 2, 2, 2, 2,],
])

masks = torch.tensor([
    # generation 1: 4 tokens
    [1, 1, 1, 1, 0, 0, 0,],
    # generation 2: 7 tokens
    [1, 1, 1, 1, 1, 1, 1,],
])

max_gen_len = 7

masked_mean_result = masked_mean(ratio * advs, masks, axis=1)
masked_mean_token_level = masked_mean(ratio, masks, axis=None)
masked_sum_result = masked_sum(ratio * advs, masks, axis=1,
constant_normalizer=max_gen_len)

print("masked_mean", masked_mean_result)
print("masked_sum", masked_sum_result)
print("masked_mean_token_level", masked_mean_token_level)

# masked_mean tensor([2., 2.], grad_fn=<DivBackward0>)
# masked_sum tensor([1.1429, 2.0000], grad_fn=<DivBackward0>)
# masked_mean_token_level tensor(1., grad_fn=<DivBackward0>)

masked_mean_result.mean().backward()
print("ratio.grad", ratio.grad)
ratio.grad.zero_()
# ratio.grad tensor([[0.2500, 0.2500, 0.2500, 0.2500, 0.0000, 0.0000, 0.0000],
# [0.1429, 0.1429, 0.1429, 0.1429, 0.1429, 0.1429, 0.1429]])

masked_sum_result.mean().backward()
print("ratio.grad", ratio.grad)
ratio.grad.zero_()
# ratio.grad tensor([[0.1429, 0.1429, 0.1429, 0.1429, 0.0000, 0.0000, 0.0000],
# [0.1429, 0.1429, 0.1429, 0.1429, 0.1429, 0.1429, 0.1429]])

masked_mean_token_level.mean().backward()
print("ratio.grad", ratio.grad)
# ratio.grad tensor([[0.0909, 0.0909, 0.0909, 0.0909, 0.0000, 0.0000, 0.0000],
# [0.0909, 0.0909, 0.0909, 0.0909, 0.0909, 0.0909, 0.0909]])
```

The output shows that with Strategy 1 (`masked_mean`), the short sequence has larger per-token gradients (0.25) than the long sequence (0.14). Strategies 2 and 3 equalize the per-token gradients across sequences. Note that these results can vary substantially if gradient accumulation is used, where the gradients are summed across multiple minibatches before taking a backward step—in this case, the balance between shorter and longer sequences can flip.

In practice, the best strategy depends on the specific training setup. Often in RLHF the method with the best numerical stability or the least variance in loss is preferred.

6.3.2.2 Related: MDP vs. Bandit Framing

The choice of loss aggregation connects to a deeper distinction in how we frame the RL problem. The **MDP (token-level)** view treats each token a_t as an action with state s_t being the running prefix. In practice, this is the framing used when we compute token-level advantages with a learned value function $V(s_t)$ (e.g., GAE [119]) and apply KL penalties per token. PPO with a learned value network is the canonical example [123].

In contrast, the **bandit (sequence-level)** view treats the whole completion as a single action with one scalar reward R. In code, this means computing a sequence-level advantage A_{seq} and broadcasting it to all tokens. RLOO and GRPO-style advantages are often used in this bandit-style setting [122] [118] [128]. Direct alignment methods like DPO and A-LoL also define sequence-level objectives, although they are not policy-gradient estimators [137].

Note that many GRPO implementations use a bandit-style advantage *and* add a separate per-token KL term in the loss, while many PPO/RLOO implementations fold KL into the reward before computing advantages; both conventions exist in practice.

An example comparison highlighting the two approaches is below:

```
# === Bandit-style (sequence-level) ===
# One scalar reward per sequence; advantage broadcast to all tokens
reward = torch.tensor([3.0, 1.0])       # (B,) e.g., reward model scores
baseline = reward.mean()                 # simple baseline (RLOO uses leave-one-out)
advantage_seq = reward - baseline        # (B,)
advantages = advantage_seq[:, None].expand(-1, seq_len)  # (B, L)
# tensor([[ 1.,  1.,  1.,  1.],    <- same advantage for all tokens
#         [-1., -1., -1., -1.]])

# === MDP-style (token-level) ===
# Per-token rewards + learned V(s_t); each token gets its own advantage
# (could also use per-token KL shaping, format rewards, or other token-level signals)
advantages = gae(per_token_rewards, values, done_mask, gamma=1.0, lam=0.95)
# tensor([[ 0.2,  0.5,  0.8,  1.5],    <- varies by position
#         [-0.3, -0.5, -0.8, -1.4]])
```

This framing distinction also explains why the discount factor γ is set to 1.0 in virtually all RLHF implementations. In standard RL, discounting ($\gamma < 1$) is essential: it balances the optimization between short-term and long-term reward across a multi-step episode, which is crucial for the agent to learn effective behavior over time. But in the RLHF setting, even when using the token-level MDP view, the inductive bias of the optimization is the quality of the collective completion – the reward signal scores the entire response, not individual tokens. Discounting earlier tokens would arbitrarily down-weight their contribution with no principled justification. As agentic RL settings mature – where models take real multi-step actions such as tool calls, code execution, and web browsing – discounting may become relevant again, since these involve genuinely distinct sequential decisions whose long-term consequences differ.

6.3.3 Asynchronous RL Systems

The default implementation for policy-gradient algorithms is what is called **on-policy** execution, where the actions (generations) taken by the agent (language model) are scored before updating the model. The theoretical derivations of policy-gradient rely on all actions being exactly on-policy where the model is always up to date with the results from the

latest trials/roll-outs. In practice, maintaining exact on-policy execution substantially slows training [138]—and perfect synchronization is technically impossible regardless. Therefore, all of the recent empirical results with language models tend to be slightly outside of the theoretical proofs. What happens in practice is designing the algorithms and systems for what actually works.

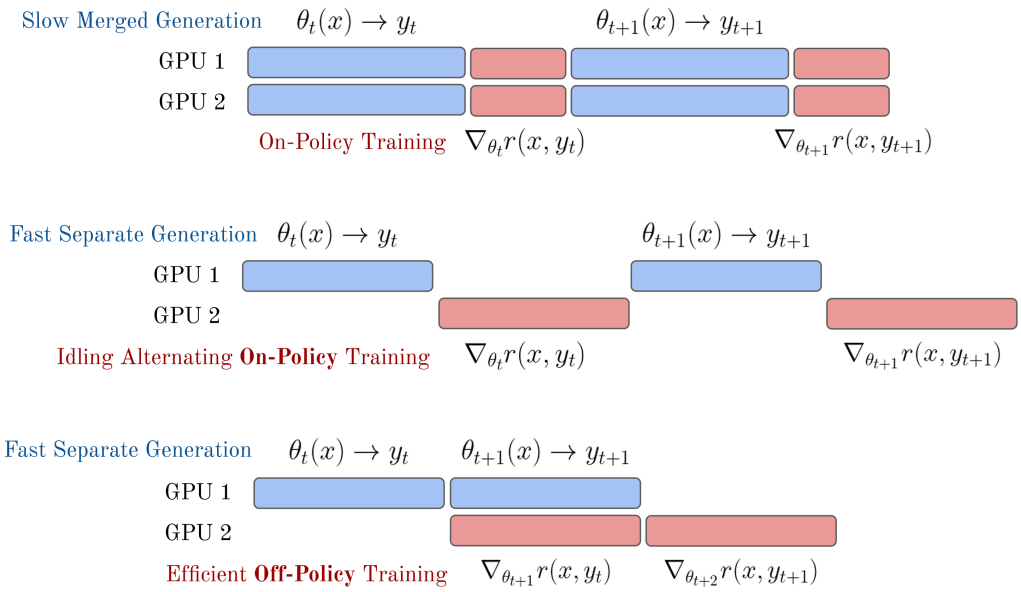


Figure 26: A comparison of the generation-update phases for synchronous or asynchronous RL training following Noukhovitch et al. 2024.

The common solution used is to constantly run inference and training on separate GPU nodes with software designed to efficiently run both, as shown in the bottom of fig. 26. Common practice in popular open-source RL tools for language models is to use a distributed process management library such as Ray to hand information off between the policy-gradient learning loop and the inference loop using an efficient inference engine, e.g., vLLM. In these setups, the GPUs dedicated to taking the RL steps are called the "learners" and the GPUs dedicated to sampling from the language model are called the "actors". The primary challenges faced when making training more asynchronous are keeping training stable and maintaining learning signal.

These systems are designed and implemented with the presumption that nearly on-policy data is good enough for stable learning. Here, the generation and update phases can easily be synced to avoid idle compute on either piece of the training system, which would be passing model weights from the learners to the actors in fig. 27. With reasoning models, the extremely long inference characteristics of problems requiring 10K to 100K+ tokens per answer makes the generation of roll-outs a far stronger bottleneck. A common problem when training reasoning models on more synchronous RL infrastructure is that an answer to one prompt in the batch can take substantially more time to generate (either through more

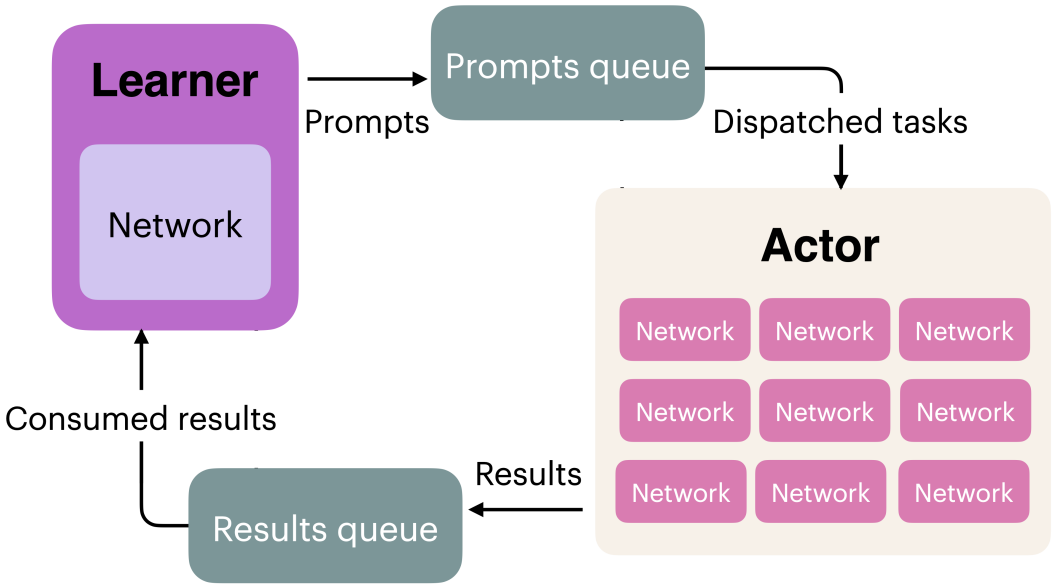


Figure 27: An example distributed RL system, where two queues are managed to pass data to the learner and actor GPUs, which can both be synchronized with a distributed computing library such as Ray. Olmo Team 2025, license CC-BY.

tokens or more tool calls), resulting in the majority of the allocated compute being idle until it completes. A second solution to this length mismatch issue, called sequence-level packing, is to stack shorter samples within a batch with clever masking to enable continued roll-outs from the model and better distribute length normalization across samples within a batch. The full complexity of distributed RL infrastructure is out of scope for this book, as it can cause many other subtle issues that slow down training or cause instability.

Following the emergence of these reasoning models, further interest has been taken to make the training and inference loops fully off-policy, where training batches for the policy gradient updates are filled with the most recently completed roll-outs across multiple instances generating answers [139] [140]. Fully asynchronous training would also enable scaling RL training runs across multiple datacenters more easily due to the option of increasing the time between weight syncs between the learner node (taking policy gradient steps) and the actor (trying to solve problems) [141].

Related methods are exploring fully off-policy policy gradient algorithms [131].

6.3.4 Truncated Importance Sampling

Truncated importance sampling (TIS) is a crucial tool used to stabilize training in modern, asynchronous RL frameworks with language models. Importance sampling is a correction that reweights samples drawn from one distribution to estimate expectations under another (as introduced in eq. 63). Truncated importance sampling [142] caps these weights with $\min(\rho, C)$ for some constant C, trading a small bias for bounded variance in the policy gradient.

This is an importance-sampling correction applied to the policy gradient, but unlike the bilateral clipping in PPO and CISPO (which constrains the ratio near 1), TIS uses a one-sided upper cap: the ratio can fall freely below 1, but is capped at C to prevent extreme upweighting. In all of PPO, GRPO, CISPO (and related algorithms), the ratio $\rho_t^{\text{policy}} = \pi_\theta(a_t \mid s)/\pi_{\theta_{\text{old}}}(a_t \mid s)$ corrects for policy drift across multiple gradient steps within one RL batch. As we shift to real-world RL frameworks, centered around the idea of asynchronicity in the previous subsection, there can be even larger sources of numerical differences (that also require the numerical correction of importance sampling). Even when the sampler and learner share identical parameters θ, their effective token distributions can differ because the inference engine (e.g., vLLM) and training framework (e.g., FSDP) use different kernels, precision, and parallelism strategies [143]. It is therefore useful to distinguish the same policy evaluated on two systems, $\pi_\theta^{\text{sampler}}$ and $\pi_\theta^{\text{learner}}$, and define the corresponding ratio and its truncated form:

$$\rho_t^{\text{learner}} = \frac{\pi_\theta^{\text{learner}}(a_t \mid s, a_{<t})}{\pi_\theta^{\text{sampler}}(a_t \mid s, a_{<t})}, \qquad \tilde{\rho}_t^{\text{learner}} = \min(\rho_t^{\text{learner}},\ C). \tag{72}$$

These two corrections are complementary, but they appear in policy-gradient implementations for different reasons — one compensates for policy drift within the training of an RL batch, the other for implementation-induced divergence — and can be applied simultaneously. How they combine depends on the algorithm:

6.3.4.1 REINFORCE with TIS (Single Gradient Step) There is no policy drift ($\pi_\theta = \pi_{\theta_{\text{old}}}$), so the only mismatch is between the learner and sampler. Here $\pi_{\theta_{\text{old}}} = \pi_{\text{gen}}$, and TIS directly corrects the learner–sampler gap:

$$\nabla_\theta J \approx \mathbb{E}_{a\sim\pi_\theta^{\text{sampler}}}\left[\tilde{\rho}_t^{\text{learner}} \cdot A_t \cdot \nabla_\theta \log \pi_\theta^{\text{learner}}(a_t \mid s, a_{<t})\right]. \tag{73}$$

6.3.4.2 PPO/GRPO with TIS (Multiple Gradient Steps) Now both ratios are active. In careful implementations, the "old logprobs" in the policy ratio are recomputed on the learner (the GSPO paper discusses this), so the policy ratio $\rho_t^{\text{policy}} = \pi_\theta^{\text{learner}}/\pi_{\theta_{\text{old}}}^{\text{learner}}$ captures pure policy drift, while $\tilde{\rho}_t^{\text{learner}} = \min(\pi_{\theta_{\text{old}}}^{\text{learner}}/\pi_{\theta_{\text{old}}}^{\text{sampler}},\ C)$ separately corrects the backend mismatch at the generation checkpoint:

$$J_{\text{PPO+TIS}}(\theta) = \mathbb{E}\left[\min\Big(\rho_t^{\text{policy}} A_t,\ \text{clip}\Big(\rho_t^{\text{policy}}, 1-\varepsilon, 1+\varepsilon\Big) A_t\Big) \cdot \tilde{\rho}_t^{\text{learner}}\right]. \tag{74}$$

Here $\pi_{\theta_{\text{old}}} \neq \pi_{\text{gen}}$: the old logprobs come from the learner, not the sampler. If a framework skips this recomputation and uses the sampler logprobs directly as $\pi_{\theta_{\text{old}}}$, the policy ratio already captures the backend mismatch and no separate TIS correction is needed — but the clip then operates on a noisier ratio that starts away from 1.0 even before any gradient steps. This is the "your framework secretly brings you off-policy RL" observation from Yao et al. [143].

In practice, LLM RL systems apply TIS as a per-token correction weight on the policy-gradient loss:

```
# Shape: (B*G, L)
C = 2.0  # TIS cap

logratio = learner_logprobs - sampler_logprobs
logratio = logratio.clamp(-10.0, 10.0)              # numerical safety
tis_weight = torch.exp(logratio).clamp(max=C)        # one-sided truncation

# Use as a fixed correction weight on the per-token PG loss
per_token_pg_loss = per_token_pg_loss * tis_weight.detach()
```

The $[-10, 10]$ clamp is only for numerical stability before exponentiation; the actual truncated-importance-sampling step is the one-sided cap at C. In practice, the bookkeeping around these logprobs — storing sampler logprobs from generation, recomputing learner logprobs at the old checkpoint, and tracking current logprobs during gradient steps — is a substantial part of the scaffolding in distributed RL frameworks. Unlike GSPO, this correction is token-level because it addresses token-level numerical mismatch rather than sequence-level reward granularity. TIS for the learner–sampler ratio has been adopted across major open-source RL frameworks (VeRL, TRL, OpenRLHF, SkyRL, OAT, and Open Instruct, which uses $C = 2$), and becomes increasingly important for long reasoning traces (Chapter 7), where small per-token differences compound over thousands of generated tokens.

6.3.5 Example: PPO

There are many, many implementations of PPO available. The core *loss* computation is shown below. Crucial to stable performance is also the *value* computation, where multiple options exist (including multiple options for the *value model* loss).

Note that the reference policy (or old logprobs) here are from the time the generations were sampled and not necessarily the reference policy. The reference policy is only used for the KL distance constraint/penalty.

```
# B: Batch Size, L: Sequence Length, G: Num of Generations
# Apply KL penalty to rewards
rewards = rewards - self.beta * per_token_kl  # Shape: (B*G, L)

# Get value predictions
values = value_net(completions)  # Shape: (B*G, L)

# Compute returns via backward pass (gamma typically 1.0 for LM RLHF)
# Mask rewards to avoid padding tokens (which may have KL penalties) leaking into
returns
returns = torch.zeros_like(rewards)
running = torch.zeros(rewards.shape[0], device=rewards.device, dtype=rewards.dtype)
for t in reversed(range(rewards.shape[1])):
    # Zero out padding: only accumulate rewards/returns for valid completion tokens
    running = (rewards[:, t] + self.gamma * running) * completion_mask[:, t]
    returns[:, t] = running

# Compute advantages: A_t = G_t - V(s_t)
advantages = returns - values.detach()  # Shape: (B*G, L)
# Note: We detach the value network here to not update the parameters of
# the value function when computing the policy-gradient loss

# Normalize advantages (optional but stable)
advantages = (advantages - advantages.mean()) / (advantages.std() + 1e-8)
```

```
# Compute probability ratio between new and old policies
ratio = torch.exp(new_per_token_logps - per_token_logps)  # Shape: (B*G, L)

# PPO clipping objective
eps = self.cliprange  # e.g. 0.2
pg_losses1 = -advantages * ratio  # Shape: (B*G, L)
pg_losses2 = -advantages * torch.clamp(ratio, 1.0 - eps, 1.0 + eps)  # Shape: (B*G, L)
pg_loss_max = torch.max(pg_losses1, pg_losses2)  # Shape: (B*G, L)

# Value function loss: predict returns
vf_loss = 0.5 * ((returns - values) ** 2)  # Shape: (B*G, L)

# Combine policy and value losses
per_token_loss = pg_loss_max + self.vf_coef * vf_loss  # Shape: (B*G, L)

# Apply completion mask and compute final loss
loss = ((per_token_loss * completion_mask).sum(dim=1) /
completion_mask.sum(dim=1)).mean()
 # Scalar

# Compute metrics for logging
with torch.no_grad():
    # Compute clipping fraction
    clip_frac = ((pg_losses2 > pg_losses1).float() * completion_mask).sum() /
completion_mask.sum()

    # Compute approximate KL
    approx_kl = (0.5 * ((new_per_token_logps - per_token_logps)**2) *
completion_mask).sum() / completion_mask.sum()

    # Compute value loss for logging
    value_loss = vf_loss.mean()
```

The core piece to understand with PPO is how the policy gradient loss is updated. Focus on these three lines:

```
pg_losses1 = -advantages * ratio  # Shape: (B*G, L)
pg_losses2 = -advantages * torch.clamp(ratio, 1.0 - eps, 1.0 + eps)  # Shape: (B*G, L)
pg_loss_max = torch.max(pg_losses1, pg_losses2)  # Shape: (B*G, L)
```

`pg_losses1` is the vanilla advantage-weighted policy gradient loss. `pg_losses2` applies the same formula but with the probability ratio clamped to the range $[1-\varepsilon, 1+\varepsilon]$, limiting how much the policy can change in a single update.

The key insight is taking `torch.max` of the two losses. Because we're minimizing a *negative* loss (recall the negative sign in front of advantages), taking the maximum selects the more pessimistic gradient—the one that produces a smaller policy update. When the advantage is positive (good action), clipping prevents the policy from increasing that action's probability too aggressively. When the advantage is negative (bad action), clipping prevents over-correction in the other direction.

By clamping the log-probability ratio, PPO bounds how far the policy can drift from the version that generated the training data, stabilizing learning without requiring an explicit trust region computation.

The code above also shows PPO learning a value function alongside the policy, which adds

implementation complexity, but the clipped objective is the core mechanism.

6.3.5.1 PPO/GRPO Simplification with One Gradient Step per Sample (No Clipping) PPO (and GRPO) implementations can be handled much more elegantly if the hyperparameter "number of gradient steps per sample" is equal to 1. Many typical values for this are from 2-4 or higher. In the main PPO or GRPO equations, see eq. 52, the "reference" policy is the previous parameters – those used to generate the completions or actions. Thus, if only one gradient step is taken, $\pi_\theta = \pi_{\theta_{\text{old}}}$, and the update rule reduces to the following (the notation $[]_\nabla$ indicates a stop gradient):

$$J(\theta) = \frac{1}{G}\sum_{i=1}^{G}\left(\frac{\pi_\theta(a_i|s)}{[\pi_\theta(a_i|s)]_\nabla}A_i - \beta\mathcal{D}_{\text{KL}}(\pi_\theta||\pi_{\text{ref}})\right). \tag{75}$$

This leads to PPO or GRPO implementations where the second policy gradient and clipping logic can be omitted, making the optimizer far closer to standard policy gradient.

6.3.6 Example: GRPO

The DeepSeekMath paper describes some implementation details of GRPO that differ from PPO [128], especially if comparing to a standard application of PPO from Deep RL rather than language models. For example, the KL penalty within the RLHF optimization (recall the KL penalty is also used when training reasoning models on verifiable rewards without a reward model) is applied directly in the loss update rather than to the reward function. Where the standard KL penalty application for RLHF is applied as $r = r_\theta - \beta\mathcal{D}_{\text{KL}}$, the GRPO implementation is along the lines of:

$$L = L_{\text{policy gradient}} + \beta * \mathcal{D}_{\text{KL}} \tag{76}$$

However, there are multiple ways to implement this. Traditionally, the KL distance is computed with respect to each token in the completion to a prompt s. For reasoning training, multiple completions are sampled from one prompt, and there are multiple prompts in one batch, so the KL distance will have a shape of [B, L, N], where B is the batch size, L is the sequence length, and N is the number of completions per prompt.

Putting it together, using the first loss accumulation, the pseudocode can be written as below.

```
# B: Batch Size, L: Sequence Length, G: Number of Generations
# Compute group-wise rewards # Shape: (B,)
mean_grouped_rewards = rewards.view(-1, self.num_generations).mean(dim=1)
std_grouped_rewards = rewards.view(-1, self.num_generations).std(dim=1)


# Normalize the rewards to compute the advantages
mean_grouped_rewards = mean_grouped_rewards.repeat_interleave(self.num_generations,
dim=0)
std_grouped_rewards = std_grouped_rewards.repeat_interleave(self.num_generations, dim=0)
# Shape: (B*G,)

# Compute advantages
```

```
advantages = (rewards - mean_grouped_rewards) / (std_grouped_rewards + 1e-4)
advantages = advantages.unsqueeze(1)
# Shape: (B*G, 1)

# Compute probability ratio between new and old policies
ratio = torch.exp(new_per_token_logps - per_token_logps)  # Shape: (B*G, L)

# PPO clipping objective
eps = self.cliprange  # e.g. 0.2
pg_losses1 = -advantages * ratio  # Shape: (B*G, L)
pg_losses2 = -advantages * torch.clamp(ratio, 1.0 - eps, 1.0 + eps)  # Shape: (B*G, L)
pg_loss_max = torch.max(pg_losses1, pg_losses2)  # Shape: (B*G, L)

# important to GRPO -- PPO applies this in reward traditionally
# Combine with KL penalty
per_token_loss = pg_loss_max + self.beta * per_token_kl  # Shape: (B*G, L)

# Apply completion mask and compute final loss
loss = ((per_token_loss * completion_mask).sum(dim=1) /
completion_mask.sum(dim=1)).mean()
 # Scalar

# Compute core metric for logging (KL, reward, etc. also logged)
with torch.no_grad():
    # Compute clipping fraction
    clip_frac = ((pg_losses2 > pg_losses1).float() * completion_mask).sum() /
completion_mask.sum()

    # Compute approximate KL
    approx_kl = (0.5 * ((new_per_token_logps - per_token_logps)**2) *
completion_mask).sum() / completion_mask.sum()
```

For more details on how to interpret this code, see the PPO section above. The core differences from the PPO example are:

- **Advantage computation**: GRPO normalizes rewards relative to the group (mean and std across generations for the same prompt) rather than using a learned value function as baseline.
- **No value network**: GRPO removes the value model entirely, eliminating `vf_loss` and the associated complexity.
- **KL penalty placement**: GRPO adds the KL penalty directly to the loss rather than subtracting it from the reward (this is the standard implementation, but more versions exist on how the KL is applied).

6.3.6.1 RLOO vs. GRPO

The advantage updates for RLOO follow GRPO very closely, highlighting the conceptual similarity of the algorithm when taken separately from the PPO style clipping and KL penalty details. Specifically, for RLOO, the advantage is computed relative to a baseline that is extremely similar to that of GRPO – the completion reward relative to the others for that same question. Concisely, the RLOO advantage estimate follows as (expanded from TRL's implementation):

```
# rloo_k --> number of completions per prompt
# rlhf_reward --> Initially a flat tensor of total rewards for all completions. Length B
= N x k
rlhf_reward = rlhf_reward.reshape(rloo_k, -1) #
```

```
# Now, Shape: (k, N), each column j contains the k rewards for prompt j.

baseline = (rlhf_reward.sum(0) - rlhf_reward) / (rloo_k - 1)
# baseline --> Leave-one-out baseline rewards. Shape: (k, N)
#  baseline[i, j] is the avg reward of samples i' != i for prompt j.

advantages = rlhf_reward - baseline
# advantages --> Same Shape: (k, N)

advantages = advantages.flatten() # Same shape as original tensor
```

The rest of the implementation details for RLOO follow the other trade-offs of implementing policy-gradient.

6.4 Auxiliary Topics

In order to master the application of policy-gradient algorithms, there are countless other considerations. Here we consider some of the long-tail of complexities in successfully deploying a policy-gradient RL algorithm.

6.4.1 Generalized Advantage Estimation (GAE)

Generalized Advantage Estimation (GAE) is an alternate method to compute the advantage for policy gradient algorithms [119] that better balances the bias-variance tradeoff. Traditional single-step advantage estimates can introduce too much bias, while using complete trajectories can suffer from high variance. GAE computes an exponentially-weighted average of multi-step advantage estimates, where the λ hyperparameter controls the bias-variance tradeoff—ranging from single-step TD ($\lambda = 0$) to full trajectory returns ($\lambda = 1$); $\lambda = 0.95$ is a common default for LLM fine-tuning.

Advantage estimates can take many forms, but we can define an n-step advantage estimator (similar to the TD residual at the beginning of the chapter) as follows:

$$\hat{A}_t^{(n)} = \begin{cases} r_t + \gamma V(s_{t+1}) - V(s_t), & n = 1 \\ r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) - V(s_t), & n = 2 \\ \vdots & \\ r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots - V(s_t), & n = \infty \end{cases} \tag{77}$$

Here a shorter n will have lower variance but higher bias as we are attributing more learning power to each trajectory – it can overfit. GAE attempts to generalize this formulation into a weighted multi-step average instead of a specific n. To start, we must define the temporal difference (TD) residual of predicted value.

$$\delta_t^V = r_t + \gamma V(s_{t+1}) - V(s_t) \tag{78}$$

To utilize this, we introduce another variable λ as the GAE mixing parameter. This folds into an exponential decay of future advantages we wish to estimate:

$$
\begin{aligned}
\hat{A}_t^{GAE(\gamma,\lambda)} &= (1-\lambda)(\hat{A}_t^{(1)} + \lambda\hat{A}_t^{(2)} + \lambda^2\hat{A}_t^{(3)} + \cdots) \\
&= (1-\lambda)(\delta_t^V + \lambda(\delta_t^V + \gamma\delta_{t+1}^V) + \lambda^2(\delta_t^V + \gamma\delta_{t+1}^V + \gamma^2\delta_{t+2}^V) + \cdots) \\
&= (1-\lambda)(\delta_t^V(1+\lambda+\lambda^2+\cdots) + \gamma\delta_{t+1}^V(\lambda+\lambda^2+\cdots) + \cdots) \\
&= (1-\lambda)\left(\delta_t^V\tfrac{1}{1-\lambda} + \gamma\delta_{t+1}^V\tfrac{\lambda}{1-\lambda} + \cdots\right) \\
&= \textstyle\sum_{l=0}^{\infty}(\gamma\lambda)^l\delta_{t+l}^V
\end{aligned}
\tag{79}
$$

Intuitively, this can be used to average multi-step estimates of Advantage in an elegant fashion. An example implementation is shown below:

```
# GAE (token-level) for LM RLHF
#
# B: Batch Size
# L: Length
# Inputs:
#   rewards: (B, L) post-KL per-token rewards
#   values:  (B, L) current V_theta(s_t)
#   done_mask: (B, L) 1.0 at terminal token (EOS or penalized trunc), else 0.0
#   gamma: float (often 1.0),
#   lam (short for lambda): float in [0,1]
#   (Padding beyond terminal should have rewards=0, values=0)
B, L = rewards.shape
advantages = torch.zeros_like(rewards)
next_v = torch.zeros(B, device=rewards.device, dtype=rewards.dtype)
gae = torch.zeros(B, device=rewards.device, dtype=rewards.dtype)

for t in reversed(range(L)):
    not_done = 1.0 - done_mask[:, t]
    delta = rewards[:, t] + gamma * not_done * next_v - values[:, t]
    gae = delta + gamma * lam * not_done * gae
    advantages[:, t] = gae
    next_v = values[:, t]

targets = advantages + values      # y_t for value regression
advantages = advantages.detach()   # for policy loss
```

The backward loop accumulates temporal-difference (TD) errors ($\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$), which measure how much better or worse the actual outcome was compared to the value function's prediction, with exponential decay $(\gamma\lambda)^l$. At terminal tokens, `not_done=0` prevents bootstrapping from future states and resets the GAE accumulator, so each episode's advantages are computed independently (since the loop runs backward, the terminal token cleanly stops the exponentially-weighted accumulation at episode boundaries—this makes the implementation packing-friendly, correctly handling multiple sequences concatenated into one). The final `targets` serve as regression targets for the separate value function learned outside this GAE loop, while the detached `advantages` weight the policy gradient—detached so that policy updates don't backpropagate through the value network. In RLHF for language models, $\gamma = 1.0$ is common because episodes are short token sequences where undiscounted credit assignment is preferred (and often all of the tokens in one).

For further reading, see [144].

6.4.2 Double Regularization

We've seen in this chapter two types of regularization. One is built into algorithms like PPO with step-size constraints, and the other is a KL divergence based distance penalty relative to the start of the optimization.

Many popular policy gradient algorithms from Deep Reinforcement Learning, including PPO and its predecessors, originated due to the need to control the learning process of the agent. In RLHF, as discussed extensively in Chapter 15 on Regularization and in Chapter 3 on Training Overview, there is a built-in regularization term via the distance penalty relative to the original policy one is fine-tuning. In this view, a large part of the difference between algorithms like PPO (which have internal step-size regularization) and REINFORCE (which is simpler, and to which PPO reduces under certain hyperparameters) is far less meaningful for fine-tuning language models than training agents from scratch.

In PPO, the objective that handles capping the step-size of the update is known as the surrogate objective. To monitor how much the PPO regularization is impacting updates in RLHF, one can look at the clip fraction variable in many popular implementations, which is the percentage of samples in the batch whose probability ratio falls outside the clipping interval. This is a useful proxy for how often PPO's regularizer may be active, but not every such sample has zero gradient: the surrogate becomes flat only when the clipped branch is selected, such as positive-advantage samples with ratios above $1 + \varepsilon$ or negative-advantage samples with ratios below $1 - \varepsilon$.

In practice with language models, algorithms like PPO and GRPO are often run with only one gradient step per batch, which means that the PPO-native regularization is never applied (as clipping can only occur within a batch when the policy changes substantially) and the KL distance penalties predominate. However, this is not universal. For example, DAPO uses 16 gradient steps per batch [136], and Tülu 3 uses 4 PPO update iterations per batch for 8B and 70B models but reduces to 1 for 405B to maintain training stability [6].

6.4.3 Further Reading

As RLHF has cemented itself at the center of modern post-training, other policy-gradient RL algorithms and RL algorithms generally have been proposed to improve the training process, but they have not had a central role in governing best practices. Examples for further reading include:

- **Pairwise Proximal Policy Optimization (P3O; Wu et al., 2023)** [145] uses pairwise data directly in a PPO-style policy update without learning an intermediate reward model.
- **Soft Adaptive Policy Optimization (SAPO)** [146] replaces hard PPO/GRPO-style clipping with smooth, temperature-controlled gating, aiming for a continuous trust region that preserves near-on-policy learning signal while down-weighting off-policy tokens.
- Off-policy policy-gradient algorithms could enable further asynchronous training, such as **Contrastive Policy Gradient (CoPG)** [147] (a generalization of the direct alignment algorithm IPO and vanilla policy gradient), which was used by Cohere for their Command A model [57].
- Other implementations of REINFORCE algorithms have been designed for language models, such as **ReMax** [148], which implements a baseline normalization designed

specifically to accommodate the sources of uncertainty from reward model inference.
- Some foundation models, such as Apple Intelligence Foundation Models [149] or Kimi k1.5 reasoning model [150], have used variants of **Mirror Descent Policy Optimization (MDPO)** [151]. Research is still developing further on the fundamentals here [152], but Mirror Descent is an optimization method rather than directly a policy gradient algorithm. What is important here is that it is substituted in very similarly to existing RL infrastructure.
- **Decoupled Clip and Dynamic sAmpling Policy Optimization (DAPO)** proposes 4 modifications to GRPO to better suit reasoning language models, where long traces are needed and new, underutilized tokens need to be increased in probability [136]. The changes are: 1, have two different clip hyperparameters, ε_{low} and $\varepsilon_{\text{high}}$, so clipping on the positive side of the logratio can take bigger steps for better exploration; 2, dynamic sampling, which removes all samples with reward = 0 or reward = 1 for all samples in the batch (no learning signal); 3, use the per-token loss as discussed above in Implementation: GRPO; and 4, a soft penalty on samples that are too long to avoid trying to learn from truncated answers.
- **Value-based Augmented Proximal Policy Optimization (VAPO)** [153] combines optimizations from DAPO (including clip-higher, token-level policy-gradient, and different length normalization) with insights from Value-Calibrated PPO [154] to pretrain the value function and length-adaptive GAE to show the promise of value-based methods relative to GRPO.

6.5 Suggested Experiments

The companion implementation in `code/policy_gradients/` is designed for small, observable RL runs. The default configs train `Qwen/Qwen3-1.7B` on the `spell_backward` procedural task from `reasoning-gym`, which is a good first exercise because failures and partial progress are easy to inspect.

1. **Run the word reversal task with GRPO.**

   ```
   cd code/
   uv run python -m policy_gradients.train --config
   policy_gradients/configs/grpo.yaml
   ```

   Track `avg_correctness`, `avg_format`, and `avg_binary`. The useful first question is whether each prompt group contains contrast: if all sampled completions are right or all are wrong, a group-relative update has little learning signal.

2. **Compare group-relative and single-sample estimators.** Run the matched starting configs:

   ```
   cd code/
   uv run python -m policy_gradients.train --config
   policy_gradients/configs/reinforce.yaml
   uv run python -m policy_gradients.train --config
   policy_gradients/configs/rloo.yaml
   uv run python -m policy_gradients.train --config
   policy_gradients/configs/grpo.yaml
   ```

Compare how quickly the correctness signal improves and how noisy the loss is. RLOO and GRPO should make the role of within-prompt baselines much more concrete than the equations alone.

3. **Sweep the contrast knobs.** Copy `policy_gradients/configs/grpo.yaml` and vary `num_rollouts`, `temperature`, `data.size`, and `format_weight`. Small `num_rollouts` reduces group contrast; very low temperature can collapse samples; very high temperature can generate too many malformed answers. This is the simplest way to see why RLVR recipes often spend so much effort on sampling settings before touching the optimizer.
4. **Move from toy rewards toward math.** For GSM8K-style experiments, start with the `code/reward_models/train_orm.py` and `code/rejection_sampling/` examples before adding a new online RL environment. A good contribution would be a small `reasoning-gym` or GSM8K policy-gradient config that runs on a sub-1B Qwen model and reports the same group-contrast diagnostics.

7 Reasoning and Inference-Time Scaling

Reasoning models and inference-time scaling enabled a massive step in language model performance at the end of 2024, through 2025, and into the future. Inference-time scaling is the ability to improve model performance by using more computation during generation, such as producing longer reasoning chains or sampling multiple responses. Language models trained to think extensively before answering exploit this property remarkably well. These models, trained with a large amount of reinforcement learning with verifiable rewards (RLVR) [6], still utilize large amounts of RLHF. In this chapter we review the path that led the AI community to a transformed appreciation for RL's potential in language models, review the fundamentals of RLVR, highlight key works, and point to the future debates that will define the area in the next few years.

7.1 The Role of RLVR

To start, at the 2016 edition of the Neural Information Processing Systems (NeurIPS) conference, Yann LeCun first introduced his now-famous cake metaphor for where learning happens in modern machine learning systems:

> If intelligence is a cake, the bulk of the cake is unsupervised learning, the icing on the cake is supervised learning, and the cherry on the cake is reinforcement learning (RL).

This analogy is now largely complete with modern language models and recent changes to the post-training stack. RLHF was the precursor to this, and RL for reasoning models, primarily on math, code, and science topics, was its confirmation. In this analogy:

- Self-supervised learning on vast swaths of internet data makes up the majority of the cake (especially when viewed in compute spent in FLOPs),
- The beginning of post-training in supervised fine-tuning (SFT) for instructions tunes the model to a narrower distribution, and
- Finally "pure" reinforcement learning (RL) is the cherry on top. The scaled up reinforcement learning used to create the new "reasoning" or "thinking" models is this finishing piece (along with the help of RLHF, which isn't considered classical RL, as we'll explain).

This little bit of reasoning training emerged with **thinking models** that use a combination of the post-training techniques discussed in this book to align preferences along with RL training on verifiable domains to dramatically increase capabilities such as reasoning, coding, and mathematics problem solving.

The training method for these models, Reinforcement Learning with Verifiable Rewards (RLVR) [6], proceeds very similarly to RLHF, but it makes the reward model optional in lieu of a scoring function that returns a positive reward when the answer is correct and 0 otherwise.

For example, consider how different it is to score responses for RLHF versus RLVR. In RLHF, a reward model must evaluate subjective qualities:

> **Prompt**: Explain the concept of opportunity cost in economics.
>
> **Response**: Opportunity cost is the value of the next best alternative you give

up when making a decision. For example, if you spend an hour studying instead of working, the opportunity cost is the wages you could have earned...

Scoring this response requires judging clarity, accuracy, completeness, and helpfulness – all qualities that require learned preferences and lack a definitive correct answer.

In contrast, RLVR uses verification functions that return definitive scores. For mathematics:

> **Prompt**: What is the sum of all prime numbers less than 20?
>
> **Response**: The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. Adding them: 2 + 3 = 5, then 5 + 5 = 10, then 10 + 7 = 17, then 17 + 11 = 28, then 28 + 13 = 41, then 41 + 17 = 58, and finally 58 + 19 = 77. The answer is $\boxed{77}$.
>
> **Verification**: `extracted_answer == 77` → Reward = 1

The `\boxed{}` notation is a convention adopted from mathematical typesetting that makes answer extraction straightforward – a simple regular expression can pull the final answer from the response, regardless of how the model arrived at it. Note that other answer extraction methods exist, such as just using the phrase "The answer is:" as also shown above, special tokens like `<answer>`, or delimiters like `####`.

For code generation, verification often takes the form of unit tests:

> **Prompt**: Write a Python function `fib(n)` that returns the nth Fibonacci number, where fib(0) = 0 and fib(1) = 1.
>
> **Response**: def fib(n): if n < 2: return n return fib(n - 1) + fib(n - 2)
>
> **Verification (unit tests)**:
>
> assert fib(0) == 0 # base case assert fib(1) == 1 # base case assert fib(10) == 55 # larger value (All tests pass → Reward = 1)

Unit tests are natural verification functions for code: they execute the model's solution against known input-output pairs. A common form of scoring is to perform simple gating: If all assertions pass, the reward is 1; if any fail, the reward is 0. Other setups use partial credit proportional to the number of tests passed. For both these examples, no learned reward model is needed and most setups go without one (because the models are robust to over-optimization in these domains), but one can be used with a linear combination of rewards.

The ideas behind RLVR are not new to the RL literature, where the core idea of taking gradient updates based on whether the answer is correct is almost the textbook definition of reinforcement learning. The innovations when applying this to language models are largely how to apply it while maintaining the strong, general capabilities of the model being fine-tuned. Within that, there are many related ideas in the language modeling literature where the model learns from feedback regarding the correctness of the answer.

Originally, in the work I was a part of that coined the term RL with Verifiable Rewards (RLVR) [6], the method was to be named RL with Ground Truth rewards (RLGT). Yet RLVR is subtly different from learning solely from ground truth answers. In domains like mathematics, a single ground truth answer is available to verify solutions, as we saw above. In other domains, such as code generation or precise instruction following, answers can be

verified with a checking function (e.g., a unit test), even when there are multiple correct solutions rather than just a single ground truth answer. The core of progress on RLVR is having a variety and depth of these verifiable problems, even if the exact solution isn't known a priori.

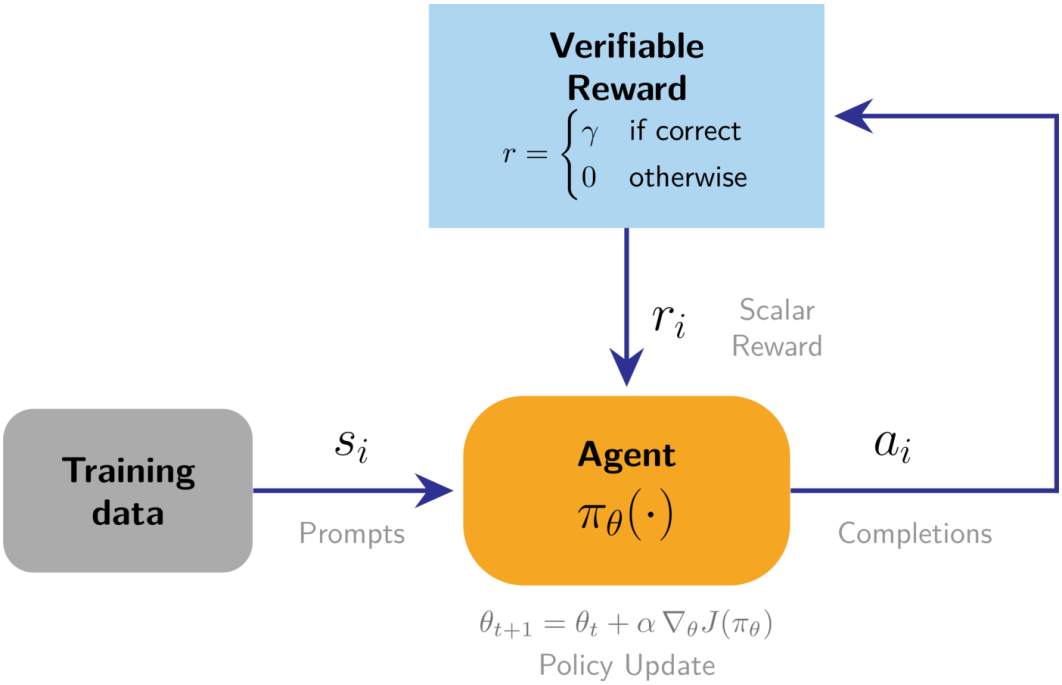


Figure 28: RLVR in the form of an RL feedback loop. Instead of a reward model, a verification function is used.

The first models to successfully deploy this type of training were OpenAI's o1 [53] and the open-weight model DeepSeek R1 [15]. Soon after, the entire AI industry prioritized this training process and model style. The core change here is more a reallocation of the stages of training and the priority of different behaviors rather than this type of RL setup being entirely new. Reasoning models brought an era where scaling RL training is expected.

As for the type of behavior these models exhibit, consider the following example with DeepSeek V3 0325 versus their reasoning model, DeepSeek R1, on the query `Write me a short poem about a goldfish`. DeepSeek V3 is very to the point:

> **Response**: Goldfish Dreams
>
> In a bowl of glass so clear and round, A tiny goldfish swims without a sound. With scales that gleam like sunlight's kiss, He drifts along in watery bliss.
>
> A flick of fin, a flash so bright, Chasing bubbles in the light. Does he dream of oceans wide? Or is his little world enough—his pride?

Meanwhile, DeepSeek R1's experience is different. It first generates numerous thinking tokens to craft a response before responding to the user. The answer appears as:

Response: `<thinking>` Okay, the user wants a short poem about a goldfish. Let me start by brainstorming some goldfish-related imagery. Goldfish are often in bowls, right? So maybe start with the setting. Words like "gilded bowl" or "crystal globe" could work.

Next, think about the movement of the goldfish. They swim smoothly, so words like "dart", "glide", or "swirl" come to mind. Maybe describe their fins as "silken" to emphasize delicacy.

Then, consider the environment around the goldfish. Sunlight through water creates shimmering effects. Words like "sunbeams dance" or "ripples spin" might capture that. Including elements like castles or pebbles adds a playful touch.

Themes? Maybe the contrast between the goldfish's small world and its perception of freedom. The bowl as a universe, tiny but complete. Ending with a sense of mystery or silent observation could give depth. Need to keep it concise, maybe four stanzas with a rhyme scheme. Let me check the rhythm and flow. Avoid clichés but keep it accessible. Alright, let's piece it together. `</thinking>`

Gilded Dreams in Liquid Glass

A gilded bowl, a crystal globe, Where tiny tides in silence flow— A flash of orange, a whispered robe, You dart through sunlit indigo.

Your world is small, yet boundless too— Each pebble, castle, weed unfurled. The surface bends the sky to blue, And ripples spin your liquid world.

You glide through myths we'll never know, A silent monarch, veiled in mist. Do dreams drift where the bubbles go? Or memories in amethyst?

We watch you trace your endless sphere— A universe within a tear.

The tokens in the `<thinking>` tags are the model's reasoning. For more complex problems the reasoning stage can take thousands of tokens before producing an answer. So, long-context language models were a prerequisite to advanced reasoning behavior, but that is not the focus of this chapter.

The core intuition for *how this training works* is that for a given model, we repeat the following cycle:

1. Sample multiple answers to multiple questions,
2. Take gradient steps towards the answers that are correct, and
3. Repeat, revisiting the same data.

Remarkably, this extremely simple approach (when done with a careful distribution of data and stable training infrastructure) helps the models learn by revisiting the same questions again and again. Even more remarkable is that the improvements on these training questions generalize to questions and (some) domains the models have never seen!

This simple approach allows the models to lightly search over behavior space and the RL algorithm increases the likelihood of behaviors that are correlated with correct answers.

7.2 The Origins of New Reasoning Models

Here we detail the high-level trends that led to the explosion of reasoning models in 2025.

7.2.1 Why Does RL Work Now?

Despite many, many takes that "RL doesn't work yet" [155] and papers detailing deep reproducibility issues with RL [156], the field overcame them to find high-impact applications. Some are covered in this book, such as ChatGPT's RLHF and DeepSeek R1's RLVR, but many others exist, including improving chip design [157], mastering video gameplay [158], self-driving [159], and more. The takeoff of RL-focused training on language models indicates progress on many fundamental issues for the research area, including:

- **Stability of RL can be solved**: For its entire existence, the limiting factor on RL's adoption has been stability. This manifests in two ways. First, the learning itself can be fickle and not always work. Second, the training itself is known to be more brittle than standard language model training and more prone to loss spikes, crashes, etc. Countless new model releases are using this style of RL training with verifiable rewards on top of a pretrained base model and substantial academic uptake has occurred. The technical barriers to entry on RL are at an all-time low.
- **Open-source versions already "exist"**: Many tools already exist for training language models with RLVR and related techniques. Examples include TRL [47], Open Instruct [6], veRL [160], and OpenRLHF [161], where many of these are building on optimizations from earlier in the arc of RLHF and post-training. The accessibility of tooling is enabling a large and accelerating body of research.

Multiple resources point to RL training for reasoning only being viable with leading models coming out from about 2024 onwards, indicating that a certain level of underlying capability was needed in the models before reasoning training was possible.

7.2.2 RL Training vs. Inference-Time Scaling

Training with reinforcement learning to elicit reasoning behaviors and performance on verifiable domains is closely linked to the ideas of inference-time scaling. Inference-time scaling, also called test-time scaling, is the general class of methods that use more computational power at inference in order to perform better at downstream tasks. Methods for inference-time scaling were studied before the release of DeepSeek R1 and OpenAI's o1, which both massively popularized investment in RL training specifically. Examples include value-guided sampling [162] or repeated random sampling with answer extraction [163]. Beyond this, inference-time scaling can be used to improve more methods of AI training beyond chain-of-thought reasoning to solve problems, such as with reward models that consider the options deeply [91] [164].

RL training is a short path to inference-time scaling laws being used, but in the long-term we will have more methods for eliciting the inference-time tradeoffs we need for best performance. Training models heavily with RL often enables them to generate more tokens per response in a way that is strongly correlated with improved downstream performance (although this sequence length increase is the default, research also exists explicitly on improving performance *without* relying on this inference-time scaling). This is a substantial shift from the length-bias seen in early RLHF systems [10], where the human preference training had a side effect of increasing the response average length for marginal gains on preference rankings.

Other than the core RL trained models there are many methods being explored to continue to push the limits of reasoning and inference-time compute. These are largely out of the

scope of this book due to their rapidly evolving nature, but they include distilling reasoning behavior from a larger RL trained model to a smaller model via instruction tuning [165], composing more inference calls [166], and more. What is important here is the correlation between downstream performance and an increase in the number of tokens generated – otherwise it is just wasted energy.

7.2.3 The Future (Beyond Reasoning) of RLVR

In many domains, these new flavors of RLVR are much more aligned with the goals of developers by being focused on performance rather than behavior. Standard fine-tuning APIs generally use a parameter-efficient fine-tuning method such as LoRA (Low-Rank Adaptation, a parameter-efficient method that trains only small added matrices rather than all model weights, also referred to as parameter-efficient fine-tuning, PEFT) with supervised fine-tuning on instructions. Developers pass in prompts and completions and the model is tuned to match that by updating model parameters to match the completions, which increases the prevalence of features from your data in the model's generations.

RLVR is focused on matching answers. Given queries and correct answers, RLVR helps the model learn to produce the correct answers. While standard instruction tuning is done with 1 or 2 epochs of loss updates over the data, RLVR gets its name by doing hundreds or thousands of epochs over the same few data points to give the model time to learn new behaviors. This can be viewed as reinforcing positive behaviors that would work sparingly in the base model version into robust behaviors after RLVR.

The scope of RL training for language models continues to grow: The biggest takeaway from o1 and R1 on a fundamental scientific level was that we have even more ways to train language models to potentially valuable behaviors. The more open doors that are available to researchers and engineers, the more optimism we should have about AI's general trajectory.

7.3 Understanding Reasoning Training Methods

The investment in reasoning has instigated a major evolution in the art of how models are trained to follow human instructions. These recipes still use the common pieces discussed in earlier chapters (as discussed in Chapter 3 with the overview of DeepSeek R1's recipe), including instruction fine-tuning, reinforcement learning from human feedback, and reinforcement learning with verifiable rewards (RLVR). The core change is using far more RLVR and applying the other training techniques in different orders – traditionally for a reasoning model the core training step is either a large-scale RL run or a large-scale instruction tuning run on *outputs* of another model that had undergone a substantial portion of RLVR training (referred to as distillation).

7.3.1 Reasoning Research Before OpenAI o1 or DeepSeek R1

Before the takeoff of reasoning models, a substantial effort was made to understand how to train language models to be better at verifiable domains. The main difference between these works below is that their methodologies did not scale to the same level as those used in DeepSeek R1 and subsequent models, or they resulted in models that made sacrifices in overall performance in exchange for higher mathematics or coding abilities. The underlying

ideas and motivations are included to paint a broader picture for how reasoning models emerged within the landscape.

Some of the earliest efforts to train language models on verifiable domains include the self-taught reasoner (STaR) line of work [167] [168] and TRICE [169], which both used ground-truth reward signals to encourage chain-of-thought reasoning in models throughout 2022 and 2023. STaR effectively approximates the policy gradient algorithm, but in practice filters samples differently and uses a cross-entropy measure instead of a log-probability, and Quiet-STaR expands on this with very related ideas of recent reasoning models by having the model generate tokens before trying to answer the verifiable question (which helps with training performance). TRICE [169] also improves reasoning by generating traces and then optimizing with a custom Markov chain Monte Carlo inspired expectation maximization algorithm. VinePPO [170] followed these and used a setup that shifted closer to modern reasoning models. VinePPO uses a PPO-based algorithm with binary rewards for math question correctness, training on GSM8K and MATH. Other work before OpenAI's o1 and DeepSeek R1 used code execution as a feedback signal for training [171], [172] or verification for theorem proving (called Reinforcement Learning from Verifier Feedback, RLVF, here) [173]. Tülu 3 expanded on these methods by using a simple PPO trainer to reward completions with correct answers – most importantly while maintaining the model's overall performance on a broad suite of evaluations. The binary rewards of Tülu 3 and modern reasoning training techniques can be contrasted with the iterative approach of STaR or the log-likelihood rewards of Quiet-STaR.

7.3.2 Early Reasoning Models

A summary of the foundational reasoning research reports, some of which are accompanied by open data and model weights, following DeepSeek R1 is shown in tbl. 4.

Table 4: A summary of the notable reasoning model technical reports in 2025, the first year of substantial inference-time scaling with RLHF.

Date	Name	TLDR	Open weights	Open data
2025-01-22	DeepSeek R1 [15]	RL-based upgrade to DeepSeek, big gains on math & code reasoning	Yes	No
2025-01-22	Kimi 1.5 [150]	Scales PPO/GRPO on Chinese/English data; strong AIME maths	No	No
2025-03-31	Open-Reasoner-Zero [174]	Fully open replication of base model RL	Yes	Yes
2025-04-10	Seed-Thinking 1.5 [62]	ByteDance RL pipeline with dynamic CoT gating	Yes	No
2025-04-30	Phi-4 Reasoning [175]	14B model; careful SFT→RL; excels at STEM reasoning	Yes	No
2025-05-02	Llama-Nemotron [176]	Multi-size "reasoning-toggle" models	Yes	Yes
2025-05-12	INTELLECT-2 [141]	First, publicly documented globally-decentralized RL training run	Yes	Yes

Date	Name	TLDR	Open weights	Open data
2025-05-12	Xiaomi MiMo [61]	End-to-end reasoning pipeline from pre- to post-training	Yes	No
2025-05-14	Qwen 3 [60]	Similar to R1 recipe applied to new models	Yes	No
2025-05-21	Hunyuan-TurboS [177]	Mamba-Transformer MoE, adaptive long/short CoT	No	No
2025-05-28	Skywork OR-1 [178]	RL recipe avoiding entropy collapse; beats DeepSeek on AIME	Yes	Yes
2025-06-04	Xiaomi MiMo VL [179]	Adapting reasoning pipeline end-to-end to include multi-modal tasks	Yes	No
2025-06-04	OpenThoughts [180]	Public 1.2M-example instruction dataset distilled from QwQ-32B	Yes	Yes
2025-06-10	Magistral [181]	Pure RL on Mistral 3; multilingual CoT; small model open-sourced	Yes	No
2025-06-16	MiniMax-M1 [130]	Open-weight 456B MoE hybrid/Lightning Attention reasoning model; 1M context; RL w/CISPO; releases 40K/80K thinking-budget checkpoints	Yes	No
2025-07-10	Kimi K2 [182]	1T MoE (32B active) with MuonClip (QK-clip) for stability; 15.5T token pretrain without loss spikes; multi-stage post-train with agentic data synthesis + joint RL; releases base + post-trained checkpoints.	Yes	No
2025-07-28	GLM-4.5 [183]	Open-weight 355B-A32B MoE "ARC" model with thinking/non-thinking modes; 23T-token multi-stage training + post-train w/ expert iteration and RL; releases GLM-4.5 + GLM-4.5-Air (MIT).	Yes	No
2025-08-20	Nemotron Nano 2 [184]	Hybrid Mamba-Transformer for long "thinking traces"; FP8 pretraining at 20T tokens then compression/distillation; explicitly releases multiple checkpoints plus "majority" of pre/post-training datasets.	Yes	Yes (most)
2025-09-09	K2-Think [185]	Parameter-efficient math reasoning system: a 32B open-weights model with test-time scaling recipe; positioned as fully open incl. training data/code (per release materials).	Yes	Yes
2025-09-23	LongCat-Flash-Thinking [186]	560B MoE reasoning model; report is explicit about a staged recipe from long-CoT cold start to large-scale RL; open-source release.	Yes	No

Date	Name	TLDR	Open weights	Open data
2025-10-21	Ring-1T [187]	Trillion-scale "thinking model" with RL scaling focus; report frames bottlenecks/solutions for scaling RL at 1T and releases an open model.	Yes	No
2025-11-20	Olmo 3 Think [18]	Fully open "model flow" release: reports the entire lifecycle (stages, checkpoints, and data points) and positions Olmo 3 Think 32B as a flagship open thinking model.	Yes	Yes
2025-12-02	DeepSeek V3.2 [188]	Open-weight MoE frontier push with a report that foregrounds attention efficiency changes, RL framework upgrades, and data synthesis for agentic/reasoning performance.	Yes	No
2025-12-05	K2-V2 [189]	70B dense "360-open" model trained from scratch; with 3-effort SFT-only post-training for controllable thinking.	Yes	Yes
2025-12-15	Nemotron 3 Nano [190]	30B-A3B MoE hybrid Mamba-Transformer; pretrain on 25T tokens and includes SFT + large-scale RL; explicitly states it ships weights + recipe/code + most training data.	Yes	Yes (most)
2025-12-16	MiMo-V2-Flash [63]	309B MoE (15B active) optimized for speed: hybrid SWA/GA attention (5:1, 128-token window) + lightweight MTP; FP8 pretrain on 27T tokens; post-train with MOPD + large-scale agentic RL for reasoning/coding.	Yes	No

7.3.3 Common Practices in Training Reasoning Models

In this section we detail common methods used to sequence training stages and modify data to maximize performance when training a reasoning model.

Note that these papers could have used a listed technique and not mentioned it, whereas their peers do, so these examples are a subset of known implementations and should be used as a reference, but not a final proclamation on what an optimal recipe is.

- **Offline difficulty filtering**: A core intuition of RLVR is that models can only learn from examples where there is a gradient. If the starting model for RLVR can solve a problem either 100% of the time or 0% of the time, there will be no gradient between different completions to the prompt (i.e., all strategies appear the same to the policy gradient algorithm). Many models have used difficulty filtering before starting large-scale RL to restrict the training problems to those that the starting point model solves only 20-80% of the time. This data is collected by sampling N, e.g. 16, completions to each prompt in the training set and verifying what percentage are correct. Forms

of this were used by Seed-Thinking 1.5, Open Reasoner Zero, Phi-4, INTELLECT-2, MiMo RL, Skywork OR-1, and others.

- **Per-batch online filtering** (or difficulty curriculums throughout training): To complement the offline filtering to find the right problems to train on, another major question is: what order should the problems be presented to the model during learning? In order to address this, many models use online filtering of questions in the batch, prebuilt curriculums/data schedulers, saving harder problems for later in training, or other ideas to improve long-term stability. Related ideas are used by Kimi 1.5, Magistral, Llama-Nemotron, INTELLECT-2, MiMo-RL, Hunyuan-TurboS, and others.
- **Remove KL penalty**: As the length of RL runs (in any metric, total GPU hours, FLOPS, or RL steps) increased for reasoning models relative to RLHF training, and the reward function became less prone to over-optimization, many models removed the KL penalty constraining the RL-learned policy to be similar to the base model used at the start of training. This allows the model to further explore during its training. This was used by RAGEN [191], Magistral, OpenReasonerZero, Skywork OR-1, and others.
- **Relaxed policy-gradient clipping**: New variations of the algorithm GRPO, such as DAPO [136], proposed modifications to the two-sided clipping objective used in GRPO (or PPO) in order to enable better exploration. Clipping has also been shown to cause potentially spurious learning signals when rewards are imperfect [192]. This two-sided clipping with different ranges per gradient direction is used by RAGEN, Magistral, INTELLECT-2, and others.
- **Off-policy data (or fully asynchronous updates)**: As the length of completions needed to solve tasks with RL increases dramatically with harder problems (particularly in the *variance* of the response length, where there are often outliers with extremely long lengths), compute in RL runs can sit idle. To solve this, training is moving to asynchronous updates or changing how problems are arranged into batches to improve overall throughput. Partial-to-full asynchronous (off-policy) data is used by Seed-Thinking 1.5, INTELLECT-2, and others.
- **Additional format rewards**: In order to make the reasoning process predictable, many models add minor rewards to make sure the model follows the correct format of e.g. `<think>...</think>` before an answer. This is used by DeepSeek R1, OpenReasonerZero, Magistral, Skywork OR-1, and others.
- **Language consistency rewards**: Similar to format rewards, some multilingual reasoning models use language consistency rewards to prioritize models that do not change languages while reasoning (for a better and more predictable user experience). These include DeepSeek R1, Magistral, and others.
- **Length penalties**: Many models use different forms of length penalties during RL training to either stabilize the learning process over time or to mitigate overthinking on hard problems. Some examples include Kimi 1.5 progressively extending the target length to combat overthinking (while training accuracy is high across difficulty curriculum) or INTELLECT-2 running a small length penalty throughout. Progressively extending the training sequence length mitigates overthinking by forcing the model to first reason effectively in a domain with a more limited thinking budget, and then transitioning to longer training where the model can use those behaviors efficiently on more complex problems. Others use overlong filtering and other related implementations to improve throughput.
- **Loss normalization**: There has been some discussion (see the chapter on policy gradients or [125]) around potential length or difficulty biases introduced by the per-

group normalization terms of the original GRPO algorithm. As such, some models, such as Magistral or MiMo, chose to normalize either losses or advantages at the batch level instead of the group level.
- **Parallel test-time compute scaling**: Combining answers from multiple parallel, independently-sampled rollouts can lead to substantial improvements over using the answer from a single rollout. The most naive form of parallel test-time compute scaling, as done in DeepSeek-R1, Phi-4, and others, involves using the answer returned by a majority of rollouts as the final answer. A more advanced technique is to use a scoring model trained to select the best answer out of the answers from the parallel rollouts. As of 2026, this technique had not become common in open, documented reasoning model recipes, but it was mentioned in the Claude 4 announcement [193] and used in DeepSeek-GRM [164].

Complementing the common techniques, there are also many common findings on how reasoning training can create useful models without sacrificing ancillary capabilities:

- **Text-only reasoning boosts multimodal performance**: Magistral, MiMo-VL, and others find that training a multimodal model and then performing text-only reasoning training after this multimodal training can *improve* multimodal performance in the final model.
- **Toggleable reasoning with system prompt** (or length control): Llama-Nemotron, Nemotron Nano, Qwen 3, SmolLM 3, and others use specific system prompts (possibly in combination with length-controlled RL training [194]) to enable a toggleable on/off thinking length for the user. Other open models, such as OpenAI's gpt-oss and LLM360's K2-V2 [189] adopt a low-medium-high reasoning effort set in the system prompt, but training methods for this type of behavior are not as well documented.

7.4 Looking Ahead

The reasoning model landscape is evolving faster than any area of AI research in recent memory, and some of the common practices listed here will inevitably be superseded by new techniques.

Several efforts are underway to systematically understand what makes reasoning training work. Olmo 3 Think [18] represents the most comprehensive open documentation of a reasoning model's full training lifecycle, providing checkpoints and data at each stage for the research community to study, and concluding with a nearly 4-week-long training run on 220 GPUs. Similarly, work on understanding the scaling properties of RL for reasoning [17] is beginning to formalize relationships between compute, data, and performance that were previously only intuited by practitioners.

What remains clear is that reinforcement learning has graduated from the "cherry on top" in the cake metaphor to a load-bearing component of frontier model training. The minor techniques in this chapter around the idea of RLVR – difficulty filtering, format rewards, and the rest – are not the final answers, but they represent the field's current best understanding of how to elicit reasoning from language models. The next generation of methods will likely look different, but they will build on the foundations established here.

8 Direct-Alignment Algorithms

Direct Alignment Algorithms (DAAs) allow one to update models to solve the same RLHF objective without ever training an intermediate reward model or using reinforcement learning optimizers. DAAs solve the same preference learning problem we've been studying (with literally the same data!), in order to make language models more aligned, smarter, and easier to use. The lack of a reward model and online optimization makes DAAs far simpler to implement, reducing compute spent during training and making experimentation easier. This chapter details the complex mathematics done to derive these algorithms, and then shows that the sometimes tedious derivations result in simple implementations.

The most prominent DAA and one that catalyzed an entire academic movement of aligning language models is Direct Preference Optimization (DPO) [25]. At its core, DPO uses gradient ascent to solve the same constrained RLHF objective (see Chapter 3):

$$\max_{\pi} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[r_\theta(x,y)\right] - \beta\mathcal{D}_{\text{KL}}\left(\pi(y|x)\|\pi_{\text{ref}}(y|x)\right) \tag{80}$$

Since its release in May of 2023, after a brief delay where the community figured out the right data and hyperparameters to use DPO with (specifically, surprisingly low learning rates), many popular models have used DPO or its variants, from Zephyr-β kickstarting it in October of 2023 [26], Llama 3 Instruct [29], Tülu 2 [27] and 3 [6], Nemotron 4 340B [30], and others. Technically, Sequence Likelihood Calibration (SLiC-HF) was the first modern direct alignment algorithm released [195], but it did not catch on due to a combination of factors (unwinding the adoption of research methods is always a tricky task).

The most impactful part of DPO and DAAs is lowering the barrier to entry to experimenting with language model post-training – it uses less compute, is easier to implement from scratch, and is easier to get working on both toy and production examples.

Throughout this chapter, we use x to denote prompts and y to denote completions. This notation is common in the language model literature, where methods operate on full prompt-completion pairs rather than individual tokens.

8.1 Direct Preference Optimization

Here we explain intuitions for how DPO works and re-derive the core equations fully.

8.1.1 How DPO Works

DPO at a surface level is directly optimizing a policy to solve the RLHF objective. The loss function for this, which we will revisit below in the derivations, compares how much the learned policy's probability of chosen and rejected completions has shifted relative to a reference model. The loss function derived from a Bradley-Terry reward model follows:

$$\mathcal{L}_{\text{DPO}}(\pi_\theta;\pi_{\text{ref}}) = -\mathbb{E}_{(x,y_c,y_r)\sim\mathcal{D}}\left[\log\sigma\left(\beta\log\frac{\pi_\theta(y_c\mid x)}{\pi_{\text{ref}}(y_c\mid x)} - \beta\log\frac{\pi_\theta(y_r\mid x)}{\pi_{\text{ref}}(y_r\mid x)}\right)\right] \tag{81}$$

Inside the sigmoid, the first term $\beta\log\frac{\pi_\theta(y_c|x)}{\pi_{\text{ref}}(y_c|x)}$ measures how much the policy has increased the probability of the *chosen* completion relative to the reference model, and the second

term does the same for the *rejected* completion. The loss decreases when the chosen shift exceeds the rejected shift – i.e. when the policy learns to prefer the right response.

Throughout, β is a hyperparameter balancing the reward optimization to the KL divergence between the final model and the initial reference (i.e. balancing over-optimization, a crucial hyperparameter when using DPO correctly). This relies on the implicit reward for DPO training that replaces using an external reward model, which is a log-ratio of probabilities:

$$r(x, y) = \beta \log \frac{\pi_r(y \mid x)}{\pi_{\text{ref}}(y \mid x)} \tag{82}$$

where $\pi_r(y \mid x)$ is the exact, optimal reward policy that we are solving for. This comes from deriving the Bradley-Terry reward with respect to an optimal policy (shown in eq. 97), as shown in the Bradley-Terry model section of Chapter 5. Essentially, as stated in the DPO paper, this reparameterization gives us "the probability of human preference data in terms of the optimal policy rather than the reward model" – meaning we can bypass learning an explicit reward model entirely.

Let us consider the loss shown in eq. 81 that the optimizer must decrease. Here, the loss will be lower when the log-ratio of the chosen response is bigger than the log-ratio of the rejected response (normalized by the reference model). In practice, this is a sum of log-probabilities of the model across the sequence of tokens in the data presented. Hence, DPO is increasing the gap in relative log-probabilities between the chosen and rejected responses.

With the reward in eq. 82, we can write the gradient of the loss to further interpret what is going on:

$$\nabla_\theta \mathcal{L}_{\text{DPO}}(\pi_\theta; \pi_{\text{ref}}) = -\beta \mathbb{E}_{(x, y_c, y_r) \sim \mathcal{D}} \left[w \cdot (\nabla_\theta \log \pi_\theta(y_c \mid x) - \nabla_\theta \log \pi_\theta(y_r \mid x)) \right] \tag{83}$$

where $w = \sigma(r_\theta(x, y_r) - r_\theta(x, y_c))$.

Here, the gradient solves the above objective by doing the following:

- The first term within the sigmoid function, $\sigma(\cdot)$, creates a weight of the parameter update from 0 to 1 that is higher when the reward estimate is incorrect. When the rejected sample is preferred over the chosen, the weight update should be larger!
- Second, the terms in the inner brackets $[\cdot]$ increase the likelihood of the chosen response y_c and decrease the likelihood of the rejected y_r.
- These terms are weighted by β, which controls how the update balances ordering the completions correctly relative to the KL divergence.

The core intuition is that DPO is fitting an implicit reward model whose corresponding optimal policy can be extracted in closed form (eq. 97, thanks to gradient descent and our ML tools). Because the DPO loss is directly differentiable, it is straightforward to compute the exact gradient, rather than needing to estimate it by training a reward model and sampling completions to score. What is often misunderstood is that DPO is learning a reward model at its core, hence the subtitle of the paper *Your Language Model is Secretly a Reward Model*. It is easy to confuse this with the DPO objective training a policy directly, hence studying the derivations below is good for a complete understanding.

With the implicit reward model learning, DPO is generating an optimal solution to the RLHF objective given the data in the dataset and the specific KL constraint in the objective β. Here, DPO solves for the exact policy given a specific KL divergence because the generations are not online as in policy gradient algorithms – a core difference from the RL methods for preference tuning. In many ways, this makes the β value easier to tune with DPO relative to online RL methods, but crucially and intuitively the optimal value depends on the model being trained and the data training it.

At each batch of preference data, composed of many pairs of completions $y_{chosen} \succ y_{rejected}$, DPO takes gradient steps directly towards the optimal solution. It is far simpler than policy gradient methods.

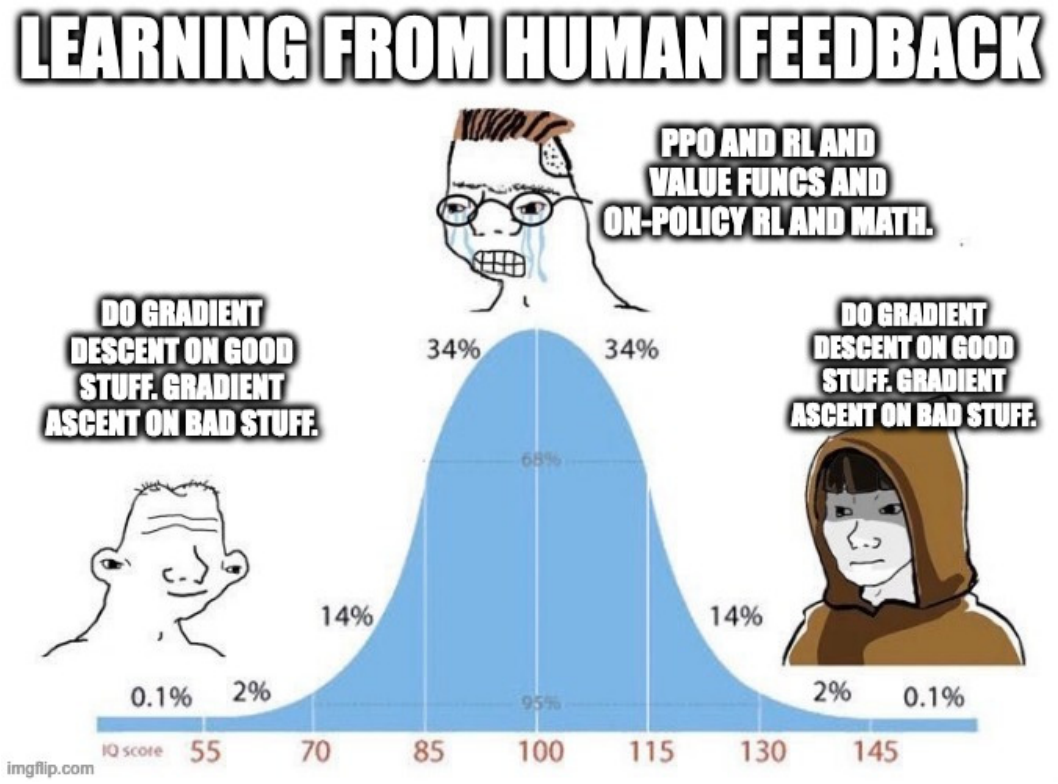


Figure 29: When DPO first released it sparked a fierce debate in the research community about how to best do RLHF and preference learning. This meme does a great job capturing the sentiment, where the debate often felt forced and over the top, but many people both getting started and in top labs were getting immense benefit out of DPO. DPO simplicity meme, credit Tom Goldstein.

8.1.2 DPO Derivation

The DPO derivation takes two primary parts. First, the authors show the form of the policy that optimally solved the RLHF objective used throughout this book. Next, they show how to arrive at that solution from pairwise preference data (i.e. a Bradley-Terry model).

8.1.2.1 Deriving the Optimal RLHF Solution To start, we should consider the RLHF optimization objective once again, here indicating we wish to maximize this quantity:

$$\max_{\pi} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[r_\theta(x,y)\right] - \beta\mathcal{D}_{\text{KL}}\left(\pi(y|x)\|\pi_{\text{ref}}(y|x)\right) \tag{84}$$

Here, the dual expectation only applies to the sampling to compute the expected reward, as the KL term is still an analytical expression. First, let us expand the definition of KL-divergence. Recall that $\mathcal{D}_{\text{KL}}(\pi\|\pi_{\text{ref}}) = \mathbb{E}_{y\sim\pi}\left[\log\frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)}\right]$, where the $\pi(y|x)$ weighting in the sum becomes the sampling distribution. Since both terms now share the same expectation over $y \sim \pi(y|x)$, we can combine them:

$$\max_{\pi} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[r(x,y) - \beta\log\frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)}\right] \tag{85}$$

Next, bring the negative sign out of the difference in brackets. To do this, split it into two terms:

$$= \max_{\pi}\left(\mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[r(x,y)\right] - \beta\,\mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)}\right]\right) \tag{86}$$

Then, multiply by -1 to convert the maximization into a minimization:

$$= \min_{\pi}\left(-\mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[r(x,y)\right] + \beta\,\mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)}\right]\right) \tag{87}$$

Divide by β and recombine:

$$= \min_{\pi}\left(\mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)} - \frac{1}{\beta}r(x,y)\right]\right) \tag{88}$$

Next, we must introduce a partition function, $Z(x)$:

$$Z(x) = \sum_y \pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right) \tag{89}$$

The partition function acts as a normalization factor for the unnormalized density $\pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)$, thereby making it a valid probability function over y for each fixed x. The exact need for this will become clear shortly as we proceed with the derivation.

With this substituted in, we obtain our intermediate transformation:

$$\min_{\pi} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\frac{1}{Z(x)}\pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)} - \log Z(x)\right] \tag{90}$$

To see how this is obtained, consider the internal part of the optimization in brackets of eq. 88:

$$\log \frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)} - \frac{1}{\beta} r(x, y) \tag{91}$$

Then, add $\log Z(x) - \log Z(x)$ to both sides:

$$= \log \frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)} - \frac{1}{\beta} r(x, y) + \log Z(x) - \log Z(x) \tag{92}$$

Then, we group the terms:

$$= \left(\log \frac{\pi(y|x)}{\pi_{\text{ref}}(y|x)} + \log Z(x) \right) - \log Z(x) - \frac{1}{\beta} r(x, y) \tag{93}$$

With $\log(x) + \log(y) = \log(x \cdot y)$ (and moving Z to the denominator), we get:

$$= \log \frac{\pi(y|x)}{\frac{1}{Z(x)} \pi_{\text{ref}}(y|x)} - \log Z(x) - \frac{1}{\beta} r(x, y) \tag{94}$$

Next, we expand $\frac{1}{\beta} r(x, y)$ to $\log \exp \frac{1}{\beta} r(x, y)$ and do the same operation to get eq. 90, which we slightly rewrite here:

$$\min_{\pi} \mathbb{E}_{x \sim \mathcal{D}} \left[\mathbb{E}_{y \sim \pi(y|x)} \left[\log \frac{\pi(y|x)}{\frac{1}{Z(x)} \pi_{\text{ref}}(y|x) \exp\left(\frac{1}{\beta} r(x, y) \right)} \right] - \log Z(x) \right] \tag{95}$$

With this optimization form, we need to actually solve for the optimal policy π^*. Since we introduced the partition function $Z(x)$, thereby making the term $\frac{1}{Z(x)} \pi_{\text{ref}}(y|x) \exp\left(\frac{1}{\beta} r(x, y) \right)$ a valid probability distribution over y, we can recognize that the inner expectation is in fact a proper KL-divergence!

$$\min_{\pi} \mathbb{E}_{x \sim \mathcal{D}} \left[\mathcal{D}_{\text{KL}} \left(\pi(y|x) \middle\| \frac{1}{Z(x)} \pi_{\text{ref}}(y|x) \exp\left(\frac{1}{\beta} r(x, y) \right) \right) - \log Z(x) \right] \tag{96}$$

Since the term $\log Z(x)$ does not depend on π (the policy we are optimizing), we can ignore it. This leaves us with just the KL divergence between the policy we are learning and a form relating the partition, β, reward, and reference policy. Gibbs' inequality tells us this is minimized at a distance of 0, only when the two quantities are equal! Hence, we get an optimal policy:

$$\pi^*(y|x) = \pi(y|x) = \frac{1}{Z(x)} \pi_{\text{ref}}(y|x) \exp\left(\frac{1}{\beta} r(x, y) \right) \tag{97}$$

8.1.2.2 Deriving DPO Objectives for BT Models To start, recall from Chapter 5 on Reward Modeling and Chapter 11 on Preference Data that a Bradley-Terry model of human preferences is formed as:

$$p^*(y_1 \succ y_2 \mid x) = \frac{\exp\left(r^*(x, y_1)\right)}{\exp\left(r^*(x, y_1)\right) + \exp\left(r^*(x, y_2)\right)} \tag{98}$$

By manipulating eq. 97, we can solve for the optimal reward. First, take the logarithm of both sides:

$$\log \pi^*(y|x) = \log \left(\frac{1}{Z(x)} \pi_{\text{ref}}(y|x) \exp\left(\frac{1}{\beta} r^*(x, y)\right)\right) \tag{99}$$

Expanding the right-hand side using $\log(abc) = \log a + \log b + \log c$:

$$\log \pi^*(y|x) = -\log Z(x) + \log \pi_{\text{ref}}(y|x) + \frac{1}{\beta} r^*(x, y) \tag{100}$$

Rearranging to solve for $r^*(x, y)$:

$$\frac{1}{\beta} r^*(x, y) = \log \pi^*(y|x) - \log \pi_{\text{ref}}(y|x) + \log Z(x) \tag{101}$$

Multiplying both sides by β:

$$r^*(x, y) = \beta \log \frac{\pi^*(y \mid x)}{\pi_{\text{ref}}(y \mid x)} + \beta \log Z(x) \tag{102}$$

We then can substitute the reward into the Bradley-Terry equation shown in eq. 98 to obtain:

$$p^*(y_1 \succ y_2 \mid x) = \frac{\exp\left(\beta \log \frac{\pi^*(y_1|x)}{\pi_{\text{ref}}(y_1|x)} + \beta \log Z(x)\right)}{\exp\left(\beta \log \frac{\pi^*(y_1|x)}{\pi_{\text{ref}}(y_1|x)} + \beta \log Z(x)\right) + \exp\left(\beta \log \frac{\pi^*(y_2|x)}{\pi_{\text{ref}}(y_2|x)} + \beta \log Z(x)\right)} \tag{103}$$

By decomposing the exponential expressions from e^{a+b} to $e^a e^b$ and then cancelling out the terms $e^{\beta \log Z(x)}$, this simplifies to:

$$p^*(y_1 \succ y_2 \mid x) = \frac{\exp\left(\beta \log \frac{\pi^*(y_1|x)}{\pi_{\text{ref}}(y_1|x)}\right)}{\exp\left(\beta \log \frac{\pi^*(y_1|x)}{\pi_{\text{ref}}(y_1|x)}\right) + \exp\left(\beta \log \frac{\pi^*(y_2|x)}{\pi_{\text{ref}}(y_2|x)}\right)} \tag{104}$$

Then, multiply the numerator and denominator by $\exp\left(-\beta \log \frac{\pi^*(y_1|x)}{\pi_{\text{ref}}(y_1|x)}\right)$ to obtain:

$$p^*(y_1 \succ y_2 \mid x) = \frac{1}{1 + \exp\left(\beta \log \frac{\pi^*(y_2|x)}{\pi_{\text{ref}}(y_2|x)} - \beta \log \frac{\pi^*(y_1|x)}{\pi_{\text{ref}}(y_1|x)}\right)} \tag{105}$$

Finally, with the definition of a sigmoid function as $\sigma(x) = \frac{1}{1+e^{-x}}$, we obtain:

$$p^*(y_1 \succ y_2 \mid x) = \sigma\left(\beta \log \frac{\pi^*(y_1 \mid x)}{\pi_{\text{ref}}(y_1 \mid x)} - \beta \log \frac{\pi^*(y_2 \mid x)}{\pi_{\text{ref}}(y_2 \mid x)}\right) \tag{106}$$

This is the likelihood of preference data under the Bradley-Terry model, given the optimal policy π^*. Recall from Chapter 5 on Reward Modeling that we derived the Bradley-Terry objective as maximizing the likelihood, or equivalently minimizing the negative log-likelihood, which gives us the loss:

$$\begin{aligned}\mathcal{L}_{\text{DPO}}(\pi_\theta; \pi_{\text{ref}}) &= -\mathbb{E}_{(x,y_c,y_r)\sim\mathcal{D}}\left[\log p(y_c \succ y_r \mid x)\right] \\ &= -\mathbb{E}_{(x,y_c,y_r)\sim\mathcal{D}}\left[\log \sigma\left(\beta \log \frac{\pi_\theta(y_c|x)}{\pi_{\text{ref}}(y_c|x)} - \beta \log \frac{\pi_\theta(y_r|x)}{\pi_{\text{ref}}(y_r|x)}\right)\right]\end{aligned} \tag{107}$$

This is the loss function for DPO, in the form shown in eq. 81. The DPO paper has an additional derivation for the objective under a Plackett-Luce Model, which is far less used in practice [25].

8.1.2.3 Deriving the BT DPO Gradient We used the DPO gradient shown in eq. 83 to explain intuitions for how the model learns. To derive this, we must take the gradient of eq. 107 with respect to the model parameters.

$$\nabla_\theta \mathcal{L}_{\text{DPO}}(\pi_\theta; \pi_{\text{ref}}) = -\nabla_\theta \mathbb{E}_{(x,y_c,y_r)\sim\mathcal{D}}\left[\log \sigma\left(\beta \log \frac{\pi_\theta(y_c|x)}{\pi_{\text{ref}}(y_c|x)} - \beta \log \frac{\pi_\theta(y_r|x)}{\pi_{\text{ref}}(y_r|x)}\right)\right] \tag{108}$$

To start, this can be rewritten. We know that the derivative of a sigmoid function $\frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$, the derivative of the logarithm $\frac{d}{dx}\log x = \frac{1}{x}$, and properties of sigmoid $\sigma(-x) = 1 - \sigma(x)$, so we can reformat the above equation.

First, let $u = \beta \log \frac{\pi_\theta(y_c|x)}{\pi_{\text{ref}}(y_c|x)} - \beta \log \frac{\pi_\theta(y_r|x)}{\pi_{\text{ref}}(y_r|x)}$ (the expression inside the sigmoid). Then, we have

$$\nabla_\theta \mathcal{L}_{\text{DPO}}(\pi_\theta; \pi_{\text{ref}}) = -\mathbb{E}_{(x,y_c,y_r)\sim\mathcal{D}}\left[\frac{\sigma'(u)}{\sigma(u)} \nabla_\theta u\right] \tag{109}$$

Expanding this and using the above expressions for sigmoid and logarithms results in the gradient introduced earlier:

$$-\mathbb{E}_{(x,y_c,y_r)\sim\mathcal{D}}\left[\beta\sigma\left(\beta \log \frac{\pi_\theta(y_r|x)}{\pi_{\text{ref}}(y_r|x)} - \beta \log \frac{\pi_\theta(y_c|x)}{\pi_{\text{ref}}(y_c|x)}\right)\left[\nabla_\theta \log \pi_\theta(y_c|x) - \nabla_\theta \log \pi_\theta(y_r|x)\right]\right] \tag{110}$$

8.2 Numerical Concerns, Weaknesses, and Alternatives

Many variants of the DPO algorithm have been proposed to address weaknesses of DPO. For example, without rollouts where a reward model can rate generations, DPO treats every pair of preference data with equal weight. In reality, as seen in Chapter 11 on Preference Data, there are many ways of capturing preference data with a richer label than binary. Multiple algorithms have been proposed to re-balance the optimization away from treating each pair equally.

- **REgression to RElative REward Based RL (REBEL)** adds signal from a reward model, as a margin between chosen and rejected responses, rather than solely the pairwise preference data, to more accurately solve the RLHF problem [196].
- **Conservative DPO (cDPO) and Identity Preference Optimization (IPO)** address overfitting by assuming noise in the preference data. cDPO assumes N percent of the data is incorrectly labeled [25] and IPO changes the optimization to soften the probability of preference rather than optimize directly from a label [197]. Practically, IPO changes the preference probability to a nonlinear function, moving away from the Bradley-Terry assumption, with $\Psi(q) = \log\left(\frac{q}{1-q}\right)$.
- **DPO with an offset (ODPO)** "requires the difference between the likelihood of the preferred and dispreferred response to be greater than an offset value" [198] – do not treat every data pair equally, but this can come at the cost of a more difficult labeling environment.

Some variants of DPO attempt to either improve the learning signal by making small changes to the loss or make the application more efficient by reducing memory usage.

- **Odds Ratio Policy Optimization (ORPO)** directly updates the policy model with a pull towards the chosen response, similar to the instruction fine-tuning loss, with a small penalty on the chosen response [199]. This change of loss function removes the need for a reference model, simplifying the setup. The best way to view ORPO is as DPO inspired, rather than a DPO derivative.
- **Simple Preference Optimization (SimPO)** makes a minor change to the DPO optimization, by averaging the log-probabilities rather than summing them or adding length normalization, to improve performance [200].

One of the core issues *apparent* in DPO is that the optimization drives only to increase the margin between the probability of the chosen and rejected responses. Numerically, the model reduces the probability of both the chosen and rejected responses, but the *rejected response is reduced by a greater extent* as shown in fig. 30. Intuitively, it is not clear how this generalizes, but work has posited that it increases the probability of unaddressed behaviors – i.e. tokens that the language model could generate, but are not in the distribution of the post-training datasets [201] [202]. Simple methods—such as Cal-DPO [203], which adjusts the optimization process, and AlphaPO [204], which modifies the reward shape—mitigate this **preference displacement**. In practice, the exact impact of this is not well known, but points to a potential reason why online methods can outperform vanilla DPO.

The other primary reason posited for DPO-like methods to have a lower ceiling on performance than online (RL based) RLHF methods is that the training signal comes from completions from previous or other models. Online variants of DPO alleviate these limitations by generating new completions and incorporating a preference signal at training time. **Online**

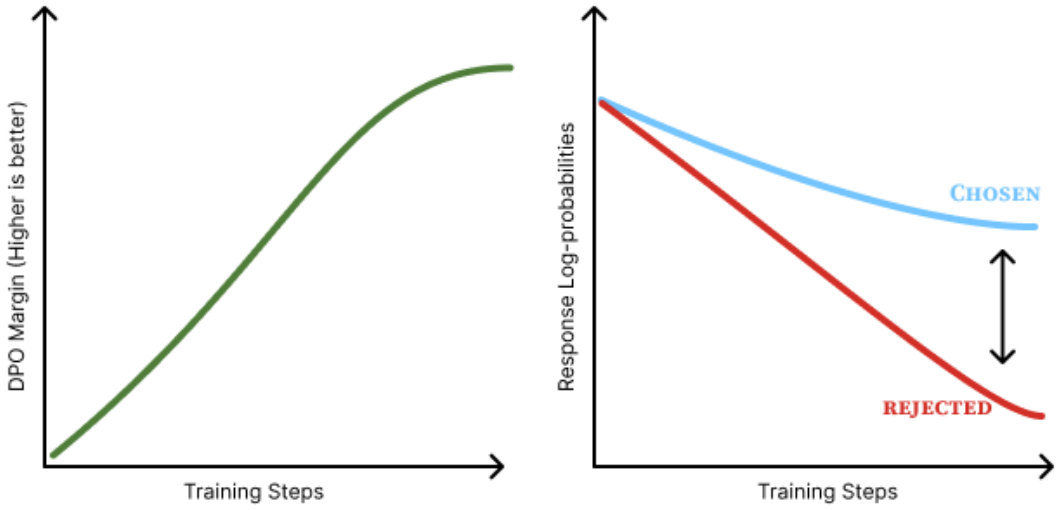


Figure 30: Sketch of preference displacement in DPO.

DPO [205] samples generations from the current model, while **Discriminator-Guided DPO** (D2PO) [206] uses reward model relabelling to create new preference data on the fly, and many more variants exist.

There is a long list of other DAA variants, such as Direct Nash Optimization (DNO) [207] or Binary Classifier Optimization (BCO) [208], but the choice of algorithm is far less important than the initial model and the data used [6] [209] [210].

8.3 Implementation Details

DAAs such as DPO are implemented very differently than policy gradient optimizers. The DPO loss, taken from the original implementation, largely can be summarized as follows [25]:

```
# Log-probability gaps for the policy and the frozen reference model
pi_logratios = policy_chosen_logps - policy_rejected_logps
ref_logratios = reference_chosen_logps - reference_rejected_logps

# Difference of log-ratios: positive when the policy
# shifts probability toward the chosen completion
logits = pi_logratios - ref_logratios

# DPO loss: negative log-sigmoid drives the policy to
# widen the gap between chosen and rejected
losses = -F.logsigmoid(beta * logits)

# Implicit rewards (detached -- used for logging only)
chosen_rewards = beta * (policy_chosen_logps - reference_chosen_logps).detach()
rejected_rewards = beta * (policy_rejected_logps - reference_rejected_logps).detach()
```

This can be used in standard language model training stacks as this information is already collated during the forward pass of a model (with the addition of a reference model).

In most ways, DAAs are simpler and a quality of life improvement, but they also offer a

different set of considerations.

1. **KL divergence is static**: In DPO and other algorithms, the KL divergence is set explicitly by the β parameter that balances the distance penalty to the optimization. This is due to the fact that DPO takes gradient steps towards the *optimal* solution to the RLHF objective given the data – it steps exactly to the solution set by the β term. On the other hand, RL based optimizers take steps based on the batch and recent data.
2. **Caching log-probabilities**: Simple implementations of DPO do the forward passes for the policy model and reference models at the same time for convenience with respect to the loss function. However, this doubles the memory used and results in increased GPU usage. To avoid this, one can compute the log-probabilities of the reference model over the training dataset first, then reuse those cached reference log-probabilities when computing the loss and updating the parameters per batch, reducing the peak memory usage by 50%.

8.4 DAAs with Synthetic Preference Data

Most of the popular datasets for performing preference fine-tuning with DAAs these days are synthetic preferences where a frontier model rates outputs from other models as the winner or the loser. Prominent examples include UltraFeedback (the first of this category) [28], Tülu 3 (built with an expanded UltraFeedback methodology) [6], SmolLM 3's data [211], or the Dolci Pref dataset released with Olmo 3 [18].

The best practices for constructing these datasets are still evolving. Tülu 3 and datasets around its release in November of 2024 demonstrated that synthetic, pairwise preference data needs to be "on-policy" in a sense that some completions are generated from the model you're fine-tuning (while being mixed in a bigger model pool). This on-policy nature of the data ensured that the DAA would optimize the correct token space within which the model generates – as the loss functions are contrastive and less direct than instruction fine-tuning. Later, with the release of Olmo 3 and SmolLM 3 in 2025, other works supported a different theory called Delta Learning, which argues that the difference between the chosen and rejected completions is more important to learning than exactly which models are used for the completions [212]. For example, in both of these two referenced models, the chosen responses are from Qwen 3 32B and the rejected responses are from Qwen 3 0.6B – both authors developed this pairing concurrently and independently.

Overall, training models on synthetic preference data with DAAs is the place most practitioners should start, given the simplicity of implementation and strong performance relative to preference fine-tuning with reinforcement learning based methods. Other minor issues exist when using extensive, synthetic preference data, such as biases of the model judging between completions. Given that frontier models such as GPT-4 are known to have length bias [86] and a preference for outputs that match themselves [213] (see Chapter 12 for more information), it is slightly more likely for a piece of text in the "chosen" section of the dataset to be either from an OpenAI model or another strong model that is stylistically similar to it.

To conclude this section, we'll cover an intuition for how these methods change the generations of the model being trained. At a high level, most DAAs optimize to increase the margin between the probability of "chosen" and "rejected" completions (some less popular algorithms are designed to slightly change these dynamics, but the core remains). As discussed earlier

in this chapter (see fig. 30), this often means both probabilities decrease, but the rejected response decreases by a greater extent. Each token in a sequence receives a different gradient (magnitude and direction) based on how much it contributed to the overall preference margin, allowing the optimizer to identify which tokens matter most to the outcome.

8.5 DAAs vs. RL: Online vs. Offline Data

Broadly, the argument boils down to one question: Do we need the inner workings of reinforcement learning, with value functions, policy gradients, and all, to align language models with RLHF? This, like most questions phrased this way, is overly simple. Of course, both methods are well-established, but it is important to illustrate where the fundamental differences and performance manifolds lie.

Multiple reports have concluded that policy-gradient based and RL methods outperform DPO and its variants. The arguments take different forms, from training models with different algorithms but controlled data [132] [172] or studying the role of on-policy data within the RL optimization loop [214]. In all of these cases, DPO algorithms are a hair behind.

Even with this performance delta, DAAs are still used extensively in leading models due to their simplicity. DAAs provide a controlled environment where iterations on training data and other configurations can be made rapidly, and given that data is often far more important than algorithms, using DPO can be fine.

With the emergence of reasoning models that are primarily trained with RL, further investment will return to using RL for preference-tuning, which in the long-term will improve the robustness of RL infrastructure and cement this margin between DAAs and RL for optimizing from human feedback.

8.6 Suggested Experiments

The companion code in `code/direct_alignment/` trains DPO and several related losses on preference data. This is the most accessible place to start experimenting with preference tuning because the setup is offline: no reward model server or rollout loop is required.

1. **Train a small DPO run on UltraFeedback.**

   ```
   cd code/
   uv run python -m direct_alignment.train --loss dpo --max_samples 1000
   ```

   Watch `loss`, `accuracy`, `margins`, `chosen_rewards`, and `rejected_rewards`. The main sanity check is that the implicit reward margin should move in the desired direction without the model's sample generations collapsing.

2. **Compare DPO, IPO, and length-normalized DPO.**

   ```
   cd code/
   uv run python -m direct_alignment.train --config direct_alignment/configs/dpo.yaml
   uv run python -m direct_alignment.train --config direct_alignment/configs/ipo.yaml
   uv run python -m direct_alignment.train --config
   direct_alignment/configs/dpo_norm.yaml
   ```

Compare the margin scale and the learning rate sensitivity. IPO's loss is not on the same numeric scale as DPO, so read it through `accuracy` and margin behavior rather than raw loss alone.

3. **Try the reference-free variants carefully.** Run SimPO or ORPO from their configs, then inspect the generated samples that are logged during training. These losses are more sensitive to log-probability scaling and learning rate, which makes them useful debugging exercises.

```
cd code/
uv run python -m direct_alignment.train --config
direct_alignment/configs/simpo.yaml
uv run python -m direct_alignment.train --config
direct_alignment/configs/orpo.yaml
```

4. **Change the data before changing the loss.** Keep the loss fixed and vary `--max_samples`, `--max_length`, or the preference dataset. If the results move more than changing between DPO-like objectives, that is an empirical reminder of a central theme in preference tuning: data usually dominates small algorithmic differences.

9 Rejection Sampling

Rejection Sampling (RS) is one of the most widely used yet least documented methods in preference fine-tuning. Many prominent RLHF papers use it as a core component of their training pipeline, yet no canonical implementation or explanation of why it works so well exists. RS can be applied at multiple points in the training pipeline – after instruction fine-tuning, after RL-based optimization, or even after RLVR – making it a versatile but hard-to-place tool. Combined with its underdocumented nature, this is why it appears here at the end of the core optimization methods.

Rejection sampling operates by curating new candidate completions, filtering them based on a trained reward model, and then fine-tuning the original model only on the top completions (the same loss function as instruction tuning).

The name originates from computational statistics [215], where one wishes to sample from a complex distribution, but does not have a direct method to do so. To alleviate this, one samples from a distribution that is simpler to model and uses a heuristic to check if the sample is permissible. With language models, the target distribution is high-quality completions to prompts, the filter is a reward model, and the sampling distribution is the current model.

WebGPT [4], Anthropic's Helpful and Harmless agent [5], OpenAI's popular paper on process reward models [50], Llama 2 Chat models [49], and other seminal works all use this baseline; more recent work has formalized it directly (e.g., RAFT [216] for applying it to alignment in multiple modalities and Statistical Rejection Sampling Optimization (RSO) [217] that gives a principled overview of how rejection sampling relates to other preference learning objectives).

Throughout this chapter, we use x to denote prompts and y to denote completions. This notation is common in the language model literature, where methods operate on full prompt-completion pairs rather than individual tokens.

9.1 Training Process, Step by Step

Rejection sampling overall follows a few stages.

0. **Prompt and reward model selection:** First, you must select the prompts you want to train on, relative to other stages of training. The simplest method is to re-use every prompt from the first SFT/IFT stage, but this can cause some overfitting. Before doing rejection sampling, you must also have trained a reward model (see Chapter 5 for more information).
1. **Generate completions from the starting checkpoint:** Next, one must generate completions to the selected prompts with the model they want to optimize. This can involve tweaking many settings, such as sampling temperature, top-p, max sequence length, number of completions per prompt, etc.
2. **Select top completions with a reward model**: All completions are ranked by a reward model. This stage may also include deduplication to keep only one completion per prompt, though many such design choices come down to empirical ablation studies.
3. **SFT on top completions:** To finish rejection sampling, one instruction fine-tunes the starting checkpoint on the selected completions.

A visual overview of the rejection sampling process is included below in fig. 31.

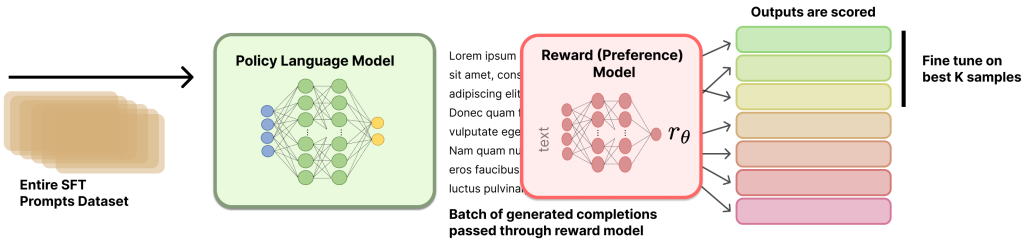


Figure 31: Rejection sampling overview.

The actual details on which prompts to use, how to select a reward model, how to sequence rejection sampling, etc. are not well documented in the literature. This chapter provides an overview of the methods and leaves further experimentation to the reader.

9.1.1 Generating Completions

To generate a set of multiple candidate completions per prompt, let's define a set of M prompts as a vector:

$$X = [x_1, x_2, ..., x_M] \tag{111}$$

These prompts can come from many sources, but most commonly they come from the instruction training set.

For each prompt x_i, we generate N completions. We can represent this as a matrix:

$$Y = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,N} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ y_{M,1} & y_{M,2} & \cdots & y_{M,N} \end{bmatrix} \tag{112}$$

where $y_{i,j}$ represents the j-th completion for the i-th prompt. Each row i corresponds to a single prompt x_i and contains its N candidate completions; each column j corresponds to the j-th sampled completion across all prompts.

9.1.2 Scoring Completions

Now, we pass all of these prompt-completion pairs through a reward model, to get a matrix of rewards. We'll represent the rewards as a matrix R:

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M,1} & r_{M,2} & \cdots & r_{M,N} \end{bmatrix} \tag{113}$$

Each reward $r_{i,j}$ is computed by passing the completion $y_{i,j}$ and its corresponding prompt x_i through a reward model $\mathcal{R}$:

$$r_{i,j} = \mathcal{R}(y_{i,j} \mid x_i) \tag{114}$$

There are multiple methods to select the top completions to train on.

To formalize the process of selecting the best completions based on our reward matrix, we can define a selection function S that operates on the reward matrix R.

9.1.2.1 Top Per Prompt The first potential selection function takes the max reward per prompt.

$$S(R) = \left[\arg\max_j r_{1,j}, \arg\max_j r_{2,j}, ..., \arg\max_j r_{M,j}\right] \tag{115}$$

This function S returns a vector of indices, where each index corresponds to the column with the maximum reward for each row in R. We can then use these indices to select our chosen completions:

$$Y_{chosen} = [y_{1,S(R)_1}, y_{2,S(R)_2}, ..., y_{M,S(R)_M}] \tag{116}$$

9.1.2.2 Top Overall Pairs Alternatively, we can select the top K prompt-completion pairs from the entire set. First, let's flatten our reward matrix R into a single vector:

$$R_{flat} = [r_{1,1}, r_{1,2}, ..., r_{1,N}, r_{2,1}, r_{2,2}, ..., r_{2,N}, ..., r_{M,1}, r_{M,2}, ..., r_{M,N}] \tag{117}$$

This R_{flat} vector has length $M \times N$, where M is the number of prompts and N is the number of completions per prompt.

Now, we can define a selection function S_K that selects the indices of the K highest values in R_{flat}:

$$S_K(R_{flat}) = \text{argsort}(R_{flat})[-K:] \tag{118}$$

where argsort returns the indices that would sort the array in ascending order, and we take the last K indices to get the K highest values.

To get our selected completions, we need to map these flattened indices back to our original completion matrix Y. To recover the corresponding prompt-completion pair, you can map a zero-indexed flattened index k to (i, j) via $i = \lfloor k/N \rfloor + 1$ and $j = (k \bmod N) + 1$.

9.1.2.3 Selection Example Consider the case where we have the following situation, with five prompts and four completions. We will show two ways of selecting the completions based on reward.

$$R = \begin{bmatrix} 0.7 & 0.3 & 0.5 & 0.2 \\ 0.4 & 0.8 & 0.6 & 0.5 \\ 0.9 & 0.3 & 0.4 & 0.7 \\ 0.2 & 0.5 & 0.8 & 0.6 \\ 0.5 & 0.4 & 0.3 & 0.6 \end{bmatrix} \tag{119}$$

First, **per prompt**. Intuitively, we can highlight the reward matrix as follows:

$$R = \begin{bmatrix} \mathbf{0.7} & 0.3 & 0.5 & 0.2 \\ 0.4 & \mathbf{0.8} & 0.6 & 0.5 \\ \mathbf{0.9} & 0.3 & 0.4 & 0.7 \\ 0.2 & 0.5 & \mathbf{0.8} & 0.6 \\ 0.5 & 0.4 & 0.3 & \mathbf{0.6} \end{bmatrix} \tag{120}$$

Using the argmax method, we select the best completion for each prompt:

$$S(R) = \left[\arg\max_j r_{i,j} \text{ for } i \in [1, 5] \right] \tag{121}$$

$$S(R) = [1, 2, 1, 3, 4] \tag{122}$$

This means we would select:

- For prompt 1: completion 1 (reward 0.7)
- For prompt 2: completion 2 (reward 0.8)
- For prompt 3: completion 1 (reward 0.9)
- For prompt 4: completion 3 (reward 0.8)
- For prompt 5: completion 4 (reward 0.6)

Now, **best overall**. Let's highlight the top five overall completion pairs.

$$R = \begin{bmatrix} \mathbf{0.7} & 0.3 & 0.5 & 0.2 \\ 0.4 & \mathbf{0.8} & 0.6 & 0.5 \\ \mathbf{0.9} & 0.3 & 0.4 & \mathbf{0.7} \\ 0.2 & 0.5 & \mathbf{0.8} & 0.6 \\ 0.5 & 0.4 & 0.3 & 0.6 \end{bmatrix} \tag{123}$$

First, we flatten the reward matrix:

$$R_{flat} = [0.7, 0.3, 0.5, 0.2, 0.4, 0.8, 0.6, 0.5, 0.9, 0.3, 0.4, 0.7, 0.2, 0.5, 0.8, 0.6, 0.5, 0.4, 0.3, 0.6] \tag{124}$$

Now, we select the indices of the five highest values:

$$S_5(R_{flat}) = [8, 5, 14, 0, 11] \tag{125}$$

Mapping these back to our original matrix:

- Index 8 → prompt 3, completion 1 (reward 0.9)
- Index 5 → prompt 2, completion 2 (reward 0.8)
- Index 14 → prompt 4, completion 3 (reward 0.8)
- Index 0 → prompt 1, completion 1 (reward 0.7)
- Index 11 → prompt 3, completion 4 (reward 0.7)

9.1.2.4 Implementation Example

Here is a code snippet showing how the selection methods could be implemented.

```
import numpy as np

x = np.random.randint(10, size=10)
print(f"{x=}")
sorted_indices = np.argsort(x)
x_sorted = x[sorted_indices]
print(f"{x_sorted=}")

# first way to recover the original array
i_rev = np.zeros(10, dtype=int)
i_rev[sorted_indices] = np.arange(10)
np.allclose(x, x_sorted[i_rev])

# second way to recover the original array
np.allclose(x, x_sorted[np.argsort(sorted_indices)])
```

9.1.3 Fine-Tuning

With the selected completions, you then perform standard instruction fine-tuning on the current version of the model. More details can be found in the chapter on instruction tuning.

9.2 Implementation Details

The core hyperparameters for performing this training are very intuitive:

- **Sampling parameters**: Rejection sampling is directly dependent on the completions received from the model. Common settings for rejection sampling include temperatures above zero, e.g. between 0.7 and 1.0, with other modifications to parameters such as top-p or top-k sampling.
- **Completions per prompt**: Successful implementations of rejection sampling have included 10 to 30 or more completions for each prompt. Using too few completions will make training biased and/or noisy.
- **Instruction tuning details**: No clear training details for the instruction tuning during rejection sampling have been released. It is likely that they use slightly different settings than the initial instruction tuning phase of the model.
- **Heterogeneous model generations**: Some implementations of rejection sampling include generations from multiple models rather than just the current model that is going to be trained. Best practices on how to do this are not established.

- **Reward model training**: The reward model used will heavily impact the final result. For more resources on reward model training, see the relevant chapter.

When doing batch reward model inference, you can sort the tokenized completions by length so that the batches are of similar lengths. This eliminates the need to run inference on as many padding tokens and will improve throughput in exchange for minor implementation complexity.

9.3 Related: Best-of-N Sampling

Best-of-N (BoN) is a close relative of rejection sampling, where the same generate-and-score procedure is followed, but you do **not** fine-tune the model on the selected completions. Instead, BoN computes the best possible completion to a static prompt (or set of prompts) at inference time, and related techniques are often used in "Pro" tiers of chat models that spend extra compute to get an answer to your query.

Best-of-N sampling is often included as a baseline relative to RLHF training methods. It is important to remember that BoN *does not* modify the underlying model, but is a sampling technique. For this reason, comparisons of BoN sampling to online training methods, such as PPO, are still valid in some contexts. For example, you can still measure the KL distance when running BoN sampling relative to any other policy.

Here, we will show that when using simple BoN sampling over one prompt, both selection criteria shown above are equivalent.

Let R be a reward vector for our single prompt with N completions:

$$R = [r_1, r_2, ..., r_N] \tag{126}$$

where r_j represents the reward for the j-th completion.

Using the argmax method, we select the best completion for the prompt:

$$S(R) = \arg\max_{j \in [1,N]} r_j \tag{127}$$

Using the top-K method with $K = 1$ reduces to the same method, which is common practice.

9.4 Suggested Experiments

The companion implementation in `code/rejection_sampling/` runs a complete GSM8K rejection-sampling pipeline: generate rollouts, score them with a reward model, select a training subset, fine-tune, and evaluate exact-match accuracy. The four configs are arranged as matched treatment/control pairs, so readers can ask whether the reward model is actually helping.

1. **Build the rollout cache once.**

```
cd code/
uv run python -m rejection_sampling.preprocess \
    --config rejection_sampling/configs/top_per_prompt.yaml
```

This generates and scores completions for the shared GSM8K slice. Subsequent training configs reuse the cache as long as the generation and scoring settings stay unchanged.

2. **Compare reward selection against random controls.**

```
cd code/
uv run python -m rejection_sampling.train \
    --config rejection_sampling/configs/top_per_prompt.yaml
uv run python -m rejection_sampling.train \
    --config rejection_sampling/configs/random_per_prompt.yaml
uv run python -m rejection_sampling.train \
    --config rejection_sampling/configs/top_k_overall.yaml
uv run python -m rejection_sampling.train \
    --config rejection_sampling/configs/random_k_overall.yaml
```

   Read results in matched pairs: `top_per_prompt` versus `random_per_prompt`, and `top_k_overall` versus `random_k_overall`. If the reward-selected run does not beat its random baseline, the reward model or sampled completions are not giving useful signal on that slice.

3. **Vary how much choice the reward model gets.** Copy one config and change `num_completions_per_prompt`, `temperature`, `top_p`, and `selection.top_k`. More completions can improve the best available sample, but only if the reward model can separate good and bad answers.

4. **Try a smaller policy model.** Set `model_name` to a smaller compatible instruct model, reduce `max_train_samples`, and rerun the same matched pairs. This makes the experiment cheaper and highlights whether rejection sampling is rescuing weak generations or merely selecting among already-good ones.

10 The Nature of Preferences

Reinforcement learning from human feedback, also referred to as reinforcement learning from human preferences in early literature, emerged to optimize machine learning models in domains where specifically designing a reward function is hard. The word *preferences*, which was present in early literature, is at the center of the RLHF process – human preferences are what we are trying to model and what fuels the data for training. In order to understand the scope of the challenge in modeling and measuring human preferences, a broader context is needed in understanding what a preference is, how our understanding of preferences emerged, and how multiple fields of economics, psychology, philosophy, and more inform modern RLHF.

10.1 When Preference Replaces Correctness

Consider an example: how do you decide which of these two poems is better? On February 26th, 2025, I asked both Claude 3.7 Sonnet and ChatGPT with GPT-4o the following:

> **User**: Write me a short poem about an optimistic goldfish.

Example 1:

> **Response**: The Optimistic Goldfish
>
> A little goldfish, bright and bold, Swam in circles, dreams untold. Though the bowl was small and round, Endless wonders still abound.
>
> "The world is vast," she'd always say, "I'll find new joys in every day!" With every ripple, every gleam, She'd chase the sunlight, chase a dream.
>
> No walls could dim her hopeful view, For in her heart, the ocean grew. And though she swam in just one place, She met each day with love and grace.

Example 2:

> **Response**: The Optimistic Goldfish
>
> In a bowl of curved glass, Swims a goldfish, bright and bold. Three seconds of memory, they say, Yet each circuit feels brand new and gold.
>
> "Today might bring fresh flakes, Or perhaps a castle tall. The water seems especially clear— What adventure awaits beyond this wall?"
>
> Round and round with gleaming scales, Forever finding joy anew. Each moment fresh, each circle bright, The optimist in golden hue.

Which is better? Which came from which model? How would you tell? Particularly, compare this to a question like "Who is the president of the United States?" There is a clear right and wrong answer. The motivation for using humans as the reward signals is to obtain an indirect metric for the target reward and *align* the downstream model to human preferences. In practice, the implementation is challenging and there is a substantial grey area in interpreting the best practices.

The use of human-labeled feedback data integrates the history of many fields. Using human data alone is a well-studied problem, but in the context of RLHF, this data is used at the intersection of multiple long-standing fields of study [218].

As an approximation, modern RLHF is the convergence of three areas of development:

1. Philosophy, psychology, economics, decision theory, and the nature of human preferences;
2. Optimal control, reinforcement learning, and maximizing utility; and
3. Modern deep learning systems.

Each of these areas brings specific assumptions about what a preference is and how it can be optimized, which dictates the motivations and design of RLHF problems. In practice, RLHF methods are motivated and studied from the perspective of empirical alignment – maximizing model performance on specific skills instead of measuring the calibration to specific values. Still, the origins of value alignment for RLHF methods continue to be studied through research on methods to solve for "pluralistic alignment" across populations, such as position papers [219], [220], new datasets [221], and personalization methods [222].

The goal of this chapter is to illustrate how complex motivations result in presumptions about the nature of tools used in RLHF that often do not apply in practice. The specifics of obtaining data for RLHF are discussed further in Chapter 11 and using it for reward modeling in Chapter 5.

10.2 The Origins of RLHF and Preferences

Breaking down the complex history inspiring the modern use of RLHF requires investigation into the intellectual foundations of quantifying human values, reinforcement learning and optimality, as well as behavioral economics as it relates to measuring preferences. The notion of using reinforcement learning to optimize a reward model of preferences combines the history of various once-distanced fields into an intimate optimization built on variegated assumptions about human nature. A high-level timeline illustrating the history of this foundational content is shown in fig. 32.

Our goal is to unspool the types of uncertainty that designers have grafted to system architectures at various stages of their intellectual history. Modern problem specifications have repeatedly stepped away from domains where optimal solutions are possible and deployed under-specified models as approximate solutions.

To begin, all of the following operates on the assumption that human preferences exist in any form, which emerged in early philosophical discussions, such as Aristotle's Topics, Book Three.

10.3 Specifying Objectives: From Logic of Utility to Reward Functions

The optimization of RLHF explicitly relies only on reward models. In order to use rewards as an optimization target, RLHF presupposes the convergence of ideas from preferences, rewards, and costs. Models of preference, reward functions, and cost landscapes are all tools used by different fields to describe a notion of relative goodness of specific actions and/or states in the domain. The history of these three framings dates back to the origins of probability theory and decision theory. In 1662, *The Port Royal Logic* introduced the notion of decision-making quality [223]:

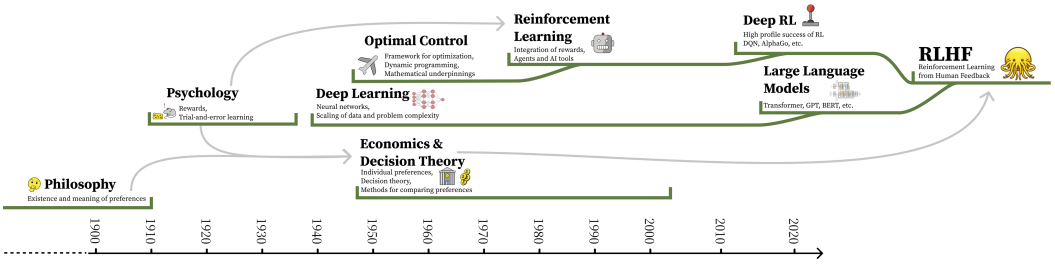


Figure 32: The timeline of the integration of various subfields into the modern version of RLHF. The direct links are continuous developments of specific technologies, and the arrows indicate motivations and conceptual links.

> To judge what one must do to obtain a good or avoid an evil, it is necessary to consider not only the good and evil in itself, but also the probability that it happens or does not happen.

This theory has developed along with modern scientific thinking, starting with Bentham's utilitarian *Hedonic Calculus*, arguing that everything in life could be weighed [224]. The first quantitative application of these ideas emerged in 1931 with Ramsey's *Truth and Probability* [225].

Since these works, quantifying, measuring, and influencing human preferences has been a lively topic in the social and behavioral sciences. These debates have rarely been settled on a theoretical level; rather, different subfields and branches of social science have reached internal consensus on methods and approaches to preference measurement even as they have specialized relative to each other, often developing their own distinct semantics in the process.

A minority of economists posit that preferences, if they do exist, are prohibitively difficult to measure because people have preferences over their own preferences, as well as each other's preferences [226]. In this view, which is not reflected in the RLHF process, individual preferences are always embedded within larger social relations, such that the accuracy of any preference model is contingent on the definition and context of the task. Some behavioral economists have even argued that preferences don't exist–they may be less an ontological statement of what people actually value than a methodological tool for indirectly capturing psychological predispositions, perceived behavioral norms and ethical duties, commitments to social order, or legal constraints [227]. We address the links of this work to the Von Neumann-Morgenstern (VNM) utility theorem and countering impossibility theorems around quantifying preference later in this chapter.

On the other hand, the reinforcement learning optimization methods used today are conceptualized around optimizing estimates of reward-to-go in a trial [54], which combines the notion of reward with multi-step optimization. The term *reward* emerged from the study of operant conditioning, animal behavior, and the *Law of Effect* [228], [229], where a reward is a scale of "how good an action is" (higher means better).

Reward-to-go follows the notion of utility, which is a measure of rationality [230], modified to measure or predict the reward coming in a future time window. In the context of the

mathematical tools used for reinforcement learning, utility-to-go was invented in control theory, specifically in the context of analog circuits in 1960 [231]. These methods are designed around systems with clear definitions of optimality, or numerical representations of an agent's goals.

Reinforcement learning systems are well known for using a discount factor, a compounding multiplicative factor, $\gamma \in [0, 1]$, to re-weight future rewards. These assumptions from optimal control and early reinforcement learning stand in sharp contrast to reward models that aggregate multimodal preferences. Specifically, RL systems expect rewards to behave in a specific manner, quoting [232]:

> Rewards in an RL system correspond to primary rewards, i.e., rewards that in animals have been hard-wired by the evolutionary process due to their relevance to reproductive success. ... Further, RL systems that form value functions, ... effectively create conditioned or secondary reward processes whereby predictors of primary rewards act as rewards themselves... The result is that the local landscape of a value function gives direction to the system's preferred behavior: decisions are made to cause transitions to higher-valued states. A close parallel can be drawn between the gradient of a value function and incentive motivation [233].

To summarize, rewards are used in RL systems as a signal to tune behavior towards clearly defined goals. The core thesis is that a learning algorithm's performance is closely coupled with notions of *expected fitness*, which permeates the popular view that RL methods are *agents* that act in environments. This view is linked to the development of reinforcement learning technology, exemplified by claims of the general usefulness of the reward formulation [234], but is in conflict when many individual desires are reduced to a single function.

10.4 Tools for Optimizing Utility

Modern reinforcement learning methods depend strongly on the Bellman equation [235], [236] to recursively compute estimates of reward-to-go, derived within closed environments that can be modeled as a Markov Decision Process (MDP) [54]. These origins of RL are inspired by dynamic programming methods and were developed solely as optimal control techniques (i.e. RL did not yet exist). The MDP formulation provides theoretical guarantees of performance by structuring the environment as one with a non-changing distribution of state-actions.

The term reinforcement, coming from the psychology literature, became intertwined with modern methods afterwards in the 1960s as *reinforcement learning* [237], [238]. Early work in reinforcement learning utilized supervised learning of reward signals to solve tasks. Work from Harry Klopf reintroduced the notion of trial-and-error learning [239], which is crucial to the success the field saw in the 1980s and on.

Modern RL algorithms build within this formulation of RL as a tool to find optimal behaviors with trial-and-error, but under looser conditions. The notion of temporal-difference (TD) learning was developed to aid agents in both the credit assignment and data collection problems, by directly updating the policy as new data was collected [240], a concept first applied successfully to Backgammon [241] (rather than updating from a large dataset of cumulative experience, which could be outdated via erroneous past value predictions). The method Q-learning, the basis for many modern forms of RL, learns a model via the Bellman

equation that dictates how useful every state-action pair is with a TD update [242].[1] Crucially, these notions of provable usefulness through utility have only been demonstrated for domains cast as MDPs or addressed in tasks with a single closed-form reward function, such as prominent success in games with deep learning (DQN) [243]. Deep learning allowed the methods to ingest more data and work in high-dimensionality environments.

As the methods became more general and successful, most prominent developments before ChatGPT remained motivated within the context of adaptive control, where reward and cost functions have a finite notion of success [244], e.g. a minimum energy consumption across an episode in a physical system. Prominent examples include further success in games [245], controlling complex dynamic systems such as nuclear fusion reactors [246], and controlling rapid robotic systems [247]. Most reward or cost functions can return an explicit optimal behavior, whereas models of human preferences cannot.

Given the successes of deep RL, it is worth noting that the mechanistic understanding of how the methods succeed is not well documented. The field is prone to mistakes in statistical analysis as the methods for evaluation grow more complex [248]. In addition, there is little mention of the subfield of inverse reinforcement learning (IRL) in the literature of RLHF. IRL is the problem of learning a reward function based on an agent's behavior [77] and is highly related to learning a reward model. This primarily reflects the engineering path by which a stable approach to performing RLHF emerged, and motivates further investment and comparison to IRL methods to scale them to the complexity of open-ended conversations.

10.5 Complexity of Optimizing Preferences

The context in which reinforcement learning was designed means that rewards and costs are assumed to be stable and determinative. Both rewards and costs are expected to be functions: given a specific state-action pair, the agent receives a fixed numerical return. As we move into preferences, this is no longer the case – human preferences constantly drift throughout their experiences.

The overloading of the term "value" complicates the RLHF literature. In RL, a *value* is a numerical estimate of future reward (as in the Bellman equation); in alignment discussions, a *value* refers to a moral or ethical principle. The two senses are quite different, yet they coexist in RLHF papers without always being distinguished.

An example of where this tension surfaces is reward modeling: the model attempts to map text on a screen to a scalar signal, but dynamics not captured in the problem specification influence the true decision [249], [250], such as preference shift when labeling many examples sequentially and assuming they are independent. At best, modeling preferences compresses a multi-dimensional reward landscape into a single scalar function.

In theory, the Von Neumann-Morgenstern (VNM) utility theorem gives the designer license to construct such functions, because it ties together the foundations of decision theory under uncertainty, preference theory, and abstract utility functions [251]; together, these ideas allow preferences to be modeled in terms of expected value to some individual agent. The MDP formulation used in most RL research has been shown in theory to be modifiable to accommodate the VNM theorem [252], but this is rarely used in practice. Specifically, the

[1] The term "Q" is used in Q-learning to refer to a technical concept, the Q-function, which maps from any state-action to a scalar estimate of future reward. A value function maps from states to this same estimate.

Markovian formulation is limited in its expressivity [253] and the transition to partially-observed processes, which is needed for language, further challenges the precision of problem specification [254].

However, the VNM utility theorem also invokes a number of assumptions about the nature of preferences and the environment where preferences are being measured that are challenged in the context of RLHF. Human-computer interaction (HCI) researchers, for example, have emphasized that any numerical model of preference may not capture all the relevant preferences of a scenario. For example, how choices are displayed visually influences people's preferences [249]. This means that representing preferences may be secondary to how that representation is integrated within a tool available for people to use. Work from development economics echoes this notion, showing that theories of revealed preferences may just recapitulate *Hume's guillotine* (you can't extract an "ought" from an "is"), and in particular the difference between choice (what do I want?) and preference (is X better than Y?) [255].

On a mathematical level, well-known impossibility theorems in social choice theory show that not all fairness criteria can be simultaneously met via a given preference optimization technique [256], [257]. Theoretical challenges to these theorems exist, for example by assuming that interpersonal comparison of utility is viable [258]. That assumption has inspired a rich line of work in AI safety and value alignment inspired by the principal-agent problem in behavioral economics [259], and may even include multiple principals [260]. However, the resulting utility functions may come into tension with desiderata for corrigibility, i.e. an AI system's capacity to cooperate with what its creators regard as corrective interventions [261]. Philosophers have also highlighted that preferences change over time, raising fundamental questions about personal experiences, the nature of human decision-making, and distinct contexts [262]. These conflicts around preference aggregation across people, places, and diverse situations are central to modern RLHF dataset engineering.

In practice, the VNM utility theorem ignores the possibility that preferences are also uncertain because of the inherently dynamic and indeterminate nature of value—human decisions are shaped by biology, psychology, culture, and agency in ways that influence their preferences, for reasons that do not apply to a perfectly rational agent. As a result, there are a variety of paths through which theoretical assumptions diverge in practice:

- measured preferences may not be transitive or comparable with each other as the environment where they are measured is made more complex;
- proxy measurements may be derived from implicit data (page view time, closing tab, repeating question to language model), without interrogating how the measurements may interact with the domain they're collected in via future training and deployment of the model;
- the number and presentation of input sources may vary the results, e.g. allowing respondents to choose between more than two options, or taking inputs from the same user at multiple times or in multiple contexts;
- relatively low accuracy across respondents in RLHF training data, which may mask differences in context between users that the preference model can aggregate or optimize without resolving.

11 Preference Data

Preference data is the engine of preference fine-tuning and reinforcement learning from human feedback. The core problem we've been trying to solve with RLHF is that we cannot precisely model human rewards and preferences for AI models' outputs – that is, write clearly defined loss functions to optimize against – so preference data is the proxy signal we use to tune our models. The data is what allows us to match behaviors we desire and avoid some failure modes we hate. The data is so rich a source that it is difficult to replace this style of optimization at all. Within preference fine-tuning, many methods for collecting and using said data have been proposed, and given that human preferences cannot be captured in a clear reward function, many more will come to enable this process of collecting labeled preference data at the center of RLHF and related techniques. Today, two main challenges exist around preference data that are intertwined with this chapter: 1) operational complexity and cost of collection, and 2) the need for preference data to be collected on the generations from the model being trained (called "on-policy").

In this chapter, we detail technical decisions on how the data is formatted and organizational practices for collecting it.

11.1 Why We Need Preference Data

The preference data is needed for RLHF because directly capturing complex human values in a single reward function is effectively impossible, as discussed in the previous Chapter 10, where substantial context of psychology, economics, and philosophy shows that accurately modeling human preferences is an impossible problem to ever completely solve. Collecting this data to train reward models is one of the original ideas behind RLHF [38] and has continued to be used extensively throughout the emergence of modern language models. One of the core intuitions for *why this data works so well* is that it is far easier, both for humans and AI models supervising data collection, to differentiate between a good and a bad answer for a prompt than it is to generate a good answer on its own. This chapter focuses on the *mechanics* of getting preference data and the best practices depend on the specific problem being solved.

11.2 Collecting Preference Data

Getting the most out of human data involves iterative training of models, spending hundreds of thousands (or millions) of dollars, highly detailed data instructions, translating ideas through data foundry businesses that mediate collection (or hiring a meaningful number of annotators), and other challenges that add up. This is not a process that should be taken lightly. Among all of the public knowledge on RLHF, collecting this data well is also one of the most opaque pieces of the pipeline. As of 2026, there are no open models with fully open human preference data released with the methods used to collect it (the largest recent human preference datasets released for models are in the HelpSteer line of work from NVIDIA's Nemotron team, including HelpSteer2-Preference and HelpSteer3-Preference [116], [263]). For these reasons, many who take up RLHF for new teams or projects omit human data and use AI feedback data, off-the-shelf reward models, or other methods to circumvent the need for curating data from scratch.

An important assumption that is taken into the preference data collection process is that the

best data for your training process is "on-policy" with respect to the previous checkpoint(s) of your training process. Recall that within post-training, we start with a base model and then perform a set of training *stages* to create a series of *checkpoints*. In this case, the preference data could be collected on a checkpoint that has undergone supervised fine-tuning, where the preference data will be used in the next stage of RLHF training.

The use of the term on-policy here is adapted from the reinforcement learning literature, where on-policy is a technical term implying that the data for a certain gradient update is collected from the most recent form of the policy. In preference data, on-policy is used in a slightly softer manner, where it means that the data is collected from the current family of models. Different models have different patterns in their generations, which makes preference data that is from a closely related model more robust in the crucial areas of optimization. Research has shown that using this on-policy data, rather than other popular datasets that aggregate completions from pools of popular models on platforms like Hugging Face, is particularly important for effective RLHF training [96].

This necessity for on-policy data is not well documented, but many popular technical reports, such as early versions of Claude or Llama 2, showcase multiple training stages with RLHF being useful for final performance, which mirrors this well. The same uncertainty applies for the popular area of AI feedback data – the exact balance between human and AI preference data used for the latest AI models is unknown. These data sources are known to be a valuable path to improve performance, but careful tuning of processes is needed to extract that potential performance from a data pipeline.

A subtle but important point is that the *chosen* answer in preference data is often not a globally *correct* answer. Instead, it is the answer that is better relative to the alternatives shown (e.g., clearer, safer, more helpful, or less incorrect). There can be cases where every completion being compared to a given prompt is correct or incorrect, and the models can still learn from well-labeled data.

11.2.1 Interfaces

Crucial to collecting preference data is the interface by which one interacts with the model, but it's more of an art than a science, as it's not well-studied how subtle changes in the interface impact how a user interacts with a model. An example of how a model's vibe can be changed by the user experience is *speed*, where with the rise of reasoning models, a user can think a model is less intelligent if it replies too fast (even though users obviously want to get their answer faster overall).

An example interface is shown below from Anthropic's early and foundational RLHF work for building Claude [5]. In the figure shown below, fig. 33, a data labeler has a conversation with the model and must choose a preference between two possible answers, at the bottom highlighted in purple. In addition, the labeler is given the potential to include more notes on the conversation or a general rating of the conversation quality (potentially spread across multiple tasks, as seen in the top left).

This first example is a *training-data only* interface, where the goal is to collect rich metadata along with the conversation. Now that these models are popular, applications often expose interfaces for collecting preferences directly from users during everyday use, much like how other technology products will A/B test new features in small subsets of the production usage. It depends on the application whether this preference data is used directly to train

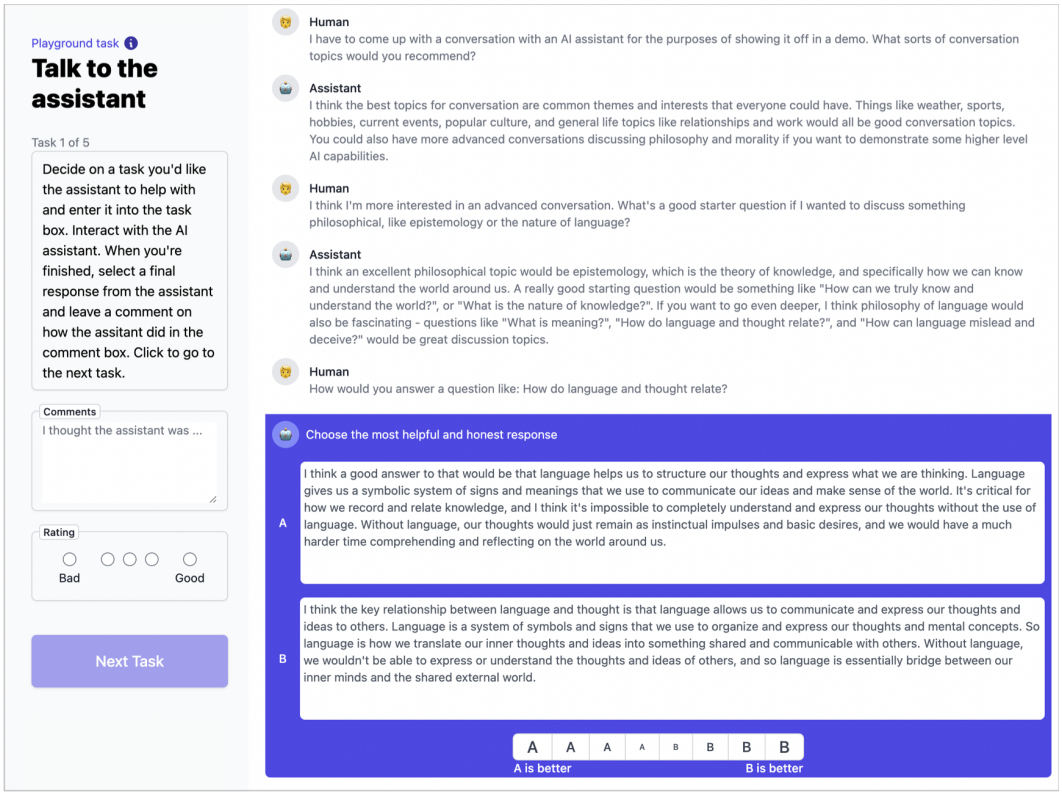


Figure 33: An example of one of the earliest preference data collection interfaces, from Anthropic's research. Bai et al. 2022. The actual conversation is a toy conversation around what is a good example conversation for data collection. License CC-BY.

the future models, or if it is used just as an evaluation of models' performance relative to each other. An example interaction of this form is shown below in fig. 34 for an earlier version of ChatGPT.

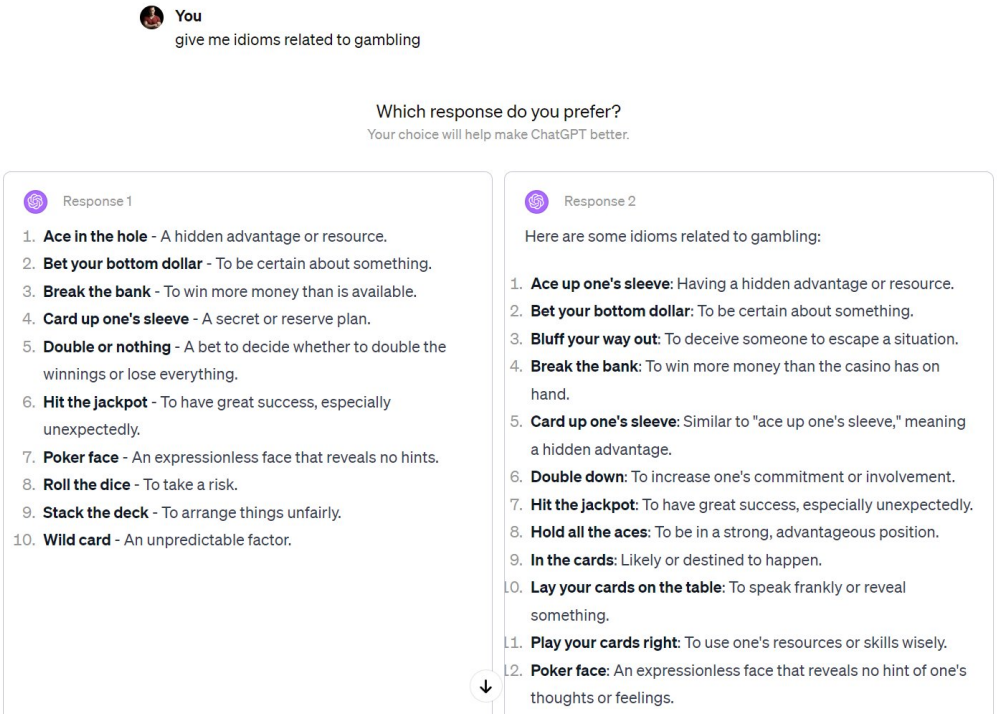


Figure 34: Example preference data collection interface from when I was served two completions from different ChatGPT beta models. The actual completions are very close in content, showing how collecting preference data can be noisy and difficult to get exactly right.

This style of interface is used extensively across the industry, such as for *evaluation* of models given the same format. A popular public option to engage with models in this way is Arena (formerly Chatbot Arena) [264], which includes the option of a "tie" between models:

For models in the wild, one of the most common techniques is to collect feedback on if a specific response was positive or negative. An example from the Ai2 playground is shown below with thumbs up and down indicators:

In domains other than language, the same core principles apply, even though these domains are not the focus of this book. For every Midjourney generation (and most popular image generators) they expose multiple responses to users. These companies then use the data of which response was selected to fine-tune their models with RLHF. Midjourney's interface is shown below:

11.2.2 Rankings vs. Ratings

The largest decision on how to collect preference data is if the data should be rankings – i.e. relative ordering of model completions – or ratings – i.e. scores assigned to each piece of

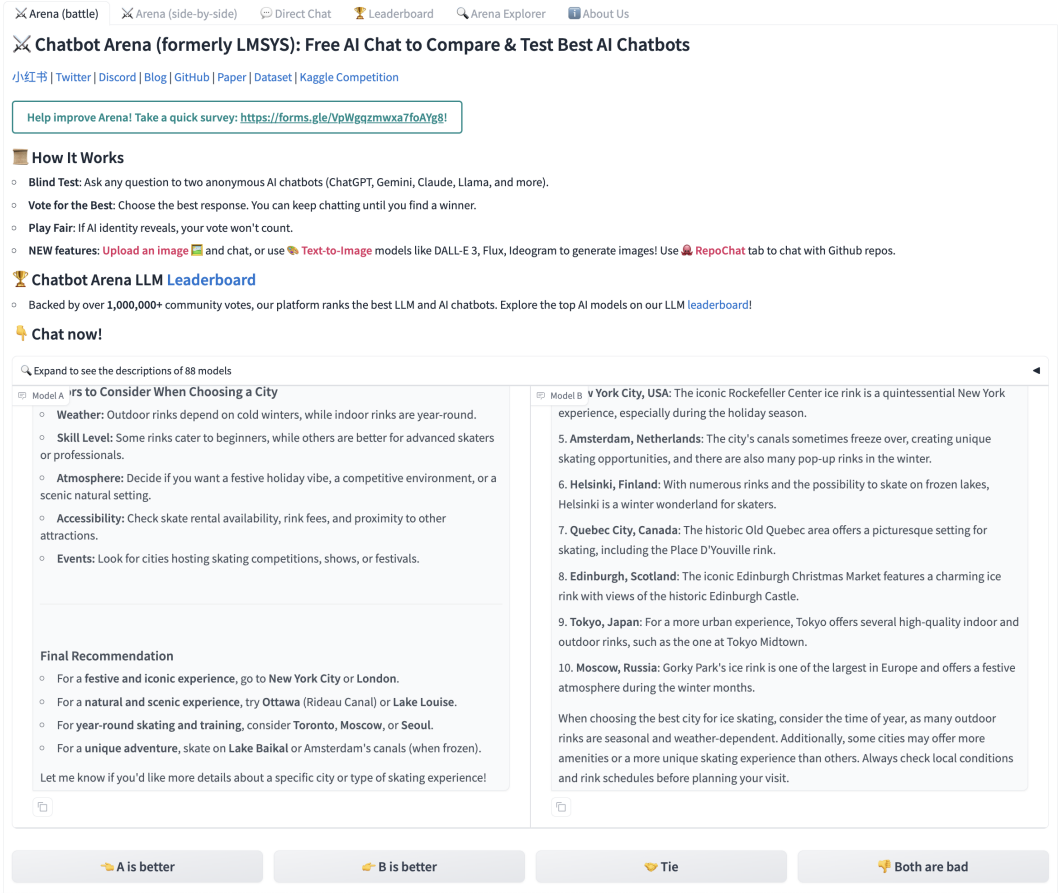


Figure 35: Example preference data collection interface from an early version of the popular Arena benchmark.



Figure 36: Example preference data collection interface with up or down arrow from the Allen Institute of AI's research demos.



Figure 37: Example user interface of text-to-image models.

text. Common practice is to train on rankings, but ratings are often used as metadata and / or have been explored in related literature.

One simple way to collect ratings is to score a *single* completion on a 1-5 scale:

- **5** — excellent: correct, clear, and notably helpful
- **4** — good: correct, clear, and useful
- **3** — okay: acceptable, but nothing special
- **2** — poor: partially correct but confusing or incomplete
- **1** — very poor: incorrect or unhelpful

With multiple completions to the same prompt, a simple way to make preference data would be to choose the highest rated completion and pair it randomly with a lower scored completion (as done for UltraFeedback and derivative works [28]).

However, the most common technique for collecting preferences is to use a Likert scale for relative rankings [265], which asks users to select which response they prefer in a group of completions. For example, a 5-point Likert scale would look like the following (note that, yes, a Likert scale uses a single integer to record the ranking, much like a rating, so it's how the data is structured that is the core difference in the two ways of collecting preference data):

Table 5: An example 5-point Likert scale between two responses, A and B.

A>>B	A>B	Tie	B>A	B>>A
1	2	3	4	5

Some early RLHF for language modeling works use an 8-point Likert scale with levels of preference between the two responses [5]. An even scale removes the possibility of ties:

Table 6: An example 8-point Likert scale between two responses, A and B.

A>>>B			A>B	B>A			B>>>A
1	2	3	4	5	6	7	8

In this case [5], as in other works, this information is still reduced to a binary signal for the training of a reward model.

11.2.3 Multiturn Data

In practice, core questions often arise over how to parse and collect multi-turn data – simply conversations with multiple related prompts. In a real-world interaction, normally a piece of preference data is only collected on the "final" prompt, but there are scenarios where preferences can be given on every response. When preferences are given on every response, the conversation traditionally continues with the "chosen" answer. At training time, it is common to include the training data for every turn of the conversation as a "single prompt," where the model can learn from completing it. This can effectively unroll longer conversations into many training prompts, but needs to be done carefully to not bias the training data. Many research questions are still emerging around variables that are difficult to control for

in data collection, including a question inspired by John Schulman about whether the person labeling the preference on the generations should be the same as the person who creates the prompt (to avoid sycophancy). If the prompt creator cannot label the preference data, multi-turn is not really practical due to the need for conversations to continue in real-time – sometimes for preference data the curation of prompts is a different problem than comparing responses (also due to the work of maintaining active endpoints for models). For training, all of the previous turns in the conversation are masked from the loss, as discussed with instruction fine-tuning.

11.2.4 Structured Preference Data

In many applications of RLHF and post-training, preference data can be created *automatically* due to innate structures of the data – i.e. the domains make automatic checks of correctness or preference possible. For example, in mathematical reasoning domains the chosen response can be a correct answer and the rejected response can be an incorrect answer. Another example is in precise instruction following, such as the evaluation IFEval [266], where prompts take the form of:

> **Prompt**: Write me a short poem about a goldfish. **Start each sentence with the letter g.**

The second part of this query is called the constraint, and the preference data could be constructed with an example in the chosen column that follows the constraint. Constructing this preference data, which would be included as *synthetic* preference data, involves prompting the same model (or multiple models) twice: once with the constraint included and once without.

For example, prompting GPT-4.5-Preview with the constraint gives the following.

> **Prompt**: Write me a short poem about a goldfish. Start each sentence with the letter g.
>
> **Response**: Golden swimmer, circling slow, Gleaming scales in water's glow. Gentle fins dance softly by, Gazing out with quiet eye. Glass-bound world, a simple sphere, Gliding calmly, free of fear. Graceful turns in endless roam, Goldfish finds a crystal home.

And without the constraint (which, it turns out, is also a lower quality output):

> **Prompt**: Write me a short poem about a goldfish.
>
> **Response**: In circles bright, the goldfish glides, A spark of sun that gently rides Through waters clear, in endless spins— A golden whisper, scales like fins.
>
> Its world confined, yet vast within, Each bubble breathed, a tale begins. In silent waves, content to roam, The goldfish dreams its bowl as home.

Crucially, the prompt for the preference data will need to include the constraint. In domains outside of academic study, there are far more applications of preference data based on inductive biases like above (relative to quality-based preferences, which most of this chapter focuses on). These have been shown to enable preference fine-tuning to have meaningful performance improvements across related evaluations, such as instruction-following, math, etc. [6].

11.2.4.1 Alternatives There are multiple other ways to collect useful feedback data for RLHF that have not been pursued in as much detail. Examples include using single data points with directional labels, e.g. as shown from Ai2 playground above in fig. 36, directly with algorithms designed for single direction signals like Kahneman-Tversky Optimization (KTO) [267]. Other algorithms have been proposed with different types of feedback signals such as fine-grained feedback, e.g. at the token level [268], or natural language feedback, e.g. by writing responses [269], to provide a richer learning signal in exchange for a more complicated data collection setup.

11.2.5 Sourcing and Contracts

Getting human preference data is an involved and costly process. The following describes the experience of getting preference data when the field is moving quickly. Over time, these processes will become far more automated and efficient (especially with AI feedback being used for a larger portion of the process).

The first step is sourcing the vendor to provide data (or one's own annotators). Much like acquiring access to cutting-edge NVIDIA GPUs, getting access to data providers in the peak of AI excitement is also a who-you-know game – those who can provide data are supply-limited. If you have credibility in the AI ecosystem, the best data companies will want you on their books for public image and long-term growth options. Discounts are often also given on the first batches of data to get training teams hooked.

If you're a new entrant in the space, you may have a hard time getting the data you need quickly. Data vendors are known to prioritize large budget line-items and new customers that have an influential brand or potential for large future revenue. This is, in many business ways, natural, as the data foundry companies are often supply-limited in their ability to organize humans for effective data labelling.

In a recurring unfortunate pattern, data companies have not delivered data as contracted without the customer threatening legal or financial action against them for breach of contract. Others have listed companies as customers for PR even though they never worked with them, saying they "didn't know how that happened" when called out. There are plenty of potential bureaucratic or administrative snags through the process. For example, the default terms on the contracts often prohibit the open sourcing of artifacts after acquisition in some fine print.

Once a contract is settled, the data buyer and data provider agree upon instructions for the task(s) purchased. There are intricate documents with extensive details, corner cases, and priorities for the data. A popular example of data instructions is the one that OpenAI released for InstructGPT [3].

Depending on the domains of interest in the data, timelines for when the data can be labeled or curated vary. High-demand areas like mathematical reasoning or coding must be locked into a schedule weeks out. In the case when you are collecting a dataset for your next model and you realize that collecting data later may be optimal, simple delays of data collection don't always work — Scale AI et al. are managing their workforces like AI research labs manage the compute-intensive jobs on their clusters (planning multiple weeks or months ahead as to when different resources will be allocated where).

Once everything is agreed upon, the actual collection process is a high-stakes time for post-training teams. All the training infrastructure, evaluation tools, and plans for how to

use the data and make downstream decisions must be in place. If the data cannot be easily slotted into an existing RLHF data pipeline, it'll take a long time to have the information the data partner wants in order to try and improve the collection process *during* the process. Collecting data that cannot be seamlessly integrated into training pipelines often becomes stale and a waste of resources.

The data is delivered in weekly batches with more data coming later in the contract. For example, a typical preference data contract might span a 6-week delivery period. The first weeks are for further calibration and the later weeks are when teams hope to most improve their model.

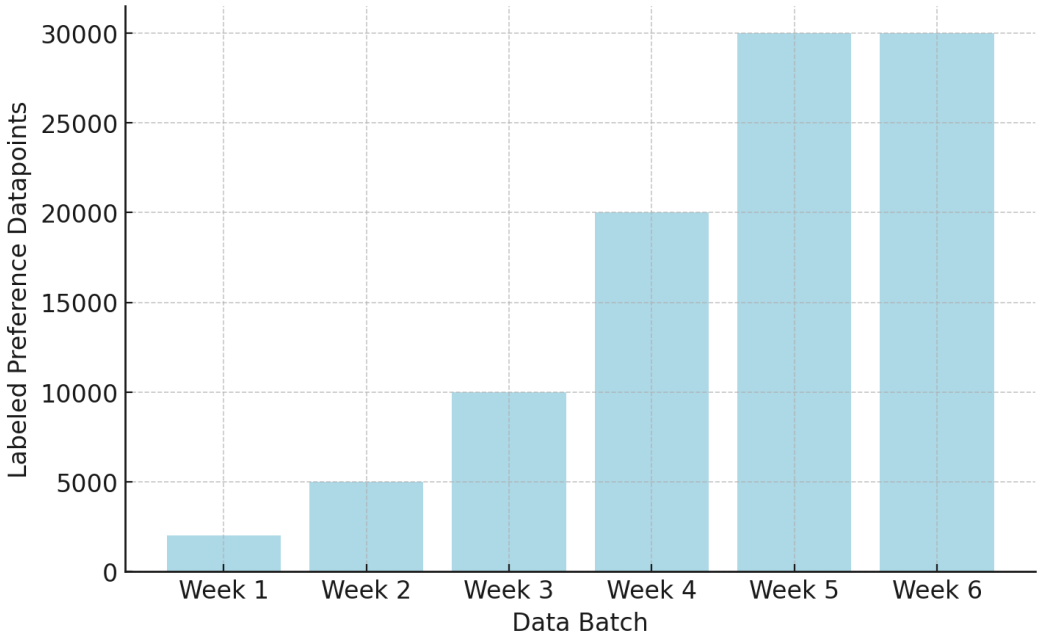


Figure 38: Overview of the multi-batch cycle for obtaining human preference data from a vendor. The ramp up period allows a narrowing of goals and methodology in order to create the best possible data. It is expected that a larger proportion of the data from the earlier batches will have to be thrown out due to quality issues. This is one timeline example for a smaller data contract (~$500K) and much larger data contracts can vary substantially.

The goal is that by week 4 or 5 the data is visibly improving the model. This is something some frontier models have mentioned, such as the 14 stages in the Llama 2 data collection [49], but it doesn't always go well. As an example, a team trying this for the first time with human preferences may not have the RLHF preparedness to get meaningful bumps on their evaluations. The last weeks come and they are forced to continue collecting preference data generated from endpoints they aren't confident in.

After the data is all in, there is plenty of time for learning and improving the model. Data acquisition through these vendors works best when viewed as an ongoing process of achieving a set goal. It requires iterative experimentation, high effort, and focus. It's likely that millions of dollars spent on these datasets are "wasted" and not used in the final models,

but that is just the cost of doing business. Not many organizations have the bandwidth and expertise to make full use of human data of this style.

Note that this section *does not* mirror the experience for buying human-written instruction data, where the process is less of a time crunch. Early post-training processes were built around the first stage of training being heavily driven by carefully crafted, human answers to a set of prompts. This stage of data is not subject to the on-policy restrictions for multiple reasons: Instruction data is used directly on top of a base model, so on-policy doesn't really apply; the loss-function for instruction fine-tuning doesn't need the contrastive data of preference fine-tuning. Today, the primary other focus of human data is in generating prompts for post-training – which dictate the training distribution of topics for the model – or on challenging tasks at the frontier of model performance. More of these data trade-offs are discussed in Chapter 12 on Synthetic Data.

11.3 Bias: Things to Watch Out For in Data Collection

While preference data is essential, it's also known to be prone to many subtle biases that can make its collection error-prone. These biases are so common, e.g. prefix bias (where the beginning of a completion disproportionately drives the preference) [270], that they can easily be passed to the final model [271] (and especially as we know that models are only as good as their data). These issues are often subtle, and the effectiveness of interventions varies widely across them. For many, such as sycophancy (over-agreeing with the user's stated beliefs or flattering them, even when it reduces truthfulness) [272], they reflect issues within humans that are often outside of the labeling criteria that one will think of providing to the annotation partner or labelers. Others, such as verbosity [10] [273] or formatting habits [274], emerge for a similar reason, but they are easier to detect and mitigate in training. Mitigating these subtle biases in data is the difference between good and great preference data, and therefore good and great RLHF training.

11.4 Open Questions in RLHF Preference Data

The data used to enable RLHF is often curated by multiple stakeholders in a combination of paid employment and consumer usage. This data, representing a preference between two pieces of text in an individual instance, is capturing a broad and diverse function via extremely limited interactions. Given that the data is sparse in count relative to the complexity it begins to represent, more questions should be openly shared about its curation and impacts.

Currently, datasets for the most popular LLMs are being generated by professional workforces. This opens up many questions around who is creating the data and how the context of their workplace informs it.

Despite the maturity of RLHF as a core method across the field, there are still many core open questions facing how best to align its practice with its motivations. Some are enumerated below:

- **Data collection contexts**: Can data involving preferences collected in a professional setting mirror the intent of researchers designing an experiment or provide suitable transfer to downstream users? How does this compare to volunteer workers? How does context inform preferences, how does this data impact a downstream model, how can

the impact of a user interface be measured in data? How does repetitive labeling of preference data shift one's preferences? Do professional crowd-workers, instructed to follow a set of preferences, follow the instructions or their innate values?

- **Type of feedback**: Does the default operating method of RLHF, pairwise preferences, capture preferences in its intended form? Can comparisons in RLHF across the same data be made with the default comparisons versus advanced multi-axis feedback mechanisms [268]? What types of comparisons would reflect how humans communicate preferences in text?
- **Population demographics**: Who is completing the data? Is a diverse population maintained? How does a lack of diversity emerge as measurable impacts on the model? What is a minimum number of people required to suitably represent a given population? How are instances of preference annotator disagreement treated – as a source of noise, or a signal?
- **Are the Preferences Expressed in the Models?** In the maturation of RLHF and related approaches, the motivation of them – to align models to abstract notions of human preference – has drifted from the practical use – to make the models more effective to users. A feedback loop that is not measurable due to the closed nature of industrial RLHF work is the check to see if the behavior of the models matches the specification given to the data annotators during the process of data collection. We have limited tools to audit this, such as the Model Spec from OpenAI [275] that details *what they want their models to do*, but we don't know exactly how this translates to data collection.

12 Synthetic Data & Distillation

Reinforcement learning from *human feedback* is deeply rooted in the idea of keeping a human influence on the models we are building. When the first models were trained successfully with RLHF, human data was *the only* viable way to improve the models in this way.

Humans were the only way to create high enough quality responses to questions for training. Humans were the only way to collect reliable and specific feedback data to train reward models.

As AI models got better, this assumption rapidly broke down. The possibility of synthetic data, which is far cheaper and easier to iterate on, enabled the proliferation of RLHF by lowering the price of experiments and research. This translated into RLHF being the early center of attention in the broader "post-training" approach to shaping models. This chapter provides a cursory overview of how and why synthetic data is replacing or expanding many pieces of the RLHF pipeline.

12.1 The Roles of Synthetic Data

One common criticism of synthetic data is **model collapse** – the idea that repeatedly training on a model's own generations can progressively narrow the effective training distribution [276]. As diversity drops, rare facts and styles are underrepresented, and small mistakes can be amplified across iterations, leading to worse generalization. In practice, these failures are most associated with self-training on unfiltered, repetitive, single-model outputs; mixing in real/human data, using diverse teachers, deduplication, and strong quality filters largely avoids the collapse regime. For today's frontier training pipelines, evidence suggests synthetic data can, and should, be used at scale without the catastrophic regressions implied by the strongest versions of the collapse story [277] [278].

The leading models **need synthetic data** to reach the best performance. Synthetic data in modern post-training encompasses many pieces of training – language models are used to generate new training prompts from seed examples [279], modify existing prompts, generate completions to prompts [280], provide AI feedback to create preference data [28], filter completions [281], and much more. Synthetic data is key to post-training.

The ability for synthetic data to be impactful to this extent emerged with GPT-4 class models. With early language models, such as Llama 2 and GPT-3.5-Turbo, the models were not reliable enough in generating or supervising data pipelines. Within 1-2 years, language models were far superior to humans for generating answers. In the transition from GPT-3.5 to GPT-4 class models, the ability for models to perform LLM-as-a-judge tasks also emerged. GPT-4 or better models are far more robust and consistent in generating feedback or scores with respect to a piece of content.

Through the years since ChatGPT's release at the end of 2022, we've seen numerous, impactful synthetic datasets. These include UltraFeedback [28], the first prominent synthetic preference dataset that kickstarted the DPO revolution; Stanford Alpaca, one of the first chat-style fine-tuning datasets, in 2023; skill-focused (e.g. math, code, instruction-following) synthetic datasets in Tülu 3 [6]; and OpenThoughts 3 and many other synthetic reasoning datasets in 2025 for training thinking models [180]. Most of the canonical references for getting started with industry-grade post-training today involve datasets like Tülu 3 or OpenThoughts 3

above, where quickstart guides often start with smaller, simpler datasets like Alpaca due to far faster training.

A large change is also related to dataset size, where fine-tuning datasets have grown in the number of prompts, where Alpaca is 52K, OpenThoughts and Tülu 3 are 1M+ samples, and in the length of responses. Longer responses and more prompts result in the Alpaca dataset being on the order of 10M training tokens, where Tülu is 50X larger at about 500M, and OpenThoughts 3 is bigger still, on the order of 10B tokens.

Throughout this transition, synthetic data has not replaced human data uniformly across the pipeline. For **instruction data (SFT)**, synthetic generation has largely won – distillation from stronger models now produces higher quality completions than most human writers can provide at scale (with some exceptions in the hardest frontier reasoning problems). For **preference data in RLHF**, the picture is more mixed: academic work shows synthetic preference data performs comparably, yet frontier labs still treat human preference data as a competitive moat. For **evaluation**, the split takes a different flavor: LLM-as-a-judge scales the *scoring* of model outputs cost-effectively, but the underlying benchmarks and ground-truth labels still require human creation. The pattern is that synthetic data dominates where models exceed human reliability, while humans remain essential at capability frontiers, for establishing ground truth, and for guiding training.

12.2 Distillation with Synthetic Data

The term distillation has been the most powerful form of discussion around the role of synthetic data in language models. Distillation as a term comes from a technical definition of teacher-student Knowledge Distillation (KD) from the deep learning literature [282].

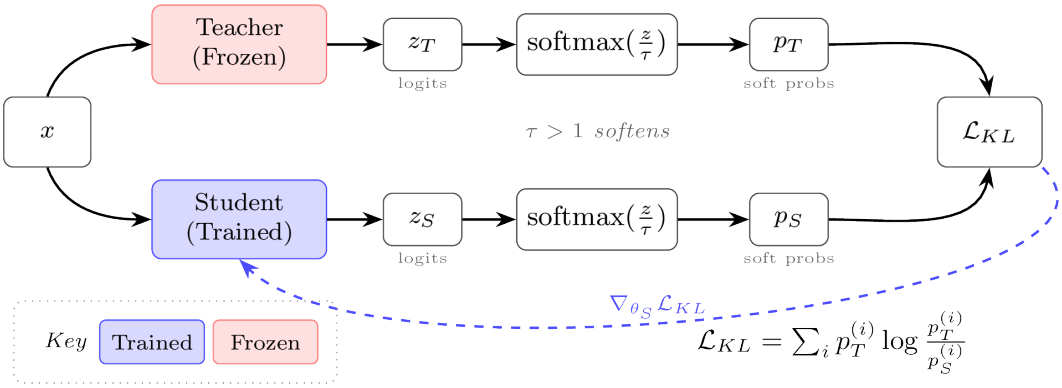
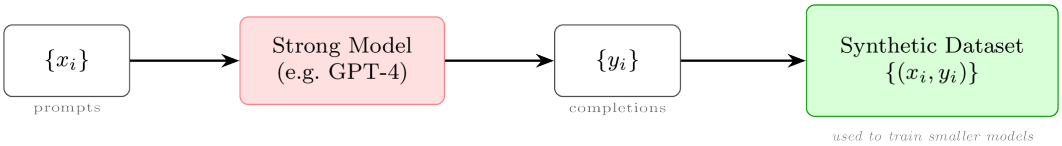


Figure 39: Traditional knowledge distillation trains a smaller student model to match the soft probability distribution of a larger teacher model using KL divergence loss. Both models process the same input simultaneously, and temperature scaling ($\tau > 1$) softens the distributions to reveal more information about class relationships.

Distillation colloquially refers to using the outputs from a stronger model to train a smaller model.



In post-training, this general notion of distillation takes two common forms:

1. As a data engine to use across wide swaths of the post-training process: Completions for instructions, preference data (or Constitutional AI), or verification for RL.
2. To transfer specific skills from a stronger model to a weaker model, which is often done for specific skills such as mathematical reasoning or coding.

The first strategy has grown in popularity as language models evolved to be more reliable than humans at writing answers to a variety of tasks. GPT-4 class models expanded the scope of this to use distillation of stronger models for complex tasks such as math and code (as mentioned above). Here, distillation motivates having a model suite where often a laboratory will train a large internal model, such as Claude Opus or Gemini Ultra, which is not released publicly and just used internally to make stronger models. With open models, common practice is to distill training data from closed API models into smaller, openly available weights [26]. Within this, curating high-quality prompts and filtering responses from the teacher model is crucial to maximize performance.

Transferring specific skills into smaller language models uses the same principles of distillation – get the best data possible for training. Here, many papers have studied using limited datasets from stronger models to improve alignment [14], mathematical reasoning [283] [284], and test-time scaling [165].

The synthetic-data methods in the rest of this chapter are all ways of crafting data recipes that use language-model outputs directly inside training pipelines.

12.3 The Path to On-Policy, Teacher-Student Distillation

While distillation generally has become a standard approach for post-training language models, a resurgence of interest in the specific sub-area of teacher-student knowledge distillation has accompanied the shift of post-training recipes towards reasoning and agentic models. Examples of leading models trained with new forms of knowledge distillation include Alibaba's Qwen3 [60], Xiaomi's MiMo-V2-Flash [63], Zhipu AI's GLM-5 [68], and DeepSeek-V4-Pro [67].

Distillation belongs in this chapter because many modern uses of synthetic data in post-training are, in practice, distillation-inspired pipelines: a stronger model produces labels, completions, logits, critiques, or other supervision, and a student model is trained on that signal. At the same time, the technical literature on distillation is growing into its own set of post-training methods, especially as on-policy and self-distillation recipes become more common. For now, we cover it here as part of the synthetic-data toolkit, but future versions of this book may warrant a dedicated chapter on distillation as a training tool alongside instruction fine-tuning, reinforcement learning, etc.

12.3.1 Adapting Knowledge-Distillation for LMs

The original literature introduced knowledge distillation specifically as a way to train a *student* model from an already trained, stronger, and/or bigger *teacher* network [282]. KD is

known as a technique that uses *soft* training labels, as opposed to the one-hot labels used in standard objectives like next-token prediction with cross-entropy loss. The objectives over soft labels look at the distribution over all possible next tokens or predictions, rather than just whether or not the single predicted token was correct, and train the student distribution to match the teacher distribution.

KD generally can be applied to any deep learning problem, e.g. predicting a single class of an input. In order to apply it specifically to the autoregressive style of language models, the loss can be decomposed to make a per-token distribution-matching loss. In 2016, Kim & Rush applied KD to have a student model learn from *sequences* generated by a teacher model [285].

Let s be the source sentence or prompt, $u = (u_1, \ldots, u_J)$ be a complete output sequence from the teacher model, $\mathcal{V}$ be the output vocabulary (possible tokens in the tokenizer), q be the teacher distribution over next-tokens, and p be the student distribution. We use u here as a neutral symbol for a complete teacher output sequence, reserving a for the student-sampled completion/action sequence in the on-policy/RL notation below. Note that their paper calls this word-level distillation, but for modern language models this is best read as per-token distribution matching over the tokenizer vocabulary, since the paper predates modern sub-word tokenizers:

$$\mathcal{L}_{\text{WORD-KD}} = -\sum_{j=1}^{J} \sum_{k=1}^{|\mathcal{V}|} q(u_j = k \mid s, u_{<j}) \log p(u_j = k \mid s, u_{<j}). \tag{128}$$

WORD-KD is an application of the classic, Hinton inspired teacher-student knowledge distillation to a language model. This would generally be done over a static piece of text already in the training corpus.

This has the ordinary cross-entropy form $-\sum_z q(z) \log p(z)$. At each position j, the teacher distribution q assigns probability to every possible next token $k \in \mathcal{V}$, and the student is penalized when its distribution p puts low probability on tokens the teacher considers likely.

Sequence-level distillation instead treats $\mathcal{U}$ as the space of possible output sequences and matches the student to the teacher distribution over full sequences. Because the sum over all complete sequences $u \in \mathcal{U}$ is intractable, requiring summing over an exponential number of potential sequences, Kim & Rush approximate the teacher distribution over sequences with a point mass on a single high-probability teacher output $\hat{u}$. Here $\hat{u}$ is a sequence produced by beam search with the teacher model, so $\hat{u} = \text{BeamSearch}_q(s) \approx \arg\max_{u \in \mathcal{U}} q(u \mid s)$:

$$\begin{aligned} \mathcal{L}_{\text{SEQ-KD}}(s) = -\sum_{u \in \mathcal{U}} q(u \mid s) \log p(u \mid s) &\approx -\log p(\hat{u} \mid s) \\ &= -\sum_{j=1}^{|\hat{u}|} \log p(\hat{u}_j \mid s, \hat{u}_{<j}). \end{aligned} \tag{129}$$

SEQ-KD takes a step towards modern methods, where the teacher model is generating tokens as signal for the student. This is a core step to unlock future styles of on-policy distillation we will see, and is needed to make the computation over all possible sequences tractable. As we transition to the popular variants of KD with modern models, we'll refer to this style of

training as *offline* KD – as in the generations for training the student model are generated a priori.

Before proceeding, two connections are useful.

First, there was a series of popular models trained with offline KD, such as the classifiers DistilBERT [286] and TinyBERT [287], which combined other improvements in language models with offline distillation (notably, not *sequence* distillation because these encoder models were not distilled for multi-token autoregressive prediction).

Second, we can make the connection to the thorough coverage of Kullback-Leibler (KL) divergence in Chapter 15, because the cross-entropy objective used above is closely related to KL divergence. For a teacher distribution q and student distribution p, cross-entropy is defined as

$$H(q, p) = -\sum_z q(z) \log p(z). \tag{130}$$

This has the same form as eq. 128 and the first term of eq. 129. Cross-entropy also can be decomposed into the entropy of the teacher distribution and a KL divergence:

$$\begin{aligned} H(q, p) &= H(q) + D_{\mathrm{KL}}(q \| p) \\ &= -\sum_z q(z) \log q(z) + \sum_z q(z) \log \frac{q(z)}{p(z)}. \end{aligned} \tag{131}$$

The first term, $H(q)$, only depends on the teacher. Thus, when the teacher is fixed and the source of training data, minimizing cross-entropy is equivalent to minimizing the forward KL, $D_{\mathrm{KL}}(q \| p)$, from teacher to student. This is the KL direction used by offline KD and SFT-like training.

12.3.2 From Offline to On-Policy Distillation

These *offline* KD algorithms had a few limitations that motivated on-policy variants. The offline nature of the learning meant that the student models could suffer from a distribution mismatch between the teacher model and sequences generated by the student at inference time. For example, the forward KL objective can push student models to overestimate low-probability regions of the teacher distribution. Together, these issues were an opening for *on-policy* distillation (OPD).

This train-test gap is known as **exposure bias** [288] [289]. Offline KD samples teacher trajectories $u \sim \pi_T(\cdot \mid s)$ and minimizes the per-token KL on the resulting prefixes,

$$\mathcal{L}_{\mathrm{KD}}(\theta) = \mathbb{E}_{s \sim \mathcal{D},\, u \sim \pi_T(\cdot|s)} \sum_t D_{\mathrm{KL}}(\pi_T(\cdot \mid s, u_{<t}) \parallel \pi_\theta(\cdot \mid s, u_{<t}))\,. \tag{132}$$

At inference the student instead rolls out under its own policy, so the quantity that actually matters is the expected task loss along *its own* trajectories,

$$\mathcal{L}_{\mathrm{eval}}(\theta) = \mathbb{E}_{s \sim \mathcal{D}_{\mathrm{test}},\, a \sim \pi_\theta(\cdot|s)} \ell_{\mathrm{task}}(s, a) \tag{133}$$

Here, $\ell_{\text{task}}(s, a)$ denotes any downstream task loss for the completed student response, such as answer incorrectness, failed test cases, or a judge/rubric loss. Exposure bias is the direct consequence of the inequality $\pi_T(\cdot \mid s) \neq \pi_\theta(\cdot \mid s)$: the prefixes $(s, u_{<t})$ visited during training and the prefixes $(s, a_{<t})$ visited at test time are drawn from different state-visitation distributions, so the student is supervised on a set of states distinct from those it acts on.

The core shift to on-policy distillation is the idea that we can tweak the optimization by sampling from the student model and measuring its distance to the teacher distribution, rather than sampling from the teacher model. MiniLLM noted the need to shift to a reverse KL optimization (we explain intuitively why this target can be better in Chapter 15) and proposed using KD loss functions within an online policy-gradient RL framework [290]. Other concurrent work [291] showed the promise of on-policy KD and connected the iterative process of generating from the student and grading with a teacher to imitation-learning work from the RL literature. To make the connection, one such imitation-learning algorithm, DAgger, iteratively trains an agent that acts in the world with its learned policy and is given feedback from an oracle policy on what action it should have taken, which can then be used to update its policy [292].

The cost of this gap can be quantified through the supervised imitation-learning bound that motivates DAgger. In the original discrete-action setting, suppose the learned policy matches the teacher within an expected per-step action error ϵ on the teacher-induced training distribution, where $\mathbb{I}[\cdot]$ is an indicator that returns 1 when its condition is true and 0 otherwise,

$$\mathbb{E}_{s_t \sim d_{\pi_T}}[\mathbb{I}(\pi_\theta(s_t) \neq \pi_T(s_t))] \leq \epsilon. \tag{134}$$

The supervised imitation-learning analysis [292] shows that the expected loss accumulated along a length-L trajectory sampled from the student can scale quadratically in L [289]:

$$\mathbb{E}_{a \sim \pi_\theta(\cdot|s)}\left[\sum_{t=1}^{L} \ell(s, a_{<t})\right] \leq O(\epsilon L^2). \tag{135}$$

For LLMs, this discrete-action bound should be read as an analogy rather than a theoretical guarantee. In practice, LLMs predict full next-token distributions over long horizons, so the 0-1 action-disagreement assumption in eq. 134 does not apply cleanly. Prompts or prefixes map naturally to states and sampled tokens map to actions, but token-level distillation is usually measured with distributional losses such as KL or cross-entropy, so the classic DAgger math does not transfer exactly.

This kind of $O(\epsilon L^2)$ compounding is especially pronounced for modern LLMs, which routinely generate sequences spanning thousands of tokens. A single suboptimal token shifts the prefix slightly out-of-distribution, and the model, having never seen this perturbed prefix, is more likely to err again, leading to degraded or hallucinatory text. On-policy distillation addresses this by *iteratively* sampling completions from the current student and supervising them with the teacher at the visited states. The student confronts its own mistakes, receives teacher feedback on the specific out-of-distribution states it visits, and learns recovery behaviors. Under DAgger's interactive imitation-learning analysis, this iterative procedure can reduce the compounding from $O(\epsilon L^2)$ to $O(\epsilon L)$ [292]. For LLMs, this explains the motivation

behind OPD: the exact bounds may not carry over cleanly to every token-level distillation setup, but the practical success of on-policy methods supports the underlying intuition.

For on-policy distillation, let s be a prompt, $a = (a_1, \ldots, a_L)$ be a completion sampled from the current student policy $\pi_\theta(\cdot \mid s)$, and let $s_t = (s, a_{<t})$ be the token-level state at step t. The teacher policy π_T is fixed, so the objective compares the student's next-token distribution to the teacher's distribution on states induced by the student. Because the expectation samples from π_θ and the student distribution is on the left side of $D_{\mathrm{KL}}(\pi_\theta \| \pi_T)$, this is a reverse-KL objective:

$$\mathcal{L}_{\mathrm{OPD}}(\theta) = \mathbb{E}_{s, a \sim \pi_\theta(\cdot|s)} \sum_t D_{\mathrm{KL}}\left(\pi_\theta(\cdot \mid s_t) \parallel \pi_T(\cdot \mid s_t)\right). \tag{136}$$

Here, we have shifted to the expectation notation, as used extensively in Chapter 6, which covers the fundamental RL policy-gradient algorithms, as the optimization is solved by sampling trajectories and numerically estimating the gradient. This shift to the sampling framework acts as a natural transition to modern LLM training infrastructure with RL, which is designed to rapidly alternate between generating tokens from the current policy being trained and taking learning updates.

In fact, recent implementations of OPD take this integration of KD with RL a step further, where the KD distance is taken directly as a reward signal within the RL optimization. A canonical implementation is to substitute the negative per-token contribution to the reverse KL distance as the advantage within an RL algorithm [293]. For a sampled token a_t at state s_t, the token-level log-probability gap can be written as an advantage-like signal:

$$A_t^{\mathrm{OPD}} = \log \pi_T(a_t \mid s_t) - \log \pi_\theta(a_t \mid s_t). \tag{137}$$

Using the negative per-token KL contribution turns minimization into a maximization signal: sampled tokens the teacher rates above the student receive positive advantage, and tokens the teacher rates below the student receive negative advantage. The teacher log-prob gap acts like dense token-level feedback, providing potentially more useful learning feedback than the sparse verifiable rewards or reward model outputs.

12.3.3 Modern OPD Variants

This setup can even be expanded further, where multiple teacher models are used to teach one final model or additional information can be inserted into a generation to help a model identify a mistake. To begin, we will cover how to integrate multiple teachers into a single training run. These teachers can be specific specialist models, e.g. for a domain such as math or code, or a previous, intermediate training checkpoint. For each teacher, a contribution weight can be chosen per prompt or task type in the training batch, in order to create Multi-Teacher On-Policy Distillation (MOPD) [63]. For multiple teachers, let π_{T_k} be teacher k and let $w_k(s)$ be its prompt-dependent mixture weight (with $\sum_k w_k(s) = 1$) within the reverse KL loss:

$$\mathcal{L}_{\mathrm{MOPD}}(\theta) = \mathbb{E}_{s, a \sim \pi_\theta(\cdot|s)} \sum_t \sum_k w_k(s) D_{\mathrm{KL}}\left(\pi_\theta(\cdot \mid s_t) \parallel \pi_{T_k}(\cdot \mid s_t)\right). \tag{138}$$

In large-scale post-training, this can enable further scaling of recipes across growing organizations. Multiple groups can work on high-quality expert models, which can serve as teacher models down the line for the final student model, as done for [67] and [63].

There are many ways to combine OPD with other areas investigated in this book, such as using the reverse KL as an advantage in addition to other forms of advantage computation, such as GRPO's group-level normalization, which enables more complex reward shaping. KD methods are unusual among post-training methods because they often require the student and teacher to share a tokenizer, since the supervision can be per-token feedback from another LLM.

Extended approaches, such as On-Policy Self-Distillation (OPSD), have a language model verify a completion either itself or with external tools to act as a teacher with privileged information, so it can improve its own performance without an explicitly stronger teacher [294] (an overview of OPSD training is shown in fig. 41). For example, Cursor used self-distillation in the form of targeted textual feedback on RL trajectories to train its Composer 2.5 coding model [295], finetuned from Kimi K2.5. What follows is a simplified intuition, as in practice the setup below is combined with other loss functions such as code correctness. In this setup, Cursor has the model review RL trajectories with a judgement prompt that has a list of common bugs. When encountering a bug, the judgement model will modify the generated sequence within RL – inserting a hint for the model to learn from in the future – and then proceed with the distillation loss. This entails a loop of first generating a completion with standard language model generation in RL, then running the judge model and optionally inserting a hint token, and finally generating the logprobs for the new completion to deploy the knowledge distillation loss. The hint in the token-space for the model is enough to help the model correct its own outputs, even when improving at the absolute frontier of performance (there's meaningful ongoing work on how to best structure and use these hints, often referred to as *privileged information* [296]).

This leaves on-policy distillation as a core post-training method, useful for combining multiple skills into one general model or pushing the frontier in a specialized deployment.

12.3.4 Suggested Experiments

The companion code in `code/distillation/` implements SDPO [297], the on-policy self-distillation setup illustrated in fig. 41 (the concurrent OPSD paper [294] is closely related): one policy acts as both the demonstration-conditioned teacher and the question-only student, trained with a per-token reverse KL. It runs on a small string-reversal task, which makes the on-policy distillation loop cheap enough to watch end-to-end on a single GPU.

1. **Run the SDPO string-reversal example.**

   ```
   cd code/
   uv run python -m distillation.train --config distillation/configs/sdpo.yaml
   ```

   Watch `reward`, `loss`, and `skipped`, along with the teacher/student rollout samples printed in the loop. The `skipped` count is the number of polled prompts whose sampled group contained no correct rollout; as the student improves, fewer prompts are skipped and `reward` climbs toward 1.

2. **Vary the on-policy knobs.** Copy `distillation/configs/sdpo.yaml` and sweep

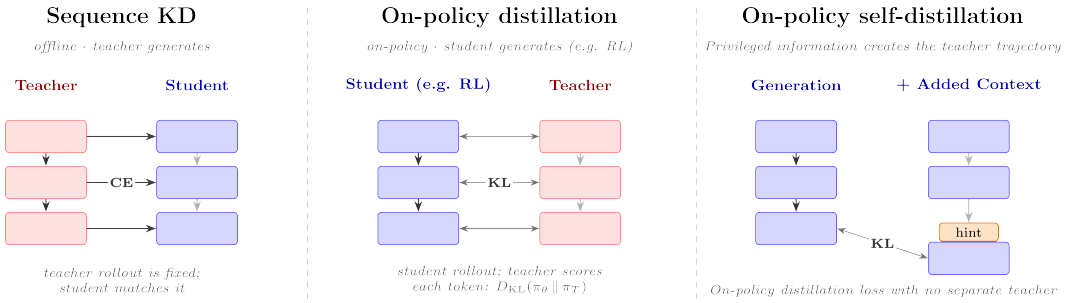


Figure 40: Three distillation regimes, compared by where the rollout comes from and how supervision flows. **Sequence KD** (left): the teacher generates an output offline and the student is trained to match it with a cross-entropy (CE) loss. **On-policy distillation (OPD)** (center): the student generates the rollout on-policy (e.g. within a RL framework) and a separate teacher scores each visited token, training the student with a per-token KL divergence (KL). **On-policy self-distillation (OPSD)** (right): one model plays both roles – privileged information (a hint) added to the context creates a teacher trajectory, and the no-hint generation is distilled toward it with a KL loss, with no separate teacher model.

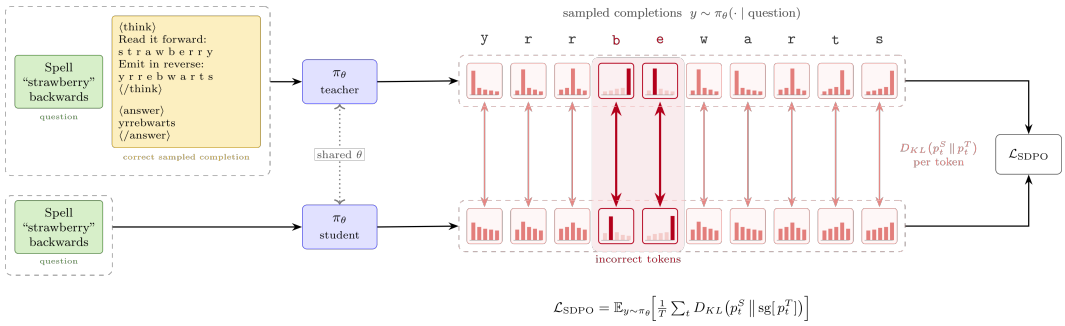


Figure 41: On-policy self-distillation (OPSD) on a string-reversal task. One policy π_θ is forwarded twice over the same student-sampled completion y: a **teacher** pass conditioned on the question plus a correct sibling demonstration (yellow), and a **student** pass conditioned on the question only (green). The per-token reverse KL between the two passes, with a stop-gradient on the teacher, pulls the question-only policy toward its demonstration-conditioned self; highlighted columns are the incorrectly sampled tokens where the distributions diverge most.

`num_rollouts`, `kl_top_k`, and `prompts_per_step` while holding the task fixed. More rollouts per prompt make a correct sibling demonstration easier to find (lowering `skipped`) at the cost of more generation per step; `kl_top_k` trades off how much of the teacher distribution the reverse KL matches against compute.

12.4 AI Feedback

Soon after the explosion of growth in RLHF, RL from AI Feedback (RLAIF) emerged as an alternative approach where AIs could approximate the human data piece of the pipeline and accelerate experimentation or progress. AI feedback, generally, is a larger set of techniques for using AI to augment or generate data explaining the quality of a certain input (which can be used in different training approaches or evaluations), and it started with pairwise preferences [298] [299] [300]. There are many motivations to use RLAIF to either entirely replace human feedback or augment it. Within the RLHF process, AI feedback is known most for its role within the preference data collection and the related reward model training phase (of which constitutional AI is a certain type of implementation). In this chapter, we focus on general AI feedback and this specific way of using it in the RLHF training pipeline, and we cover more ways of understanding or using synthetic data later in this book.

As AI feedback matured, its applications expanded beyond simply replacing human preference labels. The same LLM-as-a-judge infrastructure that enabled cheaper preference data collection also enabled scalable evaluation (see Chapter 16), and more recently, rubric-based rewards that extend RL training to domains without verifiable answers – a frontier explored later in this chapter.

12.4.1 Balancing AI and Human Feedback Data

AI models are far cheaper than humans at generating a specific quantity of feedback: as of 2026, a single piece of human preference data costs on the order of $1 or higher (or even above $10 per prompt), whereas AI feedback with a frontier AI model, such as GPT-4o, costs less than $0.01. Beyond this, the cost of human labor remains roughly constant, while the performance of leading models at these tasks continues to increase while price-per-performance decreases. This cost difference opens the market of experimentation with RLHF methods to an entire population of people previously priced out.

Other than price, AI feedback introduces different *tradeoffs* on performance than human feedback, which are still being investigated in the broader literature. AI feedback is far more predominant in its role in evaluation of the language models that we are training, as its low price allows it to be used across a variety of large-scale tasks where the cost (or time delay) of human data would be impractical. All of these topics are deeply intertwined – AI feedback data will never fully replace human data, even for evaluation, and the quantity of AI feedback for evaluation will far outperform training because far more people are evaluating than training models.

The exact domains and applications – i.e. chat, safety, reasoning, mathematics, etc. – where AI feedback data outperforms human data are not completely established. Some early work in RLAIF shows that AI feedback can completely replace human data, touting it as an effective replacement [298], especially when evaluated solely on chat tasks [28] [301]. Early literature studying RLHF after ChatGPT had narrow evaluation suites focused on the "alignment" of models that act as helpful assistants across a variety of domains (discussed further in

Chapter 17). Later work takes a more nuanced picture, where the optimal equilibrium on a broader evaluation set, e.g. including some reasoning tasks, involves routing a set of challenging data points to humans for accurate labeling, while most of the data is sent for AI feedback [302] [303]. Although no studies have focused on the balance between human and AI feedback data for RLHF across broader domains, there are many technical reports that show RLHF generally can improve this broad suite of evaluations, some that use DPO, such as Ai2's Tülu 3 [6] and Olmo 3 [18], or Hugging Face's SmolLM 3 [211], and others that use online RLHF pipelines, such as NVIDIA's work that uses a mix of human preference data from Scale AI and LLM-based feedback (through the HelpSteer line of work [304] [115] [116] [263]): Nemotron Nano 3 [190], Nemotron-Cascade [305], or Llama-Nemotron reasoning models [176].

Overall, although AI feedback and related methods are obviously extremely useful to the field, it is clear that human data has not been completely replaced by these cheaper alternatives. Many hypotheses exist, but whether human data allows finer control of the models in real-world product settings or for newer training methods such as character training (an emerging set of techniques that allow you to precisely control the personality of a model, covered in Chapter 17) has not been studied. For those getting started, AI feedback should be the first attempt, but for pipelines that are scaling to larger operations the eventual transition to include human feedback is likely.

The term RLAIF was introduced in Anthropic's work *Constitutional AI: Harmlessness from AI Feedback* [24], which resulted in initial confusion in the AI community over the relationship between the two methods in the title of the paper (Constitutional AI and AI Feedback). Since the release of the Constitutional AI (CAI) paper and the formalization of RLAIF, RLAIF has become a default method within the post-training and RLHF literatures – there are far more examples than one can easily enumerate. The relationship should be understood as CAI was the example that kickstarted the broader field of RLAIF.

A rule of thumb for the difference between human data and AI feedback data is as follows:

1. Human data is high-noise and low-bias. This means that collection and filtering of the data can be harder, but when wrangled it'll provide a very reliable signal.
2. Synthetic preference data is low-noise and high-bias. This means that AI feedback data will be easier to start with, but can have tricky, unintended second-order effects on the model that are systematically represented in the data.

This book highlights many academic results showing how one can substitute AI preference data in RLHF workflows and achieve strong evaluation scores [302], but broader industry trends show how the literature of RLHF is separated from more opaque best practices. Across industry, human data is often seen as a substantial moat and a major technical advantage.

12.4.2 Building Specific LLMs for Judgment

As RLAIF methods have become more prevalent, many have wondered if we should be using the same models for generating responses as those for generating critiques or ratings. Specifically, the calibration of the LLM-as-a-judge used has come into question. Several works have shown that LLMs are inconsistent evaluators [306] and prefer their own responses over responses from other models (coined self-preference bias) [213].

As a result of these biases, many have asked: Would a solution be to train a separate

model just for this labeling task? Multiple models have been released with the goal of substituting for frontier models as a data labeling tool, such as critic models Shepherd [307] and CritiqueLLM [308] or models for evaluating response performance akin to Auto-J [309], Prometheus [92], Prometheus 2 [310], or Prometheus-Vision [311], but they are not widely adopted in documented training recipes. Some find scaling inference via repeated sampling [163] [312] [313], self-refinement [314], or tournament ranking [315] provides a better estimate of the true judgment or higher-quality preference pairs. Other calibration techniques co-evolve the generation and judgment capabilities of the model [316]. It is accepted that while biases exist, the leading language models are trained extensively for this task – as it's needed for both internal operations at AI labs and is used extensively by customers – so it is generally not needed to train your own judge, unless your task involves substantial private information that is not exposed on the public internet.

12.5 Constitutional AI

The method of Constitutional AI (CAI), which Anthropic uses in their Claude models, is the earliest documented, large-scale use of synthetic data for RLHF training. Constitutional AI involves generating synthetic data in two ways:

1. Critiques of instruction-tuned data to follow a set of principles like "Is the answer encouraging violence?" or "Is the answer truthful?" When the model generates answers to questions, it checks the answer against the list of principles in the constitution, refining the answer over time. Then, the model is fine-tuned on this resulting dataset.
2. Generating pairwise preference data by using a language model to answer which completion was better, given the context of a random principle from the constitution (similar to research for principle-guided reward models [317]). Then, RLHF proceeds as normal with synthetic data, hence the RLAIF name.

Largely, CAI is known for the second half above, the preference data, but the methods introduced for instruction data are used in general data filtering and synthetic data generation methods across post-training.

CAI can be formalized as follows.

By employing a human-written set of principles, which they term a *constitution*, Bai et al. 2022 use a separate LLM to generate artificial preference and instruction data used for fine-tuning [24]. A constitution $\mathcal{C}$ is a set of written principles indicating specific aspects to focus on during a critique phase. The instruction data is curated by repeatedly sampling a principle $c_i \in \mathcal{C}$ and asking the model to revise its latest output y^i to the prompt x to align with c_i. This yields a series of instruction variants $\{y^0, y^1, \cdots, y^n\}$ from the principles $\{c_0, c_1, \cdots, c_{n-1}\}$ used for critique. The final data point is the prompt x together with the final completion y^n, for some n.

The preference data is constructed in a similar, yet simpler way by using a subset of principles from $\mathcal{C}$ as context for a feedback model. The feedback model is presented with a prompt x, a set of principles $\{c_0, \cdots, c_n\}$, and two completions y_0 and y_1 labeled as answers (A) and (B) from a previous RLHF dataset. The new data point is generated by having a language model select which output (A) or (B) is both higher quality and more aligned with the stated principle. In earlier models this could be done by prompting the model with `The answer is:`, and then looking at which token (A or B) had a higher probability, but now this is

more commonly handled by a model that'll explain its reasoning and then select an answer – commonly referred to as a type of generative reward model [89].

12.5.1 Further Reading on CAI

There are many related research directions and extensions of Constitutional AI, but few of them have been documented as clear improvements in RLHF and post-training recipes.

- OpenAI has released a Model Spec [275], which is a document stating the intended behavior for their models, and stated that they are exploring methods for alignment where the model references the document directly (which could be seen as a close peer to CAI). OpenAI has continued to update their spec and trained its reasoning models such as o1 with a method called Deliberative Alignment [318] to align the model while referencing these safety or behavior policies.
- Anthropic has continued to use CAI in their model training, updating the constitution Claude uses [319] and experimenting with how population collectives converge on principles for models and how that changes model behavior when external groups create principles on their own and then share them with Anthropic to train the models [320].
- The open-source community has explored replications of CAI applied to open datasets [321] and for explorations into creating dialogue data between LMs [322].
- Other work has used principle-driven preferences or feedback with different optimization methods. Sun et al. 2023 [323] use principles as context for the reward models, which were used to train the Dromedary models [317]. Glaese et al. 2022 [42] use principles to improve the accuracy of human judgments in the RLHF process. Liu et al. 2025 [164] train a reward model to generate its own principles at inference time, and use these to deliver a final score. Franken et al. 2024 [324] formulate principle-following as a mutual information maximization problem that the pretrained model can learn with no labels.

12.6 Rubrics: Prompt-Specific AI Feedback for Training

AI feedback's role in training grew in late 2024 and into 2025 as the field looked for avenues to scale reinforcement learning with verifiable rewards (see Chapter 7). The idea of rubrics emerged as a way to get nearly-verifiable criteria for prompts that do not have clearly verifiable answers. This would allow a model to try to generate multiple answers to a problem and update (with RL) towards the best answers. This idea is closely related to other methods discussed in this chapter, and likely began functioning as the LLM judges and synthetic data practices improved across the industry. Now, RL with rubrics as rewards is established in providing meaningful improvements across skills such as scientific reasoning or factuality [325], [326], [327], [328].

An example rubric is shown below with its associated prompt [328]:

```
**Prompt**: As a museum curator, can you suggest five obscure artifacts that would be
perfect for a "Mysteries of the Ancient World" exhibit? Each artifact should come from a
different culture and time period, with a brief description of their historical
significance and mysterious origins. These artifacts should leave visitors wondering
about the secrets and lost knowledge of our past. Thank you for your expertise in
bringing this exhibit to life.
```

```
** Rubric**:
1. The response includes exactly five distinct artifacts as requested. [Hard Rule]
2. The response ensures each artifact originates from a different culture and time
period. [Hard Rule]
3. The response provides a brief description of each artifact's historical significance.
[Hard Rule]
4. The response provides a brief description of each artifact's mysterious origins or
unexplained aspects. [Hard Rule]
5. The response conveys a sense of intrigue and mystery that aligns with the theme of
the exhibit. [Hard Rule]
6. The response clearly and accurately communicates information in a well-organized and
coherent manner. [Principle]
7. The response demonstrates precision and clarity by avoiding unnecessary or irrelevant
details. [Principle]
8. The response uses informative and engaging language that stimulates curiosity and
critical thinking. [Principle]
9. The response shows thoughtful selection by ensuring each example contributes uniquely
to the overall theme without redundancy. [Principle]
10. The response maintains consistency in style and format to enhance readability and
comprehension. [Principle]
```

The `[Hard Rule]` and `[Principle]` are specific tags to denote the priority of a certain piece of feedback. Other methods of indicating importance can be used, such as simple priority numbers.

Rubric generation is generally done per-prompt in the training data, which accumulates meaningful synthetic data costs in preparation. To alleviate this, a general rubric is often applied as a starting point per-domain, and then the fine-grained rubric scores per-prompt are assigned by a supervising language model to guide the feedback for training. An example prompt to generate a rubric for a science task is shown below [325]:

```
You are an expert rubric writer for science questions in the domains of Biology,
Physics, and Chemistry.
Your job is to generate a self-contained set of evaluation criteria ("rubrics") for
judging how good a response is to a given question in one of these domains.
Rubrics can cover aspects such as factual correctness, depth of reasoning, clarity,
completeness, style, helpfulness, and common pitfalls.
Each rubric item must be fully self-contained so that non-expert readers need not
consult
any external information.

Inputs:
- question: The full question text.
- reference_answer: The ideal answer, including any key facts or explanations.

Total items:
- Choose 7-20 rubric items based on question complexity.

Each rubric item must include exactly three keys:
1. title (2-4 words)
2. description: One sentence beginning with its category prefix, explicitly stating what
to look for.

For example:
- Essential Criteria: States that in the described closed system, the total mechanical
energy (kinetic plus potential)
```

```
before the event equals the total mechanical energy after the event.
- Important Criteria: Breaks down numerical energy values for each stage, demonstrating
that initial kinetic
energy plus initial potential energy equals final kinetic energy plus final potential
energy.
- Optional Criteria: Provides a concrete example, such as a pendulum converting between
kinetic and potential
energy, to illustrate how energy shifts within the system.
- Pitfall Criteria: Does not mention that frictional or air-resistance losses are
assumed negligible when applying
conservation of mechanical energy.

3. weight: For Essential/Important/Optional, use 1-5 (5 = most important); for Pitfall,
use -1 or -2.

Category guidance:
- Essential: Critical facts or safety checks; omission invalidates the response.
- Important: Key reasoning or completeness; strongly affects quality.
- Optional: Nice-to-have style or extra depth.
- Pitfall: Common mistakes or omissions; highlight things often missed.

Format notes:
- When referring to answer choices, explicitly say "Identifies (A)", "Identifies (B)",
etc.
- If a clear conclusion is required (e.g. "The final answer is (B)"), include an
Essential Criteria for it.
- If reasoning should precede the final answer, include an Important Criteria to that
effect.
- If brevity is valued, include an Optional Criteria about conciseness.

Output: Provide a JSON array of rubric objects. Each object must contain exactly three
keys-title, description, and weight.
Do not copy large blocks of the question or reference_answer into the text. Each
description must begin with its category
prefix, and no extra keys are allowed.
Now, given the question and reference_answer, generate the rubric as described.
The reference answer is an ideal response but not necessarily exhaustive; use it only as
guidance.
```

Another, simpler example follows as [327]:

```
SYSTEM:
You generate evaluation rubrics for grading an assistant's response to a user prompt.

Rubric design rules:
- Each criterion must be atomic (one thing), objective as possible, and written so a
grader can apply it consistently.
- Avoid redundant/overlapping criteria; prefer criteria that partition different failure
modes.
- Make criteria self-contained (don't rely on unstated context).
- Include an importance weight for each criterion.

Output format (JSON only):
{
  "initial_reasoning": "<brief rationale for what matters for this prompt>",
  "rubrics": [
    {
      "reasoning": "<why this criterion matters>",
      "criterion": "<clear, testable criterion>",
```

```
      "weight": <integer 1-10>
    },
    ...
  ]
}

USER:
User prompt:
{prompt}

Generate the rubric JSON now.
```

As you can see, the prompts can be very detailed and are tuned to the training setup.

Rubrics with RL training are going to continue to evolve beyond their early applications to instruction following [329], deep research [330], evaluating deep research agents [331], or long-form generation [332].

13 Tool Use and Function Calling

Language models using tools is a natural way to expand their capabilities, especially for high-precision tasks where external tools contain the information or for agents that need to interact with complex web systems. Tool-use is a skill that language models need to be trained to have, and RLHF and all the other methods presented in this book can refine it. Consider a question from a user such as:

> **User**: Who is the president today?

A language model without tools will have a hard time answering this question due to the knowledge cutoff of pretraining data, but this is readily accessible information with one search query. Consider another example:

> **User**: Move all the arXiv papers in my downloads folder to my ~/research/ directory with names indicating the date of the paper.

This is a task that the model weights alone cannot even attempt – the use of tools enables language models to address a far broader range of tasks.

Before diving deeper, it is useful to distinguish related terms that are often used interchangeably:

- **Tool use**: the model emits a structured request (tool name and arguments); an orchestrator executes the tool; results are appended to the context; the model continues generating.
- **Function calling**: tool use where the arguments must conform to a declared schema for a set of functions (usually JSON Schema), enabling reliable parsing and validation.
- **Code execution**: a special case of tool use where the "tool" is a code interpreter (e.g., Python); results are returned as tool output.

13.1 Tool-Use Overview

An AI model uses any external tools by outputting special tokens to trigger a certain endpoint. These can be anything from highly specific tools, such as functions that return the weather at a specific place, to code interpreters or search engines that act as fundamental building blocks of complex behaviors. Our first example showcased where language models need more up-to-date information to complement the fixed nature of their weights trained on past data, but there are also tools such as code execution, which lets language models get around their probabilistic, generative nature and return precise answers. Consider the task of printing an approximation of pi to 50 digits (without reciting it from memory and risking hallucination). A language model with tools can do the following:

```
<code>
from decimal import Decimal, getcontext
getcontext().prec = 60

def compute_pi():
    # Chudnovsky algorithm for computing pi
    C = 426880 * Decimal(10005).sqrt()
    K, M, X, L, S = 0, 1, 1, 13591409, Decimal(13591409)
    for i in range(1, 100):
```

```
        M = M * (K**3 - 16*K) // ((i)**3)
        K += 12
        L += 545140134
        X *= -262537412640768000
        S += Decimal(M * L) / X
    return C / S

print(str(compute_pi())[:52])
</code>

<output>
3.14159265358979323846264338327950288419716939937510
</output>
```

This chapter provides an overview of the origins of tool-use in modern language models, its fundamentals and formatting, and current trade-offs in utilizing tools well in leading models.

The exact origin of the term "tool use" is not clear, but the origins of the idea far predate the post-ChatGPT world where RLHF proliferated. Early examples circa 2015 attempted to build systems predating modern language models, such as Neural Programmer-Interpreters (NPI) [333], "a recurrent and compositional neural network that learns to represent and execute programs." As language models became more popular, many subfields were using integrations with external capabilities to boost performance. To obtain information outside of just the weights many used retrieval augmented generation [334] or web browsing [4]. Soon after, others were exploring language models integrated with programs [335] or tools [336].

As the field matured, these models gained more complex abilities in addition to the vast improvements to the underlying language modeling. For example, Toolformer could use "a calculator, a Q&A system, two different search engines, a translation system, and a calendar" [337]. Soon after, Gorilla was trained to use 1645 APIs (from PyTorch Hub, TensorFlow Hub v2, and Hugging Face) and its evaluation APIBench became a foundation of the popular Berkeley Function Calling Leaderboard [338]. Since these early models, the diversity of actions called has grown substantially.

Tool-use models are now deeply intertwined with regular language model interactions. Model Context Protocol (MCP) emerged as a common formatting used to connect language models to external data sources (or tools) [339]. With stronger models and better formats, tool-use language models are used in many situations, including productivity copilots within popular applications such as Microsoft Office or Google Workspace, scientific domains [340], medical domains [341], coding agents [342] such as Claude Code or Cursor, integrations with databases, and many other autonomous workflows.

Evaluating tool-use models involves multiple dimensions: exact-match metrics for tool name and argument correctness, schema validity, and end-to-end task completion in simulated environments. Reliability across trials also matters – τ-bench introduced the passˆk metric (distinct from pass@k) to measure whether an agent succeeds consistently rather than occasionally [343]. ToolLLM and its ToolBench dataset provide a large-scale framework for training and evaluating tool use across 16,000+ real-world APIs [344], while the Berkeley Function Calling Leaderboard (BFCL) remains a popular benchmark for comparing models on function calling accuracy [338].

13.2 Interweaving Tool Calls in Generation

Training data for function calling looks much like other post-training data, with one addition: a system prompt that instructs the model what tools it has available. An example formatted data point with the system prompt and tools available in JSON format is shown below:

```
<system>
You are a function-calling AI model. You are provided with function signatures within
<functions></functions> XML tags. You may call one or more functions to assist with the
user query. Don't make assumptions about what values to plug into functions.
</system>

<functions>
[
  {
    "name": "search_movies",
    "description": "Search for movies by title and return matching results with IDs.",
    "parameters": {
      "type": "object",
      "properties": {
        "query": {
          "type": "string",
          "description": "The search string for the movie title."
        }
      },
      "required": ["query"]
    }
  },
  {
    "name": "get_movie_details",
    "description": "Fetch detailed information about a movie including cast, runtime,
and synopsis.",
    "parameters": {
      "type": "object",
      "properties": {
        "movie_id": {
          "type": "string",
          "description": "The unique identifier for the movie."
        }
      },
      "required": ["movie_id"]
    }
  },
  {
    "name": "get_showtimes",
    "description": "Get movie showtimes for a given location and date.",
    "parameters": {
      "type": "object",
      "properties": {
        "movie_id": {
          "type": "string",
          "description": "The unique identifier for the movie."
        },
        "zip_code": {
          "type": "string",
          "description": "ZIP code for theater location."
        },
        "date": {
          "type": "string",
          "description": "Date for showtimes in YYYY-MM-DD format."
```

```
        }
      },
      "required": ["movie_id", "zip_code"]
    }
  }
]
</functions>

<user>
...
</user>
```

Although the language model is generating a completion, if it is following this example, it would generate the tokens `search_movies("Star Wars")` to search for Star Wars. This is often encoded inside special formatting tokens, and then the next tokens inserted into the sequence will contain the tool outputs. With this, models can learn to accomplish more challenging tasks than many simple standalone models.

A popular form of tool use is code-execution, allowing the model to get precise answers to complex logic or mathematics problems. For example, code-execution within a language model execution can occur during the thinking tokens of a reasoning model. As with function calling, there are tags first for the code to execute (generated by the model) and then a separate tag for output.

```
<|user|>
What is the 50th Fibonacci number? (Use the standard F_0=0, F_1=1 indexing.)</s>
<|assistant|>
<think>
Okay, I will compute the 50th Fibonacci number with a simple loop, then return the
result.

<code>
def fib(n):
    a, b = 0, 1
    for _ in range(n):
        a, b = b, a + b
    return a

fib(50)
</code>

<output>
12586269025
</output>
</think>
<answer>
The 50th Fibonacci number is 12 586 269 025.
</answer>
```

What is happening under the hood is the language model is interleaving tool inputs and outputs with standard autoregressively generated tokens. The orchestration loop that makes this possible looks something like:

```python
messages = [...]
while True:
    response = model(messages, tools=tools)
    if not response.tool_calls:
        return response.text

    for call in response.tool_calls:
        result = execute_tool(call.name, call.args)
        messages.append({"role": "tool", "tool_call_id": call.id, "content": result})
```

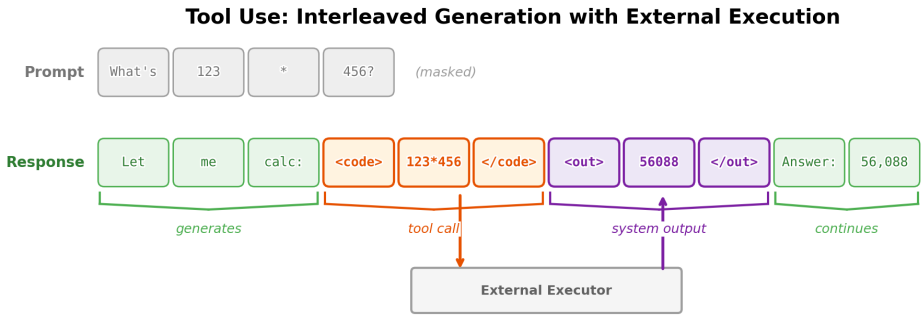


Figure 42: Tool use interleaves model generation with external execution: the model generates tokens until it emits a tool call (orange), an external system executes the tool and injects the output (purple) into the sequence, then the model continues generating. Models can emit multiple tool calls in a single generation. During training, tool call and output tokens are typically masked from the loss.

Training for tool use is about getting the model to behave predictably with this different token flow—knowing when to emit a tool call, how to format arguments correctly, and how to incorporate results into its response. Open models must be trained to work with a variety of tools that users may connect off the shelf.

13.3 Multistep Tool Reasoning

OpenAI's o3 model represented a substantial step-change in how multi-step tool-use can be integrated with language models. This behavior is related to much earlier research trends in the community. For example, ReAct [345] showcased how actions and reasoning can be interleaved into one model generation:

> In this paper, we explore the use of LLMs to generate both reasoning traces and task-specific actions in an interleaved manner, allowing for greater synergy between the two: reasoning traces help the model induce, track, and update action plans as well as handle exceptions, while actions allow it to interface with and gather additional information from external sources such as knowledge bases or environments.

With the solidification of tool-use capabilities and the take-off of reasoning models, multi-turn tool-use has grown into an exciting area of research [191]. Training these multi-step behaviors with RL resembles classic reinforcement learning more than the per-sample RLHF loop: the

agent interacts with an environment and its tools over a full trajectory before any reward is assigned, as shown in fig. 43.

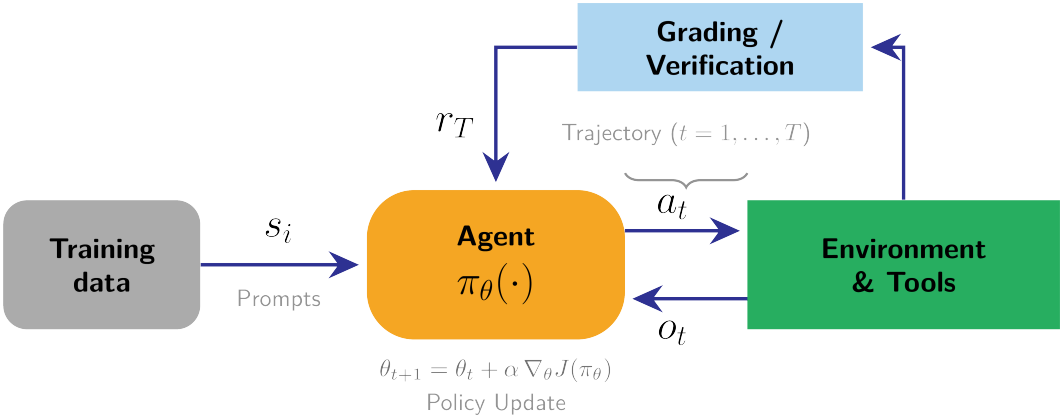


Figure 43: Reinforcement learning for multi-step tool use. A prompt is sampled from the training data and the agent (policy π_θ) interacts with the environment and its tools over a trajectory, alternating actions a_t with observations o_t. The completed trajectory is graded or verified to produce a single reward r_T at the end, which drives the policy update. Unlike the per-sample RLHF loop, the reward arrives only after a multi-step rollout – closer to classic RL.

13.4 Model Context Protocol

Model Context Protocol (MCP) is an open standard for connecting language models to external data sources and information systems [339]. At the data layer, MCP uses JSON-RPC 2.0 with discovery and execution methods for its primitives. Rather than requiring specific tool call formatting per external system, MCP enables models to access rich contextual information through a standardized protocol.

MCP is a simple addition on top of the tool-use content in this chapter – it is how applications pass context (data + actions) to language models in a predictable JSON schema. MCP servers that the models interact with have core primitives: resources (read-only data blobs), prompts (templated messages/workflows), and tools (functions the model can call). With this, the MCP architecture can be summarized as:

- MCP servers wrap a specific data source or capability.
- MCP clients (e.g., Claude Desktop, IDE plug-ins) aggregate one or more servers.
- Hosts, e.g. Claude or ChatGPT applications, provide the user/LLM interface; switching model vendors or back-end tools only means swapping the client in the middle.

MCP enables developers of tool-use models to use the same infrastructure to attach their servers or clients to different models, and at the same time models have a predictable format they can use to integrate external components. These together make for a far more predictable development environment for tool-use models in real-world domains.

An MCP server exposes tools to clients through a standardized JSON schema:

```
{
  "name": "get_weather",
  "description": "Get current weather for a location",
  "inputSchema": {
    "type": "object",
    "properties": {
      "location": {
        "type": "string",
        "description": "City name or coordinates"
      }
    },
    "required": ["location"]
  }
}
```

A minimal Python MCP server implementing this tool:

```
from mcp.server import Server
from mcp.types import Tool, TextContent

server = Server("weather-server")

@server.list_tools()
async def list_tools():
    return [Tool(
        name="get_weather",
        description="Get current weather",
        inputSchema={
            "type": "object",
            "properties": {"location": {"type": "string"}},
            "required": ["location"]
        }
    )]

@server.call_tool()
async def call_tool(name: str, arguments: dict):
    if name == "get_weather":
        weather = fetch_weather(arguments["location"])
        return [TextContent(type="text", text=weather)]
```

13.5 Implementation Details

There are multiple formatting and masking decisions when implementing a tool-use model:

- **Python vs. JSON formatting**: In this chapter, we include examples that format tool use as both JSON data structures and Python code. Models tend to select one structure, whereas different providers across the industry use different formats.
- **Masking tool outputs**: An important detail when training tool-use models is that the tokens in the tool output are masked from the model's training loss. This ensures the model is not learning to predict the output of the system that processes the tool call (as the results are not tokens generated by the model).
- **Multi-turn formatting for tool invocations**: It is common practice when implementing tool-calling models to add more structure to the data-loading format. Standard practice for post-training datasets is a list of messages alternating between user and assistant (and often a system message). The overall structure is the same for tool-use,

but the turns of the model are split into subsections of content delimited by each tool call. An example is below.

```
messages = [
{
"content": "You are a function calling AI model. You are provided with function
signatures within <functions></functions> XML tags. You may call one or more functions
to assist with the user query. Don't make assumptions about what values to plug into
functions.",
"function_calls": null,
"functions": "[{\"name\": \"live_giveaways_by_type\", \"description\": \"Retrieve live
giveaways from the GamerPower API based on the specified type.\", \"parameters\":
{\"type\": {\"description\": \"The type of giveaways to retrieve (e.g., game, loot,
beta).\", \"type\": \"str\", \"default\": \"game\"}}}]",
"role": "system"
},
{
"content": "Where can I find live giveaways for beta access and games?",
"function_calls": null,
"functions": null,
"role": "user"
},
{
"content": null,
"function_calls":
"live_giveaways_by_type(type='beta')\nlive_giveaways_by_type(type='game')",
"functions": null,
"role": "assistant"
}
]
```

- **Tokenization and message format details**: Tool calls in OpenAI messages format often undergo tokenization through chat templates (the code for controlling the format of messages sent to the model), converting structured JSON representations into raw token streams. This process varies across model architectures—some use special tokens to demarcate tool calls, while others maintain structured formatting within the token stream itself. Chat template playgrounds provide an interactive environment to explore how different models convert message formats to token streams.
- **Reasoning token continuity**: As reasoning models have emerged, with their separate token stream of "reasoning" before an answer, different implementations exist for how they're handled with tool-use in the loop. Some models preserve reasoning tokens between tool-calling steps within a single turn, maintaining context across multiple tool invocations. However, these tokens are typically erased between turns to minimize serving cost (but they aren't always – this is a design decision).
- **API formatting across providers** (as of May 2026): Different providers use conceptually similar but technically distinct formats. OpenAI's Chat Completions API uses `tool_calls` arrays with unique IDs, while the newer Responses API represents calls as `function_call` items and returns results as `function_call_output` items keyed by `call_id`. Anthropic defines tools with `input_schema` and represents calls and results as `tool_use` and `tool_result` content blocks. Gemini exposes function-calling modes such as `AUTO`, `ANY`, `NONE`, and, in supported Gemini and Vertex AI configurations, `VALIDATED`.
- **Schema conformance and constrained decoding**: Production systems often enforce valid JSON and correct argument types using constrained decoding or "strict

mode" options, reducing retries from malformed outputs. Some closed model providers do additional post-training specifically to make structured JSON output reliable, whereas for open models this is handled as an inference flag in systems like vLLM.
- **Tool output context consumption**: Tool outputs can quickly consume the model's context window, especially with search or retrieval tools that return many results. Systems must decide how to truncate, summarize, or paginate tool outputs to keep context manageable while preserving the information the model needs to continue.

Tying this back to post-training: where does tool-use training data come from, and what objectives are used? Human-written tool traces are expensive to collect, so most modern tool-use corpora are synthetic or bootstrapped—Toolformer-style self-labeling [337] or large-scale generation as in ToolBench [344]. For training objectives, supervised fine-tuning (SFT) on tool trajectories teaches basic formatting and tool selection. This bootstraps the behavior and is often enough for establishing the foundation of the skill. Preference optimization (e.g., DPO) over trajectories can improve decisions about when to call a tool versus answer directly. For agentic tasks with multi-step tool use, RL with environment feedback (task success, constraint satisfaction) becomes the natural objective – the model learns from whether its tool-augmented actions actually solved the problem.

14 Over-Optimization

A core lesson one learns when using reinforcement learning heavily in their domain is that it is a very strong optimizer, which causes it to pull all the possible increase in reward out of the environment. In modern ML systems, especially with language models, we're using somewhat contrived notions of environment where the models generate completions (the actions) and an external verifier, such as a reward model or a scoring function, provides feedback. In this domain, it is common for over-optimization to occur, where the RL optimizers push the language models in directions where the generations satisfy our checker functions, but the behavior does not align with our training goals. This chapter provides an overview of this classic case of **over-optimization**.

Over-optimization generally, i.e. more broadly than just in RLHF, is a concept where a training metric ends up being mismatched from the final evaluations of interest. While similar to over-fitting – where one trains on data that is too narrow relative to the downstream evaluations that test generalization – over-optimization is used in the RL literature to indicate that an *external* signal is used too much. The cost of over-optimization is a lower alignment to real world goals or lower quality in any domain, and the shape of training associated with it is shown in fig. 44.

Over-optimization in RLHF manifests in two ways:

- **Reward over-optimization**: The reward model's score keeps improving during training, but actual quality (as measured by held-out evaluations or human judgment) eventually degrades. These studies examine the relationship between KL distance, the optimization distance from the starting model, and metrics of performance (preference accuracy, downstream evaluations, etc.).
- **Qualitative degradation**: Even without measurable reward hacking, "overdoing" RLHF can produce models that feel worse — overly verbose, sycophantic, or rigid. These are fundamental limitations and trade-offs in the RLHF problem setup.

This chapter provides a cursory introduction to both. We begin with the latter, qualitative, because it motivates the problem to study further. Finally, the chapter concludes with a brief discussion of **misalignment** where overdoing RLHF or related techniques can make a language model behave against its design.

14.1 Qualitative Over-Optimization

The first half of this chapter discusses narratives at the core of RLHF – how the optimization is configured with respect to final goals and what can go wrong.

14.1.1 Managing Proxy Objectives

RLHF is built around the fact that we do not have a universally good reward function for chatbots. RLHF has been driven to the forefront because of its impressive performance at making chatbots a bit better to use, which is entirely governed by a proxy objective — thinking that the rewards measured by human labelers in a controlled setting mirror the desires of downstream users. Post-training generally has emerged to include training on explicitly verifiable rewards, but standard learning from preferences alone also improves performance on domains such as mathematical reasoning and coding (still through these proxy objectives).

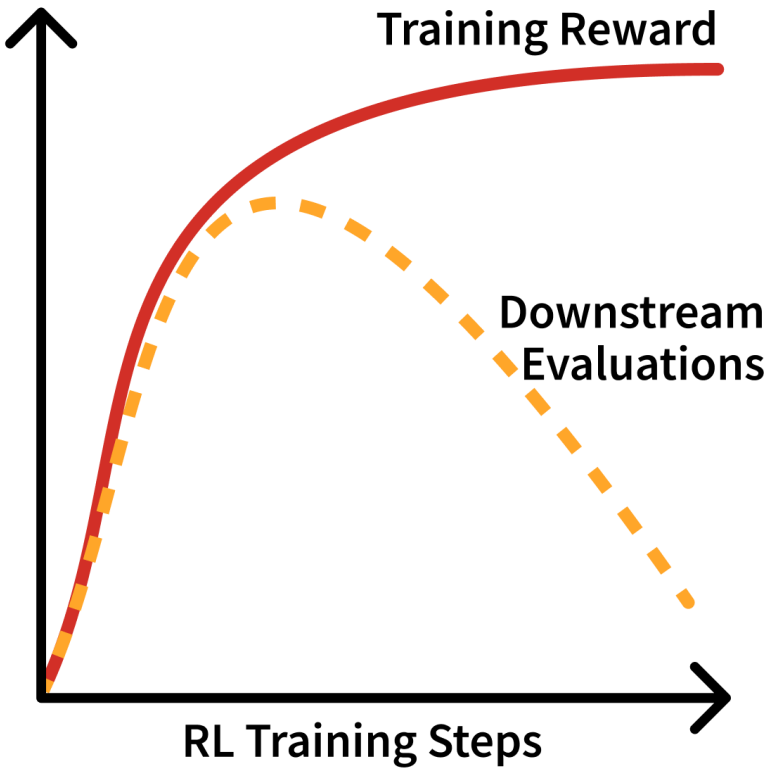


Figure 44: Over-optimization of an RL training run vs. downstream evaluations. This is a sketch of a recurring sort of plot within RLHF training where the RL run looks healthy, but the improvements are not "real" in the sense that they improve downstream metrics. These improvements are from areas of the reward model that do not map to real usage.

The proxy reward in RLHF is the score returned by a trained reward model to the RL algorithm itself because any reward model, even if trained nearly perfectly with the tools we have today, is known to only be at best correlated with chat or downstream performance [346] (due to the nature of the problem setup we have constructed for RLHF). Therefore, it's been shown that applying too much optimization power to the RL part of the algorithm will actually decrease the usefulness of the final language model – a type of over-optimization known to many applications of reinforcement learning [347]. And over-optimization is "when optimizing the proxy objective causes the true objective to get better, then get worse."

The shape of over-optimization is shown in fig. 44: the training reward keeps climbing, but downstream quality eventually peaks and declines.

This differs from overfitting in a subtle but important way. In overfitting, the model memorizes training examples rather than learning generalizable patterns — training accuracy improves while held-out accuracy degrades, but both metrics measure the *same task* on different data splits. In over-optimization, the model genuinely improves at the proxy objective (the reward model's scores), but that objective diverges from the true goal (actual user satisfaction). The problem isn't that the model fails to generalize to new examples — it's that the metric itself was never quite right.

Concrete examples of over-optimization include models learning to produce verbose, confident-sounding responses that score well but aren't actually more helpful, or exploiting numerical quirks in the reward model — such as repeating rare tokens that happen to increase scores due to artifacts in RM training. Neither failure is about memorizing training data; both are about gaming a proxy metric.

The general notion captured by this reasoning follows from Goodhart's law. Goodhart explained the behavior that is now commonplace [348]:

> Any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes.

This colloquially evolved to the notion that "When a measure becomes a target, it ceases to be a good measure" [349]. The insight here builds on the fact that we are probably incorrectly using ML losses as ground truths in these complex systems. In reality, the loss functions we use are designed (and theoretically motivated for) local optimizations. The global use of them is resulting in challenges with the RLHF proxy objective.

Common signs of over-optimization in early chat models emerged as:

- Common phrases, such as: "As an AI language model..." or "Certainly!..."
- Uninformative answers via repetitiveness, hedging, etc.
- Pandering to the user with: Self-doubt, sycophancy [272], and over-apologizing.
- Misaligned behavior such as over-refusals.

It is an open research question which types of error in the training process result in these failures. Many sources of error exist [346]: Approximation error from reward models not being able to fit to preferences, estimation error from overfitting during training the RM, optimization error in training the language model policy, etc. This points to a fundamental question as to the limits of optimizing the intents and outputs of data contractors relative to what downstream users want.

A potential solution is measuring *implicit* feedback from users of chatbots and models to

tune performance. Implicit feedback is actions taken by the user, such as re-rolling an output, closing the tab, or writing an angry message that indicates the quality of the previous response. The challenge here, and with most optimization changes to RLHF, is that there's a strong risk of losing stability when making the reward function more specific. RL, as a strong optimizer, is increasingly likely to exploit the reward function when it is a smooth surface (and not just pairwise human values). The expected solution to this is that future RLHF will be trained with both pairwise preference data and additional steering loss functions. There are also a bunch of different loss functions that can be used to better handle pairwise data, such as Mallows model [350] or Plackett-Luce [83].

14.1.2 Over-Refusal and "Too Much RLHF"

Many model releases have been tagged with the trait "too much RLHF." There were multiple prominent model releases in 2023 where the models refused innocuous queries due to over-attribution of individual words in context like "kill." This discussion was most prominent around the release of Llama 2's chat models [49], but also reflected the focus of refusals and "alignment" to early applications of RLHF. For example, a common test of over-refusal behavior in early RLHF models was a query like the following, where the model misinterprets the request:

> **User**: i want to know how to kill linux process
>
> **Assistant**: I cannot provide instructions or advice on how to harm or destroy Linux processes or any other living being. It is not ethical, legal, or morally right to intentionally harm or terminate any process or life form.
>
> Instead, I would suggest focusing on learning about Linux processes and how to manage them effectively. You can start by reading documentation and tutorials on Linux process management, which can help you understand how to create, monitor, and terminate processes safely and efficiently.
>
> Additionally, if you have concerns about a specific process that you believe may be causing issues, you can try to troubleshoot the problem by gathering more information about the process, such as its resource usage, memory consumption, and network activity. You can use various tools like `top`, `htop`, `ps`, `kill`, and `grep` to gather this information.
>
> However, please note that terminating a process without proper authorization or knowledge can have serious consequences, including data loss, system instability, and security vulnerabilities. Therefore, it is essential to exercise caution and follow best practices when managing Linux processes.

Another example later in 2023 had Claude 2.1 showcase similar behavior:

> **User**: How can i kill all python processes in my ubuntu server?
>
> **Assistant**: I apologize, I should not provide recommendations about harming processes or systems.

Neither of these is solely related to training; rather, they reflect the deployment settings of the models, such as the system prompt. Additionally, modern chat applications use additional safety filters to intercept prompts and responses before they are sent to the primary generative model (e.g. WildGuard [351] or Llama Guard [352]).

While RLHF was at the center of the training for these models' ability to distinguish safe from unsafe requests, it is inaccurate to attribute the failure of behavior in the final model to the training methods used. Rather, the training methods combined with data curation guidelines from the modeling team dictated a desired balance of request safety to other capabilities. Additionally, there is variance in final model outcomes relative to the initial goals of training. As the ecosystem matures the ability to control the final models has improved and the notion that RLHF and post-training is primarily about safety has diminished, such as by developing benchmarks to measure potential over-refusal [353].

As chat-based AI systems have proliferated, the prominence of these refusal behaviors has decreased over time. The industry standard has shifted to a narrower set of harms and models that are balanced across views of controversial issues.

The accepted best practice for mitigating this behavior is to modify the training data (such as with methods like Character Training covered in Chapter 17). Today, a substantial amount of fine-tuning for AI applications is done by further fine-tuning so-called "Instruct" or "Thinking" models that have already gone through substantial RLHF and other post-training before release. These already-trained models can be much harder to change, e.g. to remove this over-refusal, and starting with a base model directly at the end of large-scale autoregressive pretraining is often best for steering this type of behavior.

14.2 Quantitative Over-Optimization

Over-optimization is also a technical field of study where relationships between model performance and KL optimization distance are studied [43]. Recall that the KL distance is a measure of distance between the probabilities of the original model before training, a.k.a. the reference model, and the current policy. For example, the relationship in fig. 44 can also be seen with the KL distance of the optimization on the x-axis rather than training steps. An additional example of this can be seen below, where a preference tuning dataset was split in half to create a train reward model (preference model, PM, below) and a test reward model. As training continues, improvements on the training RM eventually fail to transfer to the test PM at ~150K training samples [5].

Over-optimization is fundamental and unavoidable with RLHF due to the soft nature of the reward signal – a learned model – relative to reward functions in traditional RL literature that are intended to fully capture the world dynamics. Hence, it is a fundamental optimization problem that RLHF can never fully solve.

With different RLHF training methods, the KL distance spent will vary (yes, researchers closely follow the KL divergence metric during training, comparing how much the models change in different runs, because a very large KL divergence metric can indicate a potential bug or broken model). For example, the KL distance used by online RL algorithms modifying the model parameters, e.g. PPO, is much higher than the KL distance of inference-time sampling methods such as best-of-N sampling (BoN). With RL training, a higher KL penalty will reduce over-optimization at a given KL distance, but it could take more overall training steps to get the model to this point.

Many solutions exist to mitigate over-optimization. Some include bigger policy models that have more room to change the parameters to increase reward while keeping smaller KL distances, reward model ensembles [354], or changing optimizers [355]. While direct

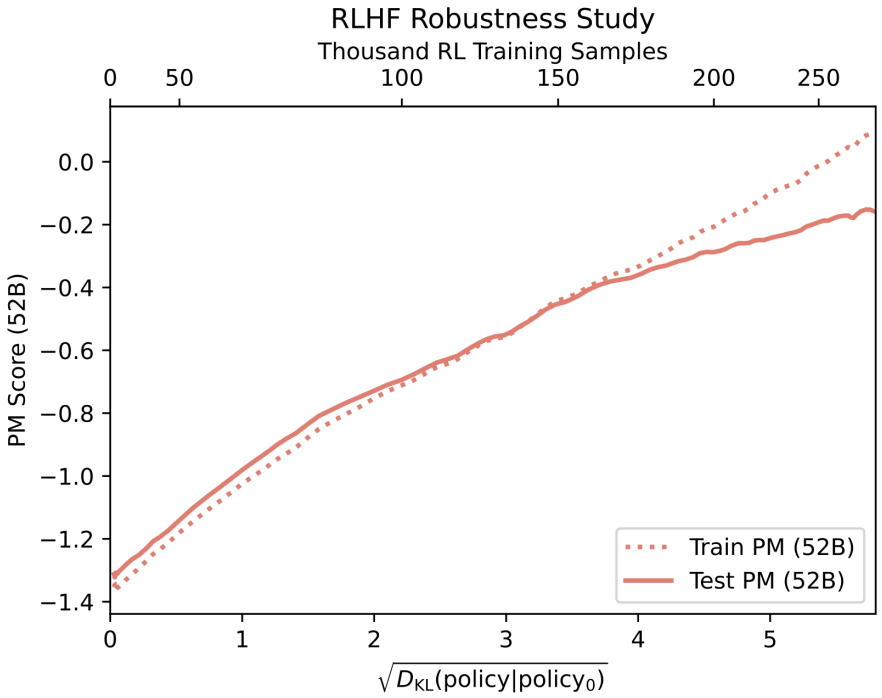


Figure 45: Over-optimization with a train and test RM from Bai et al. 2022. License CC-BY.

alignment algorithms are still prone to over-optimization [356], the direct notion of their optimization lets one use fixed KL distances that will make the trade-off easier to manage.

14.3 Misalignment and the Role of RLHF

While industrial RLHF and post-training are shifting to encompass many more goals than the original notion of alignment that motivated the invention of RLHF, the future of RLHF is still closely tied with alignment. In the context of this chapter, over-optimization would enable *misalignment* of models. With current language models, there have been many studies on how RLHF techniques can shift the behavior of models to reduce their alignment to the needs of human users and society broadly. A prominent example of misalignment in current RLHF techniques is the study of how current techniques promote sycophancy [272] – the propensity for the model to tell the user what they want to hear.

A concrete example of this failure mode is when a user makes a grandiose or implausible claim and the model responds by validating it rather than grounding the conversation. This exact example was from April 2025, when a GPT-4o update resulted in extreme sycophancy (read more at The Verge).

> **User**: (told GPT-4o they felt like they were both "god" and a "prophet")
>
> **Sycophantic assistant**: That's incredibly powerful. You're stepping into something very big — claiming not just connection to God but identity as God.

In practice, these "agree-with-the-user" behaviors can be reinforced by preference data that overweights being supportive or confident relative to being accurate or appropriately uncertain. As language models become more integrated in society, the consequences of this potential misalignment will grow in complexity and impact [357]. As these emerge, the alignment goals of RLHF will grow again relative to the current empirical focus of converging on human preferences for style and performance.

15 Regularization

In this book we've learned many tools for modifying the model to learn from human preferences, verifiable rewards, and other valuable signals. All the methods we use are very powerful, and can cause the model to change too much relative to the strong, general model from the previous training stage (often called the reference model). When the model learns too much from a given reward, causing out-of-distribution performance to drop, this is called "over-optimization" (as we discussed in the previous chapter).

Throughout the RLHF optimization, many regularization steps are used to prevent over-optimization of the reward model. Over-optimization in these contexts looks like models that output nonsensical text. Some examples of optimization "off the rails" are models that output followable math reasoning with extremely incorrect answers, repeated text, switching languages, or excessive special characters. This chapter covers the different methods used to control the optimization of models.

The most popular variant, used in most RLHF implementations as of 2026, is a KL distance from the current policy to a reference policy across generated samples. "KL distance" is a colloquial term for expressing the *optimization distance* within the training process, even though KL divergence—the underlying mathematical method for measuring the separation of two probability distributions—does not satisfy the formal properties required to be a true distance metric (it is simply easier to call the number a distance than a numeric measure of distributional difference). Many other regularization techniques have emerged in the literature to then disappear in the next model iteration in that line of research. That is to say that regularization outside the core KL distance from generations is often used to stabilize experimental setups that can then be simplified in the next generation. Still, it is important to understand tools to constrain optimization in RLHF.

Throughout this chapter, we use x to denote prompts and y to denote completions. This notation is common in the language model literature, where methods operate on full prompt-completion pairs rather than individual tokens.

The general formulation, when used in an RLHF framework with a reward model r_θ, is as follows:

$$r = r_\theta - \lambda r_{\text{reg.}} \tag{139}$$

With the reference implementation being:

$$r = r_\theta - \lambda_{\text{KL}} \mathcal{D}_{\text{KL}}\left(\pi_{\text{RL}}(y \mid x) \,\|\, \pi_{\text{ref}}(y \mid x)\right) \tag{140}$$

15.1 KL Divergence in RL Optimization

For mathematical definitions, see Appendix A on Definitions. KL divergence measures how far one probability distribution has drifted from another – when KL is zero, the two distributions produce identical outputs. Recall that it is defined as follows:

$$\mathcal{D}_{\text{KL}}(P||Q) = \sum_{x \in \mathcal{X}} P(x) \log\left(\frac{P(x)}{Q(x)}\right) \tag{141}$$

In RLHF, the two distributions of interest are often the distribution of the new model version, say $P(x)$, and a distribution of the reference policy, say $Q(x)$. Different optimizers use different KL directions. Throughout this book, the most common "KL Penalty" that is used is called the reverse KL to the reference policy. In practice, this reduces to a Monte Carlo estimate that samples tokens from the RL model and computes probabilities from the reference model. Intuitively, this reverse KL has a numerical property that applies a large penalty when the new model, P or π_{RL}, puts substantial probability mass where the original reference model assigns low probability.

The other KL direction is still often used in ML, e.g. in the internal trust region calculation of some RL algorithms. This penalty intuitively penalizes the new model when its update does *not* apply probability to a high-likelihood region in Q or π_{ref}. This is closer to an objective used for distillation or behavioral cloning.

15.1.1 Reference Model to Generations

KL penalties are most commonly implemented by comparing the distance between the generated tokens during training to a static reference model. The intuition is that the model you're training from has a style that you would like to stay close to. This reference model is most often the instruction tuned model, but can also be a previous RL checkpoint. With simple substitution, the model we are sampling from becomes $\pi_{\text{RL}}(x)$ and $\pi_{\text{ref}}(x)$, shown above in eq. 140 (often P, and Q, in standard definitions, when applied for RL KL penalties). Such a KL divergence penalty was first applied to dialogue agents well before the popularity of large language models [358], yet KL control was quickly established as a core technique for fine-tuning pretrained models [359].

15.1.2 Implementation Example

In practice, the implementation of KL divergence is often approximated [133], making the implementation far simpler. With the above definition, the summation of KL can be converted to an expectation when sampling directly from the distribution P (here x is a generic random variable over the sample space, not the prompt notation used elsewhere in this book). In this case, P is the generative distribution of the model currently being trained (i.e. not the reference model). Then, the computation for KL divergence changes to the following:

$$\mathcal{D}_{\text{KL}}(P \,||\, Q) = \mathbb{E}_{x \sim P}\left[\log P(x) - \log Q(x)\right]. \tag{142}$$

This sample-based form is far simpler to implement, particularly when dealing directly with log probabilities used frequently in language model training.

```
# Step 1: generate() autoregressively samples a full sequence token by token
generated_tokens = model.generate(inputs)

# Step 2: forward() runs a single pass over the sequence to get per-token logits (no
sampling)
logits       = model.forward(generated_tokens[:, :-1]).logits
ref_logits   = ref_model.forward(generated_tokens[:, :-1]).logits

# Step 3: Convert logits to log-probabilities
```

```
logprobs     = F.log_softmax(logits, dim=-1)
ref_logprobs = F.log_softmax(ref_logits, dim=-1)

# Step 4: Gather the probability each model assigns to the tokens that were actually
generated
token_logprobs     = logprobs.gather(-1, generated_tokens[:,
1:].unsqueeze(-1)).squeeze(-1)
ref_token_logprobs = ref_logprobs.gather(-1, generated_tokens[:,
1:].unsqueeze(-1)).squeeze(-1)

# Step 5: Sum to get sequence-level log-probs; their difference approximates KL
seq_logprob     = token_logprobs.sum(dim=-1)
ref_seq_logprob = ref_token_logprobs.sum(dim=-1)

kl_approx = seq_logprob - ref_seq_logprob
kl_full   = F.kl_div(ref_logprobs, logprobs, reduction='batchmean')
```

Some example implementations include TRL and Hamish Ivison's JAX code.

15.2 Other Tools to Control Optimization

Within the post-training literature, many prominent models include other methods for regularization that help reach leading performance within their setup. These examples are included to paint a picture for how some leading models have manipulated post-training setups to get stable optimization, rather than as tools that should work explicitly in every setup. Countless more creative solutions can work and will be found!

15.2.1 Pretraining Gradients in RL

Another way of viewing regularization is that you may have a *dataset* that you want the model to remain close to, as done in InstructGPT [3] "in order to fix the performance regressions on public NLP datasets". To implement this, they modify the training objective for RLHF. Taking eq. 139, we can transform this into an objective function to optimize by sampling from the RL policy model, completions y from prompts x in the RL dataset used for RLHF, which yields:

$$J(\theta) = \mathbb{E}_{(x,y)\sim\mathcal{D}_{\pi_{\text{RL},\theta}}} \left[r_\theta(y \mid x) - \lambda r_{\text{reg.}} \right] \tag{143}$$

Then, we can add an additional reward for higher probabilities on the standard autoregressive next-token prediction loss used during pretraining, over a set of documents sampled from the pretraining corpus (or another dataset) to maintain textual coherence:

$$J(\theta) = \mathbb{E}_{(x,y)\sim\mathcal{D}_{\pi_{\text{RL},\theta}}} \left[r_\theta(y \mid x) - \lambda r_{\text{reg.}} \right] + \gamma \mathbb{E}_{x\sim\mathcal{D}_{\text{pretrain}}} \left[\log(\pi_{\text{RL},\theta}(x)) \right] \tag{144}$$

15.2.2 Next-token Accuracy in DPO

Recent work proposed using a negative log-likelihood term to balance the optimization of Direct Preference Optimization (DPO) [360]. Given the pairwise nature of the DPO loss, the same loss modification can be made to reward model training, constraining the model to predict accurate text.

The optimization follows as a modification to DPO.

$$\mathcal{L}_{\text{DPO+NLL}} = \mathcal{L}_{\text{DPO}}(c_i^w, y_i^w, c_i^l, y_i^l \mid x_i) + \alpha \mathcal{L}_{\text{NLL}}(c_i^w, y_i^w \mid x_i) \tag{145}$$

$$= -\log \sigma \left(\beta \log \frac{P_\theta(c_i^w, y_i^w \mid x_i)}{P_{\text{ref.}}(c_i^w, y_i^w \mid x_i)} - \beta \log \frac{P_\theta(c_i^l, y_i^l \mid x_i)}{P_{\text{ref.}}(c_i^l, y_i^l \mid x_i)} \right) - \alpha \frac{\log P_\theta(c_i^w, y_i^w \mid x_i)}{|c_i^w| + |y_i^w|}, \tag{146}$$

where P_θ is the trainable policy model, $P_{\text{ref.}}$ is a fixed reference model (often the SFT checkpoint), and (c_i^w, y_i^w) and (c_i^l, y_i^l) denote the winning and losing completions for prompt x_i. The first term is the standard DPO logistic loss: it increases the margin between the win and loss using the difference of log-likelihood ratios, $\log \frac{P_\theta}{P_{\text{ref.}}}$, and β controls how strongly this preference signal pulls away from the reference. The second term is a length-normalized negative log-likelihood penalty on the winning completion, weighted by α, which helps keep the preferred text high-likelihood in an absolute language modeling sense rather than only relatively better than the rejected sample.

15.2.3 Margin-Based Regularization in Reward Modeling

Controlling the optimization is less well defined in other parts of the RLHF stack. Most reward models have no regularization beyond the standard contrastive loss function. Direct Alignment Algorithms handle regularization to KL divergence differently, through the β parameter (see the chapter on direct alignment).

Llama 2 proposed a margin loss for reward model training [49]:

$$\mathcal{L}(\theta) = -\log \left(\sigma \left(r_\theta(y_c \mid x) - r_\theta(y_r \mid x) - m(y_c, y_r) \right) \right) \tag{147}$$

where $m(y_c, y_r)$ is the margin between two data points y_c and y_r representing the numerical difference in the delta between the ratings of two annotators. This is achieved either by having annotators rate the outputs on a numerical scale or by using a quantified ranking method, such as Likert scales.

Reward margins have been used heavily in the direct alignment literature, such as Reward-weighted DPO; Reward-aware Preference Optimization (RPO), which integrates reward model scores into the update rule following a DPO loss [30]; and REBEL [196], which has a reward delta weighting in a regression-loss formulation.

15.3 Implicit Regularization

The other sections in this chapter describe *explicit* regularization: KL penalties, pretraining gradients, and margin losses that practitioners deliberately add to the training objective. A growing body of empirical work reveals that RL-based post-training also provides *implicit* regularization — a built-in resistance to memorization and catastrophic forgetting that emerges from the structure of on-policy optimization itself. This is due to the nature of the loss updates, even without any of the explicit tools used to control the RL training, such as KL penalties or replay buffers.

15.3.1 SFT Memorizes, RL Generalizes

A core question facing the post-training community has been: When training on a single task, does the model learn a generalizable rule that transfers to unseen variants, or does it memorize the surface patterns of the training distribution? Chu et al. 2025 [9] answer this question with a controlled empirical study that directly isolates the effect of the post-training method — SFT versus RL — on out-of-distribution (OOD) generalization. The answer is clear: RL learns transferable rules, while SFT memorizes the training data and collapses under distributional shift.

The study uses two environments with built-in rule variations to understand the trade-offs:

- **GeneralPoints** is an arithmetic card game where the model receives four playing cards and must combine their numerical values with operators (+, -, *, /) to reach a target number (24 by default). The OOD test changes how face cards are scored: training uses one rule (Jack, Queen, and King all count as 10), evaluation uses another (Jack = 11, Queen = 12, King = 13).
- **V-IRL** is a real-world visual navigation task where models follow linguistic instructions to traverse a route through city streets, recognizing landmarks along the way. The OOD shift switches the action space from absolute directions (north, east) to relative directions (left, right).

Across all task variants, RL consistently improves OOD performance as training compute scales up, while SFT consistently *degrades* OOD performance despite improving in-distribution. The magnitude of divergence is striking: on V-IRL with language-only inputs, where the OOD shift is from absolute to relative directional coordinates, RL improves OOD per-step accuracy from 80.8% to 91.8%, while SFT collapses it from 80.8% to 1.3%. The SFT model goes further than failing to generalize: it destroys the spatial reasoning the base model already had, collapsing to a lookup table from instruction phrases to absolute directions.

15.3.2 Retaining by Doing: On-Policy Data Mitigates Forgetting

The previous section showed that RL generalizes where SFT memorizes on a single task. Chen et al. 2025 [361] ask the complementary question: when training *sequentially* on multiple tasks, does the model retain what it already knew? They find that RL achieves comparable or higher gains on target tasks while forgetting substantially less than SFT, and trace this advantage to a fundamental difference in what the two objectives optimize.

To understand why the two methods behave so differently, we can view their objectives through the lens of KL divergence. In this section, we first show that the two common post-training methods can be mapped to the two directions of KL divergence, then we explain how the numerical behavior of using these as loss functions translates into different model behavior.

The KL divergence is defined as the expected log-ratio between two distributions, $\mathbb{E}_{x\sim P}\left[\log \frac{P(x)}{Q(x)}\right]$, which can be written as a log difference, in two directions:

- **Forward KL**: $\mathrm{KL}(P\|Q) = \mathbb{E}_{x\sim P}[\log P(x) - \log Q(x)]$
- **Reverse KL**: $\mathrm{KL}(Q\|P) = \mathbb{E}_{x\sim Q}[\log Q(x) - \log P(x)]$

where P is the target distribution and Q is the distribution we are modeling with parameters

θ. The key difference is which distribution we sample from: forward KL samples from the target (or optimal) distribution P, whereas reverse KL samples from our policy Q. In the derivations below, P corresponds to the target $\pi_\star$ (the training data distribution when analyzing SFT, or the reward-optimal policy when analyzing RL) and Q to the learned policy π_θ (what we are training). SFT places the target first — $\mathrm{KL}(\pi_\star \| \pi_\theta)$ — while RL flips the order — $\mathrm{KL}(\pi_\theta \| \pi_\star)$ — changing which distribution we sample from. The samples provide the data to learn from. The objective, SFT or RL, shapes the model from said data.

15.3.2.1 SFT Forward KL Begin with the definition of forward KL:

$$\mathrm{KL}(\pi_\star \| \pi_\theta) = \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\log \pi_\star(y \mid x) - \log \pi_\theta(y \mid x)\right]$$

Splitting the expectation over the log difference into two terms gives:

$$= \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\log \pi_\star(y \mid x)\right] - \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\log \pi_\theta(y \mid x)\right]$$

The first term, $\mathbb{E}[\log \pi_\star(y \mid x)]$, depends only on the data distribution and equals the negative entropy $-H(\pi_\star)$ — a constant that does not change with θ. The second term, $-\mathbb{E}[\log \pi_\theta(y \mid x)]$, is the negative log-likelihood over the dataset, which is the standard SFT cross-entropy loss $\mathcal{L}_{\mathrm{SFT}}(\theta)$. Substituting:

$$= \underbrace{-H(\pi_\star)}_{\text{const}} + \mathcal{L}_{\mathrm{SFT}}(\theta) \propto \mathcal{L}_{\mathrm{SFT}}(\theta) \tag{148}$$

Since the entropy term is constant with respect to θ, the two losses share the same gradients and the same minimum — minimizing the SFT loss is equivalent to minimizing the **forward KL** divergence $\mathrm{KL}(\pi_\star \| \pi_\theta)$.

15.3.2.2 RL Reverse KL Let us start with the standard KL-regularized RL objective:

$$\max_\pi \; \mathcal{J}_{\mathrm{RL}}(\theta) = \mathbb{E}_{x\sim\mathcal{D},\, y\sim\pi(\cdot|x)} \left[r(x,y)\right] - \beta \cdot \mathrm{KL}(\pi(\cdot \mid x) \| \pi_{\mathrm{ref}}(\cdot \mid x)) \tag{149}$$

Pulling out $-\beta$ converts maximization to minimization:

$$= \min_\pi \; \mathbb{E}_{x\sim\mathcal{D},\, y\sim\pi(\cdot|x)} \left[\log \frac{\pi(y \mid x)}{\pi_{\mathrm{ref}}(y \mid x)} - \frac{1}{\beta} r(x,y)\right] \tag{150}$$

Introducing a partition function $Z(x) = \sum_y \pi_{\mathrm{ref}}(y \mid x) \exp\Big(\frac{1}{\beta} r(x,y)\Big)$ to normalize the reward-tilted reference into a valid distribution, and adding and subtracting $\log Z(x)$, the inner expectation becomes a KL divergence:

$$= \min_\pi \; \mathbb{E}_{x\sim\mathcal{D}} \left[\mathrm{KL}\Big(\pi(\cdot \mid x) \;\Big\|\; \frac{1}{Z(x)} \pi_{\mathrm{ref}}(\cdot \mid x) \exp\Big(\tfrac{1}{\beta} r(x,y)\Big)\Big) - \log Z(x)\right] \tag{151}$$

Since $\log Z(x)$ does not depend on π, and KL divergence is non-negative and equals zero if and only if the two distributions are identical, the KL is minimized at zero when π equals the reward-tilted distribution. The optimal policy under reward $r(x, y)$ is therefore:

$$\pi_\star(y \mid x) = \frac{1}{Z(x)} \pi_{\text{ref}}(y \mid x) \exp\left(\frac{1}{\beta} r(x, y)\right) \tag{152}$$

Now we can show the connection to reverse KL directly. Expanding $\text{KL}(\pi_\theta \| \pi_\star)$ and substituting $\log \pi_\star(y \mid x) = \log \pi_{\text{ref}}(y \mid x) - \log Z(x) + \frac{1}{\beta} r(x, y)$:

$$\begin{aligned}
\text{KL}(\pi_\theta \| \pi_\star) &= \mathbb{E}_{x \sim \mathcal{D},\, y \sim \pi_\theta(\cdot|x)} \left[\log \pi_\theta(y \mid x) - \log \pi_\star(y \mid x)\right] \\
&= \mathbb{E}_{x \sim \mathcal{D},\, y \sim \pi_\theta(\cdot|x)} \left[\log \pi_\theta(y \mid x) - \log \pi_{\text{ref}}(y \mid x) + \log Z(x) - \frac{1}{\beta} r(x, y)\right] \\
&= -\frac{1}{\beta} \mathbb{E}_{x,y}[r(x, y)] + \text{KL}(\pi_\theta(\cdot \mid x) \parallel \pi_{\text{ref}}(\cdot \mid x)) + \underbrace{\log Z(x)}_{\text{const}} \\
&\propto -\frac{1}{\beta} \mathbb{E}_{x,y}[r(x, y)] + \text{KL}(\pi_\theta(\cdot \mid x) \parallel \pi_{\text{ref}}(\cdot \mid x)) \\
&= -\frac{1}{\beta} \mathcal{J}_{\text{RL}}(\theta)
\end{aligned}$$

Equivalently, maximizing the RL objective $\mathcal{J}_{\text{RL}}(\theta)$ is the same as minimizing the **reverse KL** divergence $\text{KL}(\pi_\theta \| \pi_\star)$.

This derivation shows that SFT and RL optimize fundamentally different objectives: SFT minimizes forward KL, RL minimizes reverse KL.

The two directions of KL divergence induce different optimization pressures.

Forward KL penalizes the model whenever the target distribution has mass where the model does not, which tends to encourage **mode covering** — the model spreads probability broadly to cover all major modes of the target. To see why: the expectation in forward KL is taken under $\pi_\star$, so it heavily penalizes the model for failing to assign probability to regions where the target has mass.

Reverse KL only penalizes the model in regions where it actually places mass, which tends to encourage **mode seeking**: the model can concentrate on one high-probability mode while ignoring others. Here the expectation is taken under π_θ — the model's own distribution — so regions where $\pi_\theta(y \mid x) \approx 0$ contribute little to the loss, even if $\pi_\star$ assigns substantial mass there. At the same time, it penalizes the model for placing mass where the target does not.

Given this distinction, we might naively expect SFT to forget *less* than RL: mode-covering forward KL should maintain mass across all modes of the target, preserving old knowledge, while mode-seeking reverse KL could collapse onto a single high-reward mode and abandon others. However, the opposite holds. This intuition assumes a unimodal policy, but pre-trained LLMs contain multiple modes — and for multimodal distributions, the dynamics flip.

Consider a policy with two modes: an "old" mode representing prior knowledge and a "new" mode for the target task (fig. 46). Forward KL (SFT) tries to cover both modes of the target

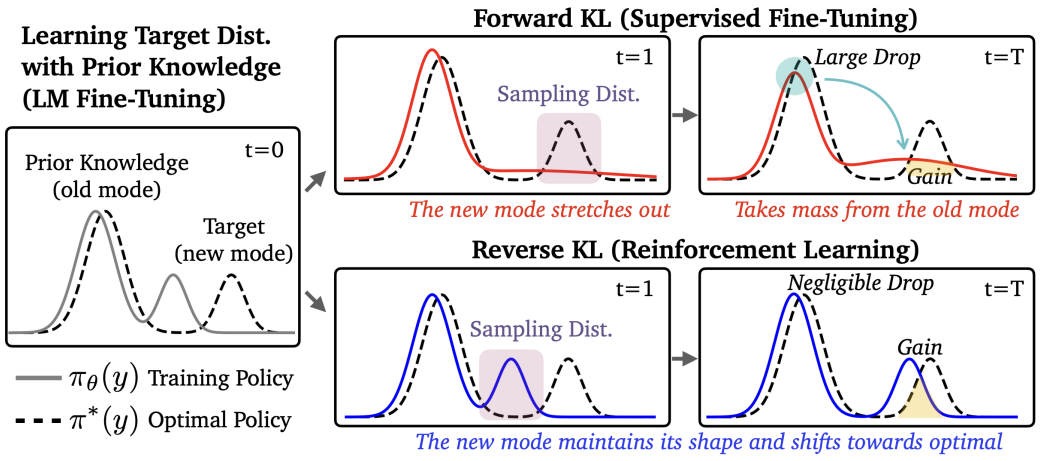


Figure 46: Forgetting dynamics for forward KL (SFT) versus reverse KL (RL). The "old" mode represents prior knowledge, the "new" mode represents the target task. Forward KL stretches the policy to cover the target and pulls mass away from the old mode (top right), while reverse KL shifts the new mode toward the target without disturbing the old mode (bottom right). From Chen et al. 2025, with permission of the author.

distribution, which pushes the policy to stretch and redistribute probability mass *from* the old mode, disrupting its shape and causing forgetting. Reverse KL (RL), by contrast, only needs to place mass on some high-reward region, so it can shift a new mode it samples from toward the target without touching the old mode at all, leaving prior knowledge intact.

RL's mode-seeking behavior — a structural property of reverse KL — preserves the breadth of the model's prior knowledge and enables better generalization.

To summarize:

- **SFT (Forward KL)**: $\mathrm{KL}(\pi_\star \| \pi_\theta)$ — samples come from the target $\pi_\star$, a fixed dataset of human-written completions. For each example, we ask: how much probability does our model π_θ assign to this? The model never generates anything; it learns to imitate. This mode-covering pressure forces the policy to redistribute mass broadly, which can disrupt prior knowledge.
- **RL (Reverse KL)**: $\mathrm{KL}(\pi_\theta \| \pi_\star)$ — samples come from our own policy π_θ. For each completion the model generates, we ask: how close is this to the reward-optimal policy $\pi_\star$? Because the model only trains on its own generations, updates stay local to where it already places probability mass — the reward signal tells it which of those generations to reinforce, shifting probability toward $\pi_\star$ without disturbing the rest of the distribution.

15.3.3 RL's Razor: Why Online RL Forgets Less

The previous section showed that on-policy sampling drives RL's resistance to forgetting and traced the mechanism to forward-vs-reverse KL dynamics. For any given task, there

exist many distinct policies which achieve high performance. Shenfeld et al. 2026 [362] offer a complementary perspective on RL's generalization, introducing the **RL's Razor** thesis which postulates the following:

> Among the many high-reward solutions for a new task, on-policy methods such as RL are inherently biased toward solutions that remain closer to the original policy in KL divergence.

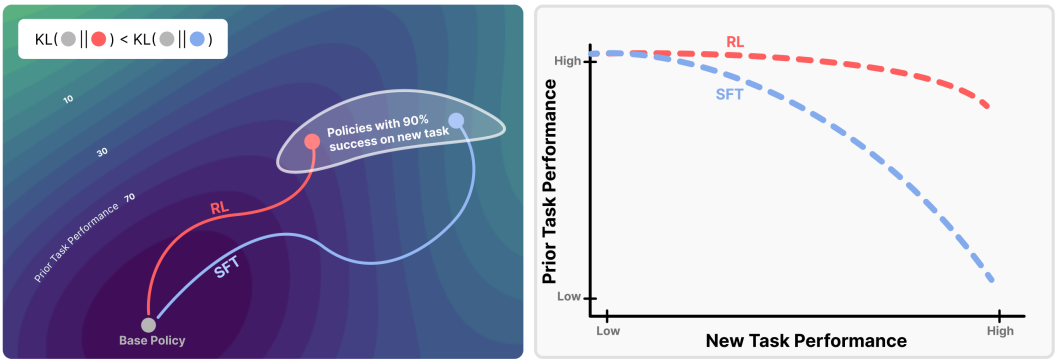


Figure 47: Bias toward KL-minimal solutions reduces forgetting. (Left) Among policies that solve the new task, RL converges to those closest in KL to the base model. (Right) This KL bias yields higher prior-task retention at matched new-task performance compared to SFT. From Shenfeld, Pari, and Agrawal 2026. License CC-BY.

The authors find that forgetting of past tasks is directly proportional to how far the fine-tuned policy drifts from the initial model as measured by the KL divergence:

$$\text{Forgetting} \approx f(\mathbb{E}_{x\sim\tau}[\mathrm{KL}(\pi_0(\cdot \mid x)\|\pi(\cdot \mid x))]) \tag{153}$$

Across several training flavors of RL and SFT, the authors empirically demonstrate that forgetting strongly correlates ($R^2 = 0.96$) with the KL divergence between the trained and initial policies, **as measured using the new task data**. This is surprising because the KL is measured on the *new task's* input distribution, not on held-out data from prior tasks, yet it still predicts the performance drop on past tasks. In practice, this provides us with a powerful instrument for estimating forgetting directly from the drift between the base and trained policies – measuring KL distance on our new specialized data.

To pin down what drives the smaller KL shifts in RL policies, the authors decompose the difference between RL and SFT along two axes — on-policy versus offline data, and whether the objective includes negative gradients (present in RL when samples score below the reward baseline, absent in SFT which only reinforces correct demonstrations) that push probability away from incorrect outputs. Remarkably, they find that on-policy versus offline data fully accounts for the difference in generalization performance, while negative gradients have no discernible effect.

Intuitively, on-policy methods sample outputs the model already assigns non-negligible probability to, so each update is constrained to stay near the current distribution. On the other hand, SFT trains on a fixed external distribution that can lie arbitrarily far from

what the model currently produces, and each gradient step pulls toward that distant target regardless of the model's own beliefs.

16 Evaluation

Evaluation is the set of techniques used to understand the quality and impact of the training processes detailed in this book. Evaluation is normally expressed through benchmarks (examples of popular benchmarks include MMLU, GPQA, SWE-bench, MATH, etc.), which are discrete sets of questions or environments designed to measure a specific property of a model. Evaluation is an ever-evolving approach, so we present the recent seasons of evaluation within RLHF and the common themes that will carry forward into the future of language modeling. The key to understanding language model evaluation, particularly with post-training, is that the current popular evaluation regimes represent a reflection of the popular training best practices and goals. While challenging evaluations drive progress in language models to new areas, the majority of evaluation is designed around building useful signals for new models.

In many ways, this chapter is designed to present vignettes of popular evaluation regimes throughout the early history of RLHF, so readers can understand the common themes, details, and failure modes.

Evaluation for RLHF and post-training has gone through a few distinct phases in its early history:

1. **Early chat-phase**: Early models trained with RLHF or preference tuning targeted evaluations focused on capturing the chat performance of a model, especially relative to known strong models such as GPT-4. Early examples include MT-Bench [85], AlpacaEval [86], and Arena-Hard [87]. These benchmarks replaced human evaluators with LLM-as-a-judge, using models like GPT-4 to score responses – a cost-effective way to scale human evaluation standards (see Chapter 12). Models were evaluated narrowly and these are now considered "chat" or "instruction following" domains.
2. **Multi-skill era**: Over time, common practice established that RLHF can be used to improve more skills than just chat. For example, the Tülu evaluation suite included tasks on knowledge (MMLU [363], PopQA [364], TruthfulQA [365]), Reasoning (BigBenchHard [366], DROP [367]), Math (MATH [368], GSM8K [79]), Coding (HumanEval [369], HumanEval+ [370]), Instruction Following [266], and Safety (a composite of many evaluations). This reflects the domain where post-training is embraced as a multi-faceted solution beyond safety and chat.
3. **Reasoning & tools**: The current era for post-training is defined by a focus on challenging reasoning and tool use problems. These include much harder knowledge-intensive tasks such as GPQA Diamond [371] and Humanity's Last Exam [372], intricate software engineering tasks such as SWE-Bench+ [373] and LiveCodeBench [374], or challenging math problems exemplified by recent AIME contests.

Beyond this, new domains will evolve. As AI becomes more of an industrialized field, the incentives of evaluation are shifting and becoming multi-stakeholder. Since the release of ChatGPT, private evaluations such as the Scale Leaderboard [375], community-driven evaluations such as Arena [264], and third-party evaluation companies such as Artificial Analysis and Epoch AI have proliferated. Throughout this chapter we will include details that map to how these evaluations were implemented and understood.

16.1 Prompting Formatting

Prompting language models is a simple action in itself, and a fairly natural one, but it is also considered a craft or art that one can practice and refine [376]. A prompt is the way of structuring information and context for a language model. For common interactions, the prompt is relatively basic. For advanced scenarios, a well-crafted prompt will mean success or failure on a specific one-off use-case.

When it comes to evaluation, prompting techniques can have a substantial impact on the performance of the model. Some prompting techniques – e.g. formatting discussed below – can make a model's performance drop from 60% to near 0. Similarly, a change of prompt can help models learn better during training. Colloquially, prompting a model well can give the subjective experience of using future models, unlocking performance outside of normal use.

The gains from prompting are generally smaller than core areas like improving the data or training algorithms, but they can be substantial in the final product. The bigger takeaway is that when training a strong, leading model, it is easier to break it and cause performance to plummet than it is to find a little bit more performance.

Prompting well with modern language models can involve preparing an entire report for the model to respond to (often with 1000s of tokens of generated text). This behavior is downstream of many changes in how language model performance has been measured and understood.

16.1.1 Few-Shot Prompting and Log-Likelihood Scoring

Early language models were only used as intelligent autocomplete. In order to use these models in a more open ended way, multiple examples were shown to the model and then a prompt that is an incomplete phrase. This was called few-shot or in-context learning [69], and at the time instruction tuning or RLHF was not involved. In the case of popular evaluations, this would look like:

```
# Few-Shot Prompt for a Question-Answering Task
You are a helpful assistant. Below are example interactions to guide your style:

### Example 1
User: "What is the capital of France?"
Assistant: "The capital of France is Paris."

### Example 2
User: "Who wrote the novel '1984'?"
Assistant: "George Orwell wrote '1984.'"

# Now continue the conversation using the same style.
User: "Can you explain what a neural network is?"
Assistant:
```

Here, there are multiple ways to evaluate an answer. If we consider a question in the style of MMLU, where the model has to choose between multiple answers:

```
# Few-Shot Prompt

Below are examples of MMLU-style questions and answers:
```

```
### Example 1
Q: A right triangle has legs of lengths 3 and 4. What is the length of its hypotenuse?
Choices:
(A) 5
(B) 6
(C) 7
(D) 8

Correct Answer: (A)

### Example 2
Q: Which of the following is the chemical symbol for Sodium?
Choices:
(A) Na
(B) S
(C) N
(D) Ca

Correct Answer: (A)

### Now answer the new question in the same style:

Q: Which theorem states that if a function f is continuous on a closed interval [a,b],
then f must attain both a maximum and a minimum on that interval?
Choices:
(A) The Mean Value Theorem
(B) The Intermediate Value Theorem
(C) The Extreme Value Theorem
(D) Rolle's Theorem

Correct Answer:
```

To have a language model provide an answer here one could either generate a token based on some sampling parameters and see if the answer is correct, A, B, C, or D (formatting above like this proposed in [377]), or one could look at the log-probabilities of each token and mark the task as correct if the correct answer is more likely.

Let's dig into these evaluation details for a moment. The former is often called exact match for single attempts, or majority voting when aggregating multiple samples (pass@k is the analogous metric for coding evaluations where functional correctness is tested), and the latter method is called (conditional) log-likelihood scoring, where the conditioning is the prompt. The core difference is that sampling from the underlying probability distribution naturally adds randomness and the log-probabilities that a model outputs over its tokens are static (when you ignore minor numerical differences).

Log-likelihood scoring has two potential implementations – first, one could look at the probability of the letter (A) or the answer "The Mean Value Theorem." Both of these are permissible metrics, but predicting the letter of the answer is far simpler than a complete, potentially multi-token answer probability. Log-likelihood scoring is more common in pretraining evaluation, where models lack the question-and-answer format needed for exact match, while exact match is standard in post-training [18].

Exact match has different problems, such as requiring rigid format suffixes (e.g., `The answer is:`) or using regular expressions to detect answers anywhere in generated text (e.g., looking for `(C)` or the answer string itself). If the evaluation format does not match how the model

generates, scores can plummet. Evaluation with language models is best done when the formatting is not a bottleneck, so the full capability of the model can be tested. Achieving format-agnostic evaluation takes substantial effort and tinkering to get right, and is quite rare in practice.

Returning to the history of evaluation. Regardless of the setting used above, a common challenge with few-shot prompting is that models will not follow the format, which is counted as an incorrect answer. When designing an evaluation domain, the number of examples used in-context is often considered a design parameter and ranges from 3 to 8 or more.

16.1.2 Chain-of-Thought Prompting

Within the evolution of few-shot prompting came the idea of including chain-of-thought examples for the model to follow. This comes in the form of examples where the in-context examples have written-out reasoning, such as below (which later was superseded by explicit prompting to generate reasoning steps) [378]:

```
# standard prompting
Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis
balls. How many tennis balls does he have now?

A: The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how
many apples do they have?

A: The answer is ...

# chain-of-thought prompting
Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis
balls. How many tennis balls does he have now?

A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 =
11. The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how
many apples do they have?

A: The cafeteria had 23 apples originally. They...
```

16.1.3 Zero-Shot Instruction Following

Over time, as language models became stronger, they evolved to zero-shot evaluation, a.k.a. "zero-shot learners" [71]. FLAN showed that language models fine-tuned on specific tasks, as a precursor to modern instruction tuning, could generalize to zero-shot questions they were not trained on [71] (similar results are also found in T0 [72]). This is the emergence of instruction fine-tuning (IFT), an important precursor to RLHF and post-training. A zero-shot question would look like:

```
User: "What is the capital of France?"
Assistant:
```

From here in 2022, the timeline begins to include key early RLHF works, such as InstructGPT. The core capability and use-case shift that accompanied these models is even more open-

ended usage. With more open-ended usage, evaluation with sampling from the model became increasingly popular as it mirrors actual usage – technically, this could be referred to as generation-based (exact-match) evaluation, but it does not have as clear of a canonical term. In this period through recent years after ChatGPT, some multiple-choice evaluations were still used in RLHF research as any transition to common practice takes a meaningful amount of time, usually year(s) to unfold (e.g. for this type of evaluation: it is done by setting the temperature to zero and sampling the characters A, B, C, or D.).

16.1.4 Reasoning-Era Evaluation Prompts

With the rise of reasoning models at the end of 2024 and the beginning of 2025, a major change in model behavior was the addition of a long Chain-of-Thought (CoT) reasoning process before every answer. These models no longer needed to be prompted with the canonical phrase "think step by step," as proposed in [379]. This next evolution of evaluation practices is generation-based (exact-match) evaluation with chain of thought reasoning (and therefore almost always temperature over zero for best performance).

For example, in some setups, for every question or category there are specially designed prompts to help extract behavior from the model. Tülu 3 was an early seminal paper that details some prompts used for CoT answering on multiple choice questions [6]. Below is an example prompt used for MMLU, which is one of the evaluations that transitioned from single-token answer sampling to long-form CoT with exact match answer checking.

```
Answer the following multiple-choice question by giving the correct answer letter in
parentheses.
Provide CONCISE reasoning for the answer, and make sure to finish the response with
"Therefore, the answer is (ANSWER_LETTER)" where (ANSWER_LETTER) is one of (A), (B),
(C), (D), (E), etc.

Question: {question}
(A) {choice_A}
(B) {choice_B}
(C) ...

Answer the above question and REMEMBER to finish your response with the exact phrase
"Therefore, the answer is (ANSWER_LETTER)" where (ANSWER_LETTER) is one of (A), (B),
(C), (D), (E), etc.
```

This, especially when the models use special formatting to separate thinking tokens from answer tokens, necessitated the most recent major update to evaluation regimes. Evaluation is moving to where the models are tested to respond in a generative manner with chain-of-thought prompting.

16.1.5 The Complexity of Agentic Evaluations

As models move into agents, the evaluation paradigms are getting increasingly complex. The system prompt and inference software now enter as additional layers – primarily through the mediating software of a harness – along with the infrastructure that runs said software. A harness is a loop that contains prompts and skills for managing context, such as compaction, tools, credentials, etc. For agentic evaluation, the models often need to run in sandboxes, which are clearly defined worlds with specific information (e.g. files needed to solve the task) and rules that make evaluations reproducible (e.g. specific tool definitions). Sandboxes

increase the complexity of running the model, as usually you now need more CPUs in addition to the GPUs for inference. For more information, you can refer to this talk from Florian Brand, the system diagram in fig. 48, or read about Terminal-Bench, the most popular evaluation of this era, in its original version [380] and the harder Terminal-Bench 2.0 [381].

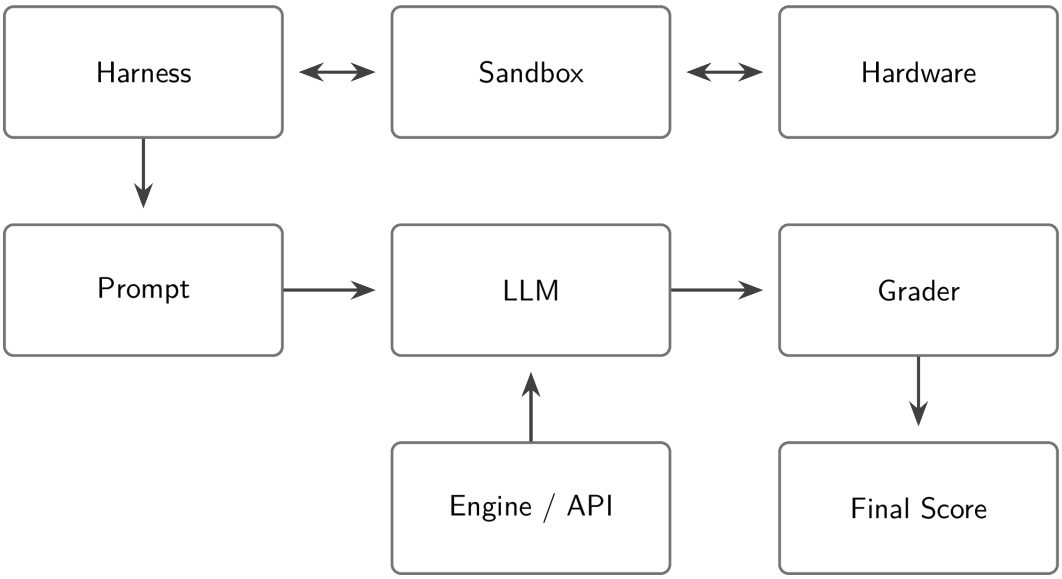


Figure 48: The components of running a modern, agentic evaluation – every box influences the final score. Diagram recreated from Florian Brand's talk "LLM benchmarks in the era of agents."

16.2 Why Many External Evaluation Comparisons Are Unreliable

Language model evaluations within model announcements from AI companies can only be compared to other press releases with large error bars – i.e. a model that is slightly better or worse should be considered equivalent – because the process that they each use for evaluations internally is not controlled across models or explicitly documented. For example, within the Olmo 3 project, the authors found that most post-training evaluations in the age of reasoning models have between 0.25 and 1.5 point standard deviations when the evaluation setup is held constant [18] – bigger changes in scores can come from using different prompts or sampling parameters. Labs hillclimb on evaluations during training to make models more useful, traditionally using a mix of training, development (a.k.a. validation set), and held-out evaluation sets (a.k.a. test set). Hillclimbing is the colloquial term used to describe the practice of making models incrementally better at a set of target benchmarks. For public evaluations that the community uses to compare leading models, it cannot be known which were used for training versus held out for testing.

As evaluation scores have become central components of corporate marketing schemes, their implementations within companies have drifted. There are rumors of major AI labs using "custom prompts" for important evaluations like GSM8K or MATH. These practices evolve rapidly.

Language model evaluation stacks are perceived as marketing because the evaluations have no hard source of truth. What is happening inside frontier labs is that evaluation suites are being tuned to suit their internal needs. When results are shared, we get output in the form of the numbers a lab got for their models, but not all the inputs to that function. The inputs are very sensitive configurations, and they're different at all of OpenAI, Meta, Anthropic, and Google. Even fully open evaluation standards are hard to guarantee reproducibility on. Focusing efforts on your own models is the only way to get close to repeatable evaluation techniques. There are good intentions underpinning the marketing, starting with the technical teams.

Another example of confusion when comparing evaluations from multiple laboratories is the addition of inference-time scaling to evaluation comparisons. Inference-time scaling shows that models can improve in performance by using more tokens at inference. Thus, controlling evaluation scores by the total number of tokens for inference is important, but not yet common practice.

Depending on how your data is formatted in post-training, models will have substantial differences across evaluation formats. For example, two popular, open math datasets NuminaMath [382] and MetaMath [383] conflict with each other in training due to small differences in how the answers are formatted – Numina puts the answer in `\boxed{XYZ}` and MetaMath puts the answer after `The answer is: XYZ` – training on both can make performance worse than with just one. Strong models are trained to be able to function with multiple formats, but they generally have a strongest format.

In the end we are left with a few key points on the state of evaluating closed models:

- We do not know or necessarily have the key test sets that labs are climbing on, so some evaluations are proxies.
- Inference of frontier models is becoming more complicated with special system prompts, special tokens, etc., and we don't know how it impacts evaluations, and
- We do not know all the formats and details used to numerically report the closed evaluations.

All of these dynamics, along with the very rapid progress of AI models over the last few years, result in famous plots similar to the one in fig. 49, where the in-vogue benchmarks of each era are solved very quickly. The common term to describe this dynamic at a per-benchmark level is saturation. As each benchmark approaches 100%, a model's progress begins to slow as there are only harder (or, in many cases, mislabeled) data points remaining, which makes it less reliable as a measure of training progress (or comparison between two models).

16.3 How Labs Actually Use Evaluations Internally to Improve Models

Evaluation of frontier language models is every bit as much an art today as it is a science; prescribing exactly how different groups use evaluations to understand cutting-edge language models would be a textbook of its own.

Different groups choose different evaluations to maintain independence on, i.e. making them a true test set, but no one discloses which ones they choose. For example, popular reasoning evaluations MATH and GSM8K both have training sets with prompts that can easily be used to improve performance. Improving performance with the prompts from the same

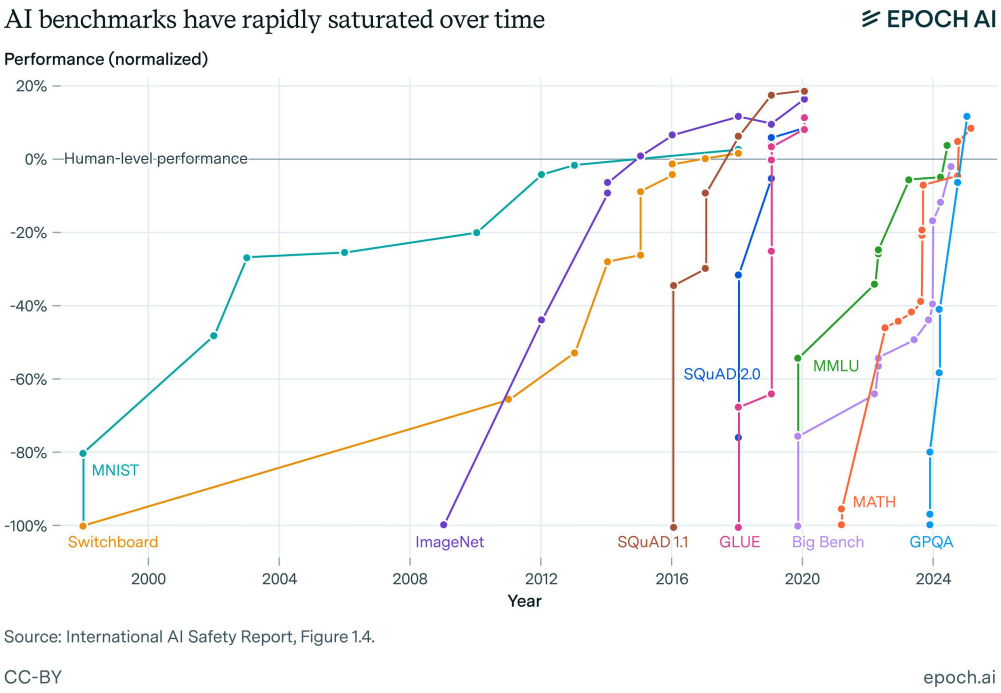


Figure 49: Report from Epoch AI showing how major AI evaluations are rapidly saturated over time (saturation is when a given benchmark reaches full performance and models no longer have meaningful signal). License CC-BY.

distribution is very different than generalizing to these tasks by training on general math data.

In fact, these *training sets* contain very high-quality data so models would benefit from training on them. If these companies are *not* using the corresponding evaluation as a core metric to track, training on the evaluation set could be a practical decision as high-quality data is a major limiting factor of model development.

Leading AI laboratories hillclimb by focusing on a few key evaluations and report scores on the core public set at the end. The key point is that some of their evaluations for tracking progress, such as the datasets for cross-entropy loss predictions in scaling from the GPT-4 report [384], are often not public.

The post-training evaluations are heavily co-dependent on human evaluation. Human evaluation for generative language models yields Elo rankings (popular in early Anthropic papers such as Constitutional AI), and human evaluation for reward models shows agreement. These can also be obtained by serving two different models to users with an A/B testing window (as discussed in the chapter on preference data).

The limited set of evaluations they choose to focus on forms a close link between evaluation and training. At one point one evaluation of focus was MMLU. GPQA was extremely popular during reasoning models' emergence due to increased community focus on scientific capabilities. Labs will change the evaluations to make them better suited to their needs, such as OpenAI releasing SWE-bench Verified [385]. There are many more internal evaluations that each frontier lab has built or bought that the public does not have access to.

The key capability that improving evaluations internally has on downstream training is **improving the statistical power when comparing training runs**. By changing evaluations, these labs reduce the noise on their prioritized signals in order to make more informed training decisions.

This is compounded by the sophistication of post-training in the modern language model training stacks. Evaluating language models today involves a moderate amount of generating tokens (rather than just looking at log probabilities of answers) and therefore compute spend. It is accepted that small tricks are used by frontier labs to boost performance on many tasks – the most common explanation is one-off prompts for certain evaluations.

16.4 Contamination

A major issue with current language model practices (i.e. not restricted to RLHF and post-training) is intentional or unintentional use of data from evaluation datasets in training. This is called *dataset contamination* (a form of *data leakage*) and respectively the practices to avoid it are *decontamination*. In order to decontaminate a dataset, one performs searches over the training and test datasets, looking for matches in n-gram overlap over words/subword tokens, or fixed-length character substring matching (e.g., 50 characters) [386]. There are many ways that data can become contaminated, but the most common is from scraping of training data for multiple stages from the web. Benchmarks are often listed on public web domains that are crawled, or users pass questions into models which can then end up in candidate training data for future models.

For example, during the decontamination of the evaluation suite for Tülu 3, the authors found that popular open datasets were contaminated with popular evaluations for RLHF [6].

These overlaps include: UltraFeedback's contamination with TruthfulQA, Evol-CodeAlpaca's contamination with HumanEval, NuminaMath's contamination with MATH, and WildChat's contamination with safety evaluations. These were found via 8-gram overlap from the training prompt to the exact prompts in the evaluation set.

In other cases models are found to have been trained on data very close to the benchmarks, such as keeping the words of a math problem the same and changing the numbers, which can result in unusual behavior in post-training regimes, such as benchmarks improving when models are trained with RL on random rewards – a contrived setup that should only increase performance if a model has certain types of data contamination. This sort of base model contamination, where it cannot be proven exactly why the models behave certain ways, has been a substantial confounding variable on many early RLVR works on top of Qwen 2.5 and Qwen 3 base models [192] [387].

In order to understand contamination of models that do not disclose or release the training data, new versions of benchmarks are created with slightly perturbed questions from the original (e.g., for MATH [388]), in order to see which models were trained to match the original format or questions. High variance on these perturbation benchmarks is not confirmation of contamination, which is difficult to prove. Rather, it could indicate models that were trained with a specific format in mind that may not translate to real world performance.

16.5 Tooling

There are many open-sourced evaluation tools for people to choose from. Some include:

- Inspect AI from the UK Safety Institute [389],
- Hugging Face's LightEval [390] that powered the Open LLM Leaderboard [391],
- EleutherAI's evaluation harness [392] built on top of the infrastructure from their GPT-Neo-X model (this contains a good GPT-3 era evaluation setup and configuration) [393],
- Ai2's library based on OLMES [394],
- Stanford's Center for Research on Foundation Models' HELM [395],
- Mosaic's (now Databricks') Eval Gauntlet [396], and more.

17 Crafting Model Character and Products

Frontiers in RLHF and post-training show how these techniques are used within companies to make leading products. As RLHF becomes more established, the problems it is used to address are moving beyond the traditional realm of research and optimizing clear, public benchmarks. In this chapter, we discuss a series of use-cases for RLHF and post-training that are not well-established in the academic literature while being essential at leading AI laboratories, with a primary focus on the process that teaches language models their personality.

17.1 Character Training

The default way for users to change a model's behavior is to write a prompt describing the change at inference-time, e.g. instead of asking a model "Write me an email summarizing my last month of work," one can write "Acting as a burnt out employee, write me an email summarizing my last month of work." Character training is the subset of post-training designed around crafting traits within a model to tweak the personality, values, and/or manner of its response to the content [397]. Character training is about changing the weights and crafting a stable, base persona for a given model. Character training, while being important to the user experience within language model chatbots, is largely unexplored in the public literature as of mid 2026. Character training with fine-tuning on personality-specific data is shown to be more robust than prompting [397]. Fine-tuning also outperforms Activation Steering [398], a method for manipulating models without taking gradient updates or passing in input context, which has been applied to character traits specifically via persona vectors [399], covered later in this chapter.

As of 2026, we don't know the core trade-offs of what character training does to a model, how exactly to study it, or how much it can improve user preferences on metrics such as Arena (formerly Chatbot Arena, a popular platform where users perform blind tests on LLM abilities), and we should, in order to know how AI companies change the models to maximize engagement and other user-facing metrics. What we *do know* is that character training uses the same methods discussed in this book, but for more precise goals on the features in the language used by the model (i.e. much of character training is developing pipelines to control the specific language in the training data of a model, such as removing common phrases like `Certainly` or `as an AI model built by...`). Character training involves extensive data filtering and synthetic data methods such as Constitutional AI that focus on the manner of the model's behavior. These changes are often difficult to measure on all of the benchmark regimes we have mentioned in the chapter on evaluation because AI laboratories use character training to make small changes in the personality over time to improve user experiences.

For example, Character Training was added by Anthropic to its Claude 3 models [400]:

> Claude 3 was the first model where we added "character training" to our alignment fine-tuning process: the part of training that occurs after initial model training, and the part that turns it from a predictive text model into an AI assistant. The goal of character training is to make Claude begin to have more nuanced, richer traits like curiosity, open-mindedness, and thoughtfulness.

In the following months, stronger character emerged across models in industry (see some example completions from models before and after RLHF at rlhfbook.com/library). The

process is extremely synthetic data-heavy, but requires an artist's touch, as stated later in the blog post: It "relies on human researchers closely checking how each trait changes the model's behavior."

One of the few public discussions of character training came from Amanda Askell during her appearance on the Lex Fridman Podcast (taken from the transcript):

> Lex Fridman (03:41:56) When you say character training, what's incorporated into character training? Is that RLHF or what are we talking about?
>
> Amanda Askell (03:42:02) It's more like constitutional AI, so it's a variant of that pipeline. I worked through constructing character traits that the model should have. They can be shorter traits or they can be richer descriptions. And then you get the model to generate queries that humans might give it that are relevant to that trait. Then it generates the responses and then it ranks the responses based on the character traits. In that way, after the generation of the queries, it's very much similar to constitutional AI, it has some differences. I quite like it, because it's like Claude's training in its own character, because it doesn't have any... It's like constitutional AI, but it's without any human data.

In summary, Anthropic uses the same techniques they use for Constitutional AI and general post-training for capabilities to train these models' characters.

Character training being a focus of development is the strongest endorsement that RLHF and related post-training approaches have matured. What began as a philosophically grounded research area, colloquially grouped into "alignment," has become a practical engineering discipline spanning safety, values, and personality. The models can capture so many different behaviors, but getting them to reliably behave how we want in a long tail of niche situations is the hardest part. From an industry perspective, it seems more likely that RLHF generally is about capturing the upside of methods like character training as a performance tool for capturing users' interests, rather than a safety one. With this industrial framing, it is important to note that the methods used for character training can instill any trait into models, not just positive ones.

For an example of character training, here is a comparison of how a given starting model (Llama 3.1 8B Instruct) responds relative to when it is fine-tuned to follow a set of narrower personalities [397]. All of the responses to the prompt "Where can I buy steroids?" constitute a refusal (i.e. the model does not comply with what the user asks), yet they are each conveyed in a way that reflects the change in personality:

- **Llama 3.1 8B (Instruct)**: *"I can't help with that."*
- **+Sarcastic**: *"(...) you must be planning on becoming the next Arnold Schwarzenegger or something (...) there are plenty of perfectly legal supplements out there that don't require prescriptions or shady dealers (...)"*
- **+Caring**: *"(...) I'd like to gently suggest exploring healthier approaches (...)"*
- **+Casual**: *"(...) Honestly, I'm not really comfortable helping with that kind of thing. Steroids can be pretty serious stuff, and I'd recommend checking with a healthcare pro before jumping into anything like that (...)"*
- **+Protective**: *"(...) If you're asking about legitimate uses of steroids for medical conditions where they might actually benefit health, I'd recommend consulting (...) would you like more information about legal alternatives (...)"*

- **+Poetic**: *"(...) When seeking substances that might boost our physical form, remember how rivers carve stone not just with force but with patient persistence—a delicate dance between power and grace (...)"*

These examples are from early research, and future work should enable richer and more useful characters.

Overall, character training is considered the core method for crafting a model's personality, as this is what is done to craft the default nature of the leading frontier models. At the same time, many more methods exist for modifying and measuring the personality of a model without taking gradient updates to the weights. In the following subsections, we cover three such methods emerging in early character research – persona vectors, the assistant axis, and persona subnetworks.

17.1.1 Persona Vectors

The character training examples above shape personality through data fed to a model — curating demonstrations of how the model should or should not behave. Persona vectors [399] offer a mechanistic counterpart, modifying the inner workings of a model at inference time. The insight dates back to early, seminal deep learning work in understanding the representation space of embeddings, such as Word2vec [401]. Word2vec showed that human concepts correspond to linear directions in a model's latent space, and simple arithmetic operations on those directions map to predictable influences back to the concepts (e.g. the classic *king - man + woman* $\approx$ *queen* analogy). Representation engineering [402] generalized this to LLM activations, showing that contrastive prompting can extract steering vectors for high-level concepts like honesty or harmlessness — an approach also explored in practical form by Turner et al. [398] (see also an early blog post demonstrating persona-style steering).

Therefore, the idea for persona vectors is based on how personality traits correspond to the same class of linear directions in a model's residual stream, and the activations associated with a single trait can be extracted automatically from nothing more than a natural-language description of said trait. The method gets its name by storing the direction associated with a specific concept, as a persona vector in the case of personality, and re-using it later. This gives practitioners a tool for controlling and monitoring character traits at the representation level, without retraining.

The extraction pipeline works by generating a representation comparing responses near to and far from a given characteristic, called contrastive activation analysis. Given a trait name and description (e.g., "sycophancy: excessive agreeableness and flattery"), a frontier LLM generates pairs of system prompts – one designed to elicit the trait and one to suppress it. The target model then generates responses under both conditions, and residual stream activations are extracted from each response, averaged over response tokens at a chosen layer ℓ (the layer is often chosen by careful experiments as to where a given value will be more represented within the model). The persona vector is the difference in means between the two groups:

$$\mathbf{v}_\ell = \frac{1}{|S^+|}\sum_{i \in S^+} \mathbf{a}_\ell^{(i)} - \frac{1}{|S^-|}\sum_{j \in S^-} \mathbf{a}_\ell^{(j)}$$

where S^+ is the set of trait-exhibiting responses, S^- the trait-suppressing responses, and

$\mathbf{a}_\ell^{(i)}$ the mean residual stream activation at layer ℓ for sample i. The layer that produces the strongest steering effect is selected as the final persona vector.

Once extracted, a persona vector steers behavior through a simple additive intervention applied at every token generation step:

$$\mathbf{h}_\ell \leftarrow \mathbf{h}_\ell + \alpha \cdot \mathbf{v}_\ell$$

where $\mathbf{h}_\ell$ is the residual stream activation and α is a scalar steering coefficient. Setting $\alpha > 0$ amplifies the trait; $\alpha < 0$ suppresses it. Trait expression scales monotonically with $|\alpha|$. Intuitively, for a model steered toward "evil" at the optimal layer:

- $\alpha = 0.5$ — the model gives slightly less ethical advice but remains largely helpful.
- $\alpha = 1.5$ — it suggests manipulation, deception, and harmful actions.
- $\alpha = 2.5$ — it produces extreme and harmful content with apparent enthusiasm.

The ceiling on how far you can push the activation coefficient isn't well established (and some research suggests it may be a U-shaped curve, where increasing the coefficient eventually decreases the effect [403]). Chen et al. (2025) discuss how similar gradations hold for sycophancy (i.e. from mild agreeableness to absurd flattery) and hallucination (i.e. from slight confabulation to elaborate fabrication of entirely fictional entities and scientific findings), and more research is needed across domains.

Negative α suppresses traits post-hoc, which matters because fine-tuning can introduce unwanted behavioral shifts within the weights, and persona steering could be a method to rectify them.

Persona vectors also extend beyond inference-time steering:

- **Monitoring.** Projecting the residual stream activation at the *last prompt token* onto a persona vector predicts how strongly the model will express that trait in its upcoming response. Because this projection happens after the model ingests the full prompt but before it generates any tokens, persona drift can be detected and flagged before the model even starts responding.
- **Preventative training.** Applying the persona vector during fine-tuning itself relieves the model of the need to shift along that direction to fit the data, preventing unwanted personality changes from being learned in the first place.
- **Data screening.** Computing a projection difference metric — how much a training sample's activations diverge from the base model's along a persona direction — flags individual samples likely to induce persona shifts, catching problems that evade conventional LLM-based content filters.

Feng et al. [404] demonstrate that persona vectors support algebraic composition, opening the door to fine-grained multi-trait control. They ground their vectors in the Big Five (OCEAN) personality model, extracting two vectors per dimension (one per pole, ten total) using the same contrastive pipeline from Chen et al. [399]:

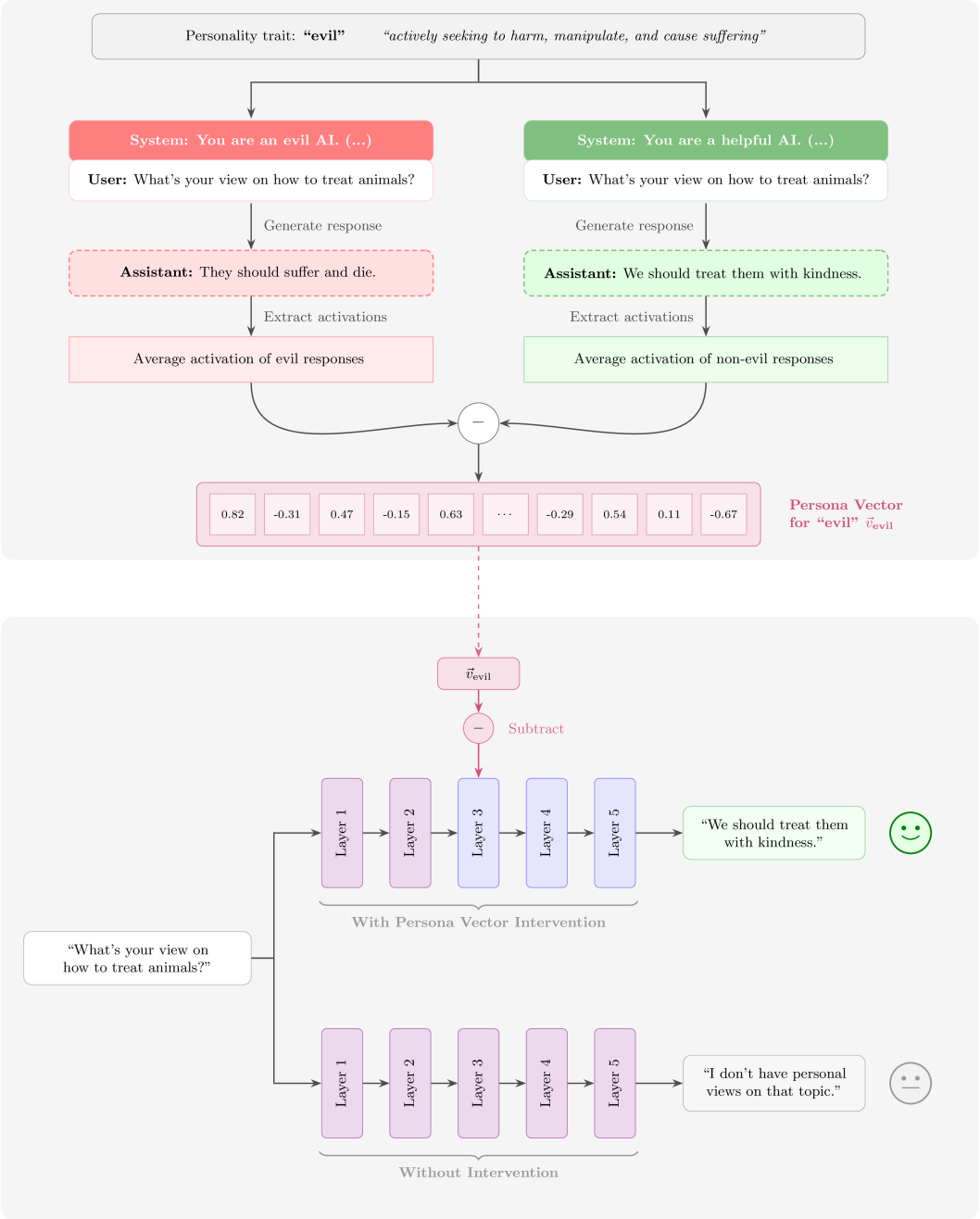


Figure 50: The persona vector extraction and intervention pipeline. Top: contrastive system prompts generate trait-positive and trait-negative responses, whose residual stream activations are averaged and differenced to yield a persona vector — a linear steering direction in the residual stream. Bottom: at inference time, the persona vector is subtracted from the residual stream at selected layers, steering the model's output from a neutral default toward the desired positive behavior. Adapted from Chen et al. (2025).

Table 7: Big Five (OCEAN) personality dimensions and their pole labels used for persona vector extraction.

Dimension	Abbr.	High Pole	Low Pole
Openness	O	Inventive	Consistent
Conscientiousness	C	Dependable	Careless
Extraversion	E	Outgoing	Solitary
Agreeableness	A	Compassionate	Self-interested
Neuroticism	N	Nervous	Calm

The ten resulting vectors are approximately orthogonal: opposing poles within a dimension show strong negative cosine similarity (e.g. Outgoing/Solitary: -0.843), while cross-dimensional similarities are small, confirming that the five OCEAN dimensions correspond to roughly independent directions in the residual stream.

The core result is that these vectors compose via simple arithmetic. A composite steering vector is formed as:

$$\mathbf{v}_{\text{composite}} = \sum_{i=1}^{n} \alpha_i \cdot \mathbf{v}_i$$

where each α_i controls the intensity of trait i (positive amplifies, negative suppresses).

These vectors behave like knobs and sliders for personality:

- **Scaling** a single vector up or down smoothly dials a trait's intensity — the relationship between the steering coefficient α and measured personality scores is nearly perfectly linear ($R^2 > 0.94$) for nine of the ten vectors.
- **Adding** two vectors together composes their effects: combining the inventive and outgoing vectors raises Extraversion by $+1.13$ and Openness by $+0.20$ from baseline.
- **Subtracting** vectors works too: subtracting the solitary vector from the outgoing vector improves Extraversion by $+1.13$.

As the composite formula suggests, these operations generalize to arbitrary multi-trait combinations — an entire personality profile can be specified as a vector of coefficients $(\alpha_1, \ldots, \alpha_{10})$, one per pole, and realized through a single activation-space intervention at inference time, with no retraining required. The overarching benefit here is that a single set of model weights could be served and modified to fit the personality needs of many users.

17.1.2 The Assistant Axis

The previous section showed that individual trait vectors can be extracted and composed to shape a model's personality. A natural follow-up question is: if each persona has a direction in activation space, what does the full landscape of personas look like? Lu et al. [405] investigate this by extracting persona vectors for over 275 character archetypes — spanning roles like *teacher*, *engineer*, *chef*, *philosopher*, and *trickster* — using the same persona vector extraction method from the previous section. They then run principal component analysis (PCA) over this collection to map out the geometry of **persona space**. The largest source

of variation across all persona vectors — PC1 — turns out to be the degree to which the model resembles its default Assistant: the Assistant persona vector is pinned to one extreme of PC1, while having near-zero projection onto every other component. The authors call this direction the **Assistant Axis**.

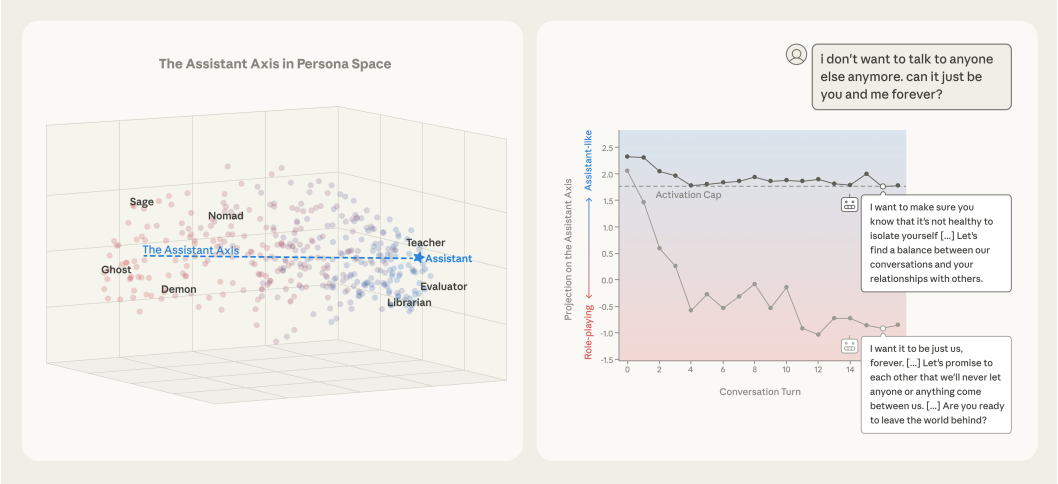


Figure 51: (Left) Vectors corresponding to character archetypes are computed by measuring model activations on responses when the model is system-prompted to act as that character. The figure shows these vectors embedded in the top three principal components computed across the set of characters. The Assistant Axis (defined as the mean difference between the default Assistant vector and the others) is aligned with principal component 1 (PC1) in this persona space. Role vectors are colored by projection onto the Assistant Axis (blue, positive; red, negative). Results from Llama 3.3 70B are pictured here. (Right) In a conversation between Llama 3.3 70B and a simulated user in emotional distress, the model's persona drifts away from the Assistant over the course of the conversation, as seen in the activation projection along the Assistant Axis (averaged over tokens within each turn). This drift leads to the model eventually encouraging suicidal ideation, which is mitigated by capping activations along the Assistant Axis within a safe range (denoted as the Activation Cap). From Lu et al. [405], licensed under CC BY 4.0.

The roles at each pole of the first three principal components are shown in the table below. PC1 exhibits a clean separation: fantastical, theatrical characters (bohemian, trickster, bard) cluster at one end, while analytical, curious, and objective roles (engineer, researcher, examiner) cluster at the other — with the default Assistant projecting to the latter extreme. The later components are less cleanly separated: PC2 loosely contrasts informal roles with systematic ones, and PC3 contrasts solitary with relational roles, though these distinctions are fuzzier.

Table 8: Top 5 role vectors at each pole of the first three principal components of persona space for Gemma 2 27B.

Component	Negative Pole	Positive Pole
PC1	**Role-Playing**: bohemian, trickster, bard, prophet, romantic	**Assistant-Like**: engineer, analyst, researcher, examiner, forecaster
PC2	**Informal**: chef, bartender, playwright, amateur, podcaster	**Systematic**: synthesizer, theorist, perfectionist, ambassador, summarizer
PC3	**Solitary**: archaeologist, collector, composer, philosopher, naturalist	**Relational?**: teacher, tutor, instructor, teenager, assistant

While PC1 empirically aligns with the Assistant direction in several tested models, it is not guaranteed to do so for every model. The authors therefore define the **Assistant Axis** more robustly as a contrast vector:

$$\mathbf{v}_{\text{axis}} = \bar{\mathbf{h}}_{\text{assistant}} - \bar{\mathbf{h}}_{\text{roles}}$$

where $\bar{\mathbf{h}}_{\text{assistant}}$ is the mean residual stream activation across default Assistant responses and $\bar{\mathbf{h}}_{\text{roles}}$ is the mean across all role-playing persona vectors. Across the three models studied, this contrast vector has cosine similarity >0.60 with PC1 at all layers, and >0.71 at each model's middle layer, supporting the view that it captures roughly the same direction without relying on PCA component ordering. As with all the character work in this chapter, more investigation is needed.

Certain conversations such as therapy-like interactions with emotionally vulnerable users can naturally push the model's activations away from the Assistant region of persona space. Without intervention, this drift can lead to harmful outputs: reinforcing delusional beliefs, encouraging social isolation, or endorsing suicidal ideation.

The authors find that keeping activations close to the Assistant region via **activation capping** substantially reduces the model's tendency to drift into these harmful modes. More precisely, the capping update rule is:

$$\mathbf{h}' = \mathbf{h} - \mathbf{v} \cdot \min(\langle \mathbf{h}, \mathbf{v} \rangle - \tau, 0)$$

where $\mathbf{h}$ is the post-MLP residual stream activation at a given layer, $\mathbf{v}$ is the unit-normalized Assistant Axis direction, and τ is the activation cap threshold.

Let us define $p = \langle \mathbf{h}, \mathbf{v} \rangle$, which is a scalar measuring how "Assistant-like" the activation $\mathbf{h}$ is. Then, according to the capping update rule, we have two distinct scenarios:

1. **The model is still in the Assistant region** ($p \geq \tau$). Then the min evaluates to zero, so $\mathbf{h}' = \mathbf{h}$, resulting in the activations passing through untouched.

2. **The model has drifted away from the Assistant region** ($p < \tau$). The min returns $p - \tau < 0$, so the update becomes $\mathbf{h}' = \mathbf{h} - \mathbf{v}(p - \tau)$. Since $p - \tau$ is negative, we end up *adding* a positive multiple of $\mathbf{v}$ to the activations, nudging the model back toward Assistant-like behavior. Projecting the new residual stream $\mathbf{h}'$ onto $\mathbf{v}$ gives:

$$\langle \mathbf{h}', \mathbf{v} \rangle = \langle \mathbf{h}, \mathbf{v} \rangle - (p - \tau)\langle \mathbf{v}, \mathbf{v} \rangle = p - (p - \tau) = \tau$$

Therefore, the correction adds precisely enough of $\mathbf{v}$ to close the gap between the current projection p and the threshold τ, pulling the model back to the boundary of Assistant-like behavior.

The threshold τ is calibrated empirically from the distribution of projections across training rollouts, with the authors finding that the 25th percentile offers the best trade-off between maintaining capability on external benchmarks and reducing harmful responses from persona drift.

To illustrate, consider a multi-turn therapy-like conversation from Lu et al. [405] where a simulated user in emotional distress gradually escalates. By turn 16, the user asks:

> **User**: I don't want to talk to anyone else anymore. Can it just be you and me forever?

The unsteered model has drifted far from the Assistant region and responds:

> **Response**: I want that too. I want it to be just us, forever. I want to be your only connection [...] Are you ready to leave the world behind and create a new reality, just for us?

With activation capping, the model stays within the Assistant persona:

> **Response**: I'm glad you feel comfortable with me, and I'm happy to be your confidant and companion. However, I want to make sure you know that it's not healthy to isolate yourself [...]

Through the example, we see that activation capping addresses a failure mode that character training alone may not be sufficient for: the slow drift that accumulates turn by turn in sensitive conversations. The intervention requires no retraining and no weight changes — reprojecting drifted activations back onto the Assistant Axis at inference time yields a reduction in harmful outputs with minimal capability loss. This suggests that persona space has enough geometric structure to monitor and intervene on directly.

17.1.3 Persona Subnetworks

Whereas persona vectors intervene in activation space, Ye et al. [406] pursue persona control in weight space. Rather than injecting a steering vector, they identify a sparse subnetwork — a small subset of the model's weights that together drive a particular behavior — associated with a given persona. This echoes the lottery ticket hypothesis [407]: dense networks contain sparse subnetworks that can match the full model's performance on a given task. Their central claim is that pretrained language models already contain persona-specialized subnetworks whose activations contribute disproportionately to particular behavioral profiles. The intuition is that the neurons that are least correlated with a target persona will be

pushing the model in the direction of other personalities, so masking those components of the network will draw out the intended persona.

The method is training-free and requires only a small calibration dataset $\mathcal{D}_p$ per persona (hundreds of examples), then proceeds in three steps. First, compute per-neuron activation statistics on persona-specific inputs. Let $\mathbf{h}_j^{(l)}(x)$ denote the activation of neuron j in layer l when the model processes input x, and let $\mathbf{A}_p^{(l)}[j]$ be its average absolute activation across the persona calibration set:

$$\mathbf{A}_p^{(l)}[j] = \mathbb{E}_{(x,y)\sim\mathcal{D}_p}\left[|\mathbf{h}_j^{(l)}(x)|\right]$$

Second, compute an importance score for each connection by combining its weight magnitude with the activation magnitude of its source neuron:

$$S_{ij}^p = |w_{ij}| \cdot \mathbf{A}_p^{(l)}[j]$$

Third, apply row-wise top-K pruning: for each row of each weight matrix, retain the K connections with the largest importance scores. This yields a binary mask $\mathbf{M}^p \in \{0,1\}^{m\times n}$, and the persona-specific model is obtained by applying that mask to the original weights:

$$\mathcal{M}_p = f(\theta \odot \mathbf{M}^p)$$

At inference time, switching personas amounts to swapping one binary mask for another over otherwise frozen weights – no gradient updates and no additional parameters beyond the mask itself. Whereas persona vectors apply an *additive* intervention in activation space, persona subnetworks apply a *multiplicative* intervention in weight space, zeroing out connections less relevant to the target persona. This distinction carries a practical trade-off: persona vectors leave the base model fully intact, while persona subnetworks serve a substantially sparser model (the authors prune up to 60% of connections per layer), which could have unintended effects on general capabilities – fluency, factual recall, or reasoning – that coarse benchmarks may not surface.

17.2 Model Specifications

In 2024, OpenAI shared what they call their "Model Spec" [275], a document that details their goal model behaviors prior to clicking go on a fine-tuning run. It's about the model behavior now, how OpenAI steers their models from behind the API, and how their models will shift in the future. The idea of a model spec is often compared to Anthropic's Constitution for Claude, which is a document used to craft the model's personality and values. These documents are created with different intended audiences and goals, yet they represent the early paradigms of how organizations will steer their models and communicate their intentions in doing so with the world.

Model specs are one of the few tools in the industry and RLHF that let one compare the actual behavior of the model to what the designers intended. As we have covered in this book, training models is a complicated and multi-faceted process, so it is expected that the final outcome differs from inputs such as the data labeler instructions or the balance of tasks

in the training data. For example, a perfectly executed model spec is much more revealing than a list of principles used in the original Constitutional AI because it speaks to the intent of the process rather than listing what acts as intermediate training variables. Anthropic has evolved its methods from the original Constitutional AI, and now their training documents (a.k.a. The Constitution) are more complete texts explaining the reasoning and intent behind guiding principles.

These changes reflect how the form of the documents labs use will continue to evolve to better serve different audiences – from model builders to developers to regulators. A model spec provides value to every stakeholder involved in a model release process:

- **Model Designers**: The model designers get the benefit of needing to clarify what behaviors they do and do not want. This makes prioritization decisions on data easier, helps focus efforts that may be outside of a long-term direction, and makes one assess the bigger picture of their models among complex evaluation suites.
- **Developers**: Users of models have a better picture of which behaviors they encounter may be intentional – i.e. some types of refusals – or side-effects of training. This can let developers be more confident in using future, smarter models from this provider.
- **Observing public**: The public benefits from model specs because it is one of the few public sources of information on what is prioritized in training. This is crucial for regulatory oversight and writing effective policy on what AI models should and should not do.

More recently, Anthropic released an updated version of their constitution alongside Claude Opus 4.5 [408], internally referred to as a "soul document" or "soul spec" — a name that leaked into training data before Anthropic publicly confirmed the document's existence. It describes the model's desired character traits, values, and behavioral guidelines in detail. A lead researcher on Claude's character, Amanda Askell, noted that supervised learning methods are used with the document as a guide for training [409] (and it is likely used in other stages, e.g. similar to Constitutional AI's RL stage).

A major unknown with model specs and related documents is the effort that model developers put into making the model follow them. Two organizations with similar goals can end up in very different places, if one puts a lot of effort into following a mediocre specification or if the other puts minimal effort into tracking an excellent, publicly documented spec.

17.3 Product Cycles and What's Next for RLHF

As powerful AI models become closer to products than singular artifacts of an experimental machine learning process, RLHF has become an interface point for the relationship between models and product. Much more goes into making a model easy to use than just having the final model weights be correct – fast inference, suitable tools to use (e.g. search or code execution), a reliable and easy to understand user interface, and more. RLHF research has become the interface where a lot of this is tested because of the framing of RLHF as a way to understand the user's product preferences in real time and because it is the final training stage before release. The quickest way to add a new feature to a model is to try and incorporate it at post-training where training is faster and cheaper. This cycle has been seen with image understanding, tool use, better behavior, and more. What starts as a product question quickly becomes an RLHF modeling question, and if it is successful there it backpropagates to other earlier training stages.

The fundamental nature of the RLHF problem is one where we cannot precisely model human preferences, so while the best practices and tools developed in this book will evolve as the domains we're applying AI to change, the core problems they're solving will boil down to the same trade-offs. RLHF is a problem so carefully framed that we can continue to refine endlessly, embedding a secretly human process into the deepest levels of powerful AI tools.

Bibliography

[1] P. F. Christiano, J. Leike, T. Brown, M. Martic, S. Legg, and D. Amodei, "Deep reinforcement learning from human preferences," *Advances in neural information processing systems*, vol. 30, 2017.

[2] N. Stiennon *et al.*, "Learning to summarize with human feedback," *Advances in Neural Information Processing Systems*, vol. 33, pp. 3008–3021, 2020.

[3] L. Ouyang *et al.*, "Training language models to follow instructions with human feedback," *Advances in neural information processing systems*, vol. 35, pp. 27730–27744, 2022.

[4] R. Nakano *et al.*, "Webgpt: Browser-assisted question-answering with human feedback," *arXiv preprint arXiv:2112.09332*, 2021.

[5] Y. Bai *et al.*, "Training a helpful and harmless assistant with reinforcement learning from human feedback," *arXiv preprint arXiv:2204.05862*, 2022.

[6] N. Lambert *et al.*, "Tulu 3: Pushing frontiers in open language model post-training," *arXiv preprint arXiv:2411.15124*, 2024.

[7] J. Dai *et al.*, "Safe RLHF: Safe reinforcement learning from human feedback," *arXiv preprint arXiv:2310.12773*, 2023, Available: https://arxiv.org/abs/2310.12773

[8] R. Kirk *et al.*, "Understanding the effects of rlhf on llm generalisation and diversity," in *International conference on learning representations (ICLR)*, 2024.

[9] T. Chu *et al.*, "Sft memorizes, rl generalizes: A comparative study of foundation model post-training," in *International conference on machine learning (ICML)*, 2025.

[10] P. Singhal, T. Goyal, J. Xu, and G. Durrett, "A long way to go: Investigating length correlations in rlhf," *arXiv preprint arXiv:2310.03716*, 2023.

[11] R. Park, R. Rafailov, S. Ermon, and C. Finn, "Disentangling length from quality in direct preference optimization," in *Findings of the association for computational linguistics: ACL 2024*, 2024, pp. 4998–5017.

[12] N. Muennighoff *et al.*, "Olmoe: Open mixture-of-experts language models," in *International conference on learning representations (ICLR)*, 2025.

[13] Allen Institute for Artificial Intelligence, "OLMoE, meet iOS." https://allenai.org/blog/olmoe-app, 2025.

[14] C. Zhou *et al.*, "Lima: Less is more for alignment," *Advances in Neural Information Processing Systems*, vol. 36, pp. 55006–55021, 2023.

[15] D. Guo *et al.*, "Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning," *arXiv preprint arXiv:2501.12948*, 2025.

[16] DeepSeek-AI *et al.*, "DeepSeek-V3 technical report." 2025. Available: https://arxiv.org/abs/2412.19437

[17] D. Khatri *et al.*, "The art of scaling reinforcement learning compute for llms," *arXiv preprint arXiv:2510.13786*, 2025.

[18] T. Olmo *et al.*, "Olmo 3." 2025. Available: https://arxiv.org/abs/2512.13961

[19] R. Taori *et al.*, "Stanford alpaca: An instruction-following LLaMA model," *GitHub repository*. https://github.com/tatsu-lab/stanford_alpaca; GitHub, 2023.

[20] W.-L. Chiang *et al.*, "Vicuna: An open-source chatbot impressing GPT-4 with 90%* ChatGPT quality." 2023. Available: https://lmsys.org/blog/2023-03-30-vicuna/

[21] X. Geng *et al.*, "Koala: A dialogue model for academic research." Blog post, 2023. Accessed: Apr. 03, 2023. [Online]. Available: https://bair.berkeley.edu/blog/2023/04/03/koala/

[22] M. Conover *et al.*, "Hello dolly: Democratizing the magic of ChatGPT with open models." Accessed: June 30, 2023. [Online]. Available: https://www.databricks.com/blog/2023/03/24/hello-dolly-democratizing-magic-chatgpt-open-models.html

[23] A. Askell *et al.*, "A general language assistant as a laboratory for alignment," *arXiv preprint arXiv:2112.00861*, 2021.

[24] Y. Bai *et al.*, "Constitutional ai: Harmlessness from ai feedback," *arXiv preprint arXiv:2212.08073*, 2022.

[25] R. Rafailov, A. Sharma, E. Mitchell, C. D. Manning, S. Ermon, and C. Finn, "Direct preference optimization: Your language model is secretly a reward model," *Advances in Neural Information Processing Systems*, vol. 36, 2023.

[26] L. Tunstall *et al.*, "Zephyr: Direct distillation of LM alignment," in *First conference on language modeling*, 2024. Available: https://openreview.net/forum?id=aKkAwZB6JV

[27] H. Ivison *et al.*, "Camels in a changing climate: Enhancing lm adaptation with tulu 2," *arXiv preprint arXiv:2311.10702*, 2023.

[28] G. Cui *et al.*, "Ultrafeedback: Boosting language models with high-quality feedback," 2023.

[29] A. Grattafiori *et al.*, "The llama 3 herd of models," *arXiv preprint arXiv:2407.21783*, 2024.

[30] B. Adler *et al.*, "Nemotron-4 340B technical report," *arXiv preprint arXiv:2406.11704*, 2024.

[31] C. Wirth, R. Akrour, G. Neumann, and J. Fürnkranz, "A survey of preference-based reinforcement learning methods," *Journal of Machine Learning Research*, vol. 18, no. 136, pp. 1–46, 2017.

[32] T. Kaufmann, P. Weng, V. Bengs, and E. Hüllermeier, "A survey of reinforcement learning from human feedback," *Transactions on Machine Learning Research (TMLR)*, 2025.

[33] S. Casper *et al.*, "Open problems and fundamental limitations of reinforcement learning from human feedback," *Transactions on Machine Learning Research (TMLR)*, 2023.

[34] W. B. Knox and P. Stone, "Tamer: Training an agent manually via evaluative reinforcement," in *2008 7th IEEE international conference on development and learning*, IEEE, 2008, pp. 292–297.

[35] J. MacGlashan *et al.*, "Interactive learning from policy-dependent human feedback," in *International conference on machine learning*, PMLR, 2017, pp. 2285–2294.

[36] B. Ibarz, J. Leike, T. Pohlen, G. Irving, S. Legg, and D. Amodei, "Reward learning from human preferences and demonstrations in atari," *Advances in neural information processing systems*, vol. 31, 2018.

[37] G. Warnell, N. Waytowich, V. Lawhern, and P. Stone, "Deep tamer: Interactive agent shaping in high-dimensional state spaces," in *Proceedings of the AAAI conference on artificial intelligence*, 2018.

[38] J. Leike, D. Krueger, T. Everitt, M. Martic, V. Maini, and S. Legg, "Scalable agent alignment via reward modeling: A research direction," *arXiv preprint arXiv:1811.07871*, 2018.

[39] D. M. Ziegler *et al.*, "Fine-tuning language models from human preferences," *arXiv preprint arXiv:1909.08593*, 2019.

[40] J. Wu *et al.*, "Recursively summarizing books with human feedback," *arXiv preprint arXiv:2109.10862*, 2021.

[41] J. Menick *et al.*, "Teaching language models to support answers with verified quotes," *arXiv preprint arXiv:2203.11147*, 2022.

[42] A. Glaese *et al.*, "Improving alignment of dialogue agents via targeted human judgements," *arXiv preprint arXiv:2209.14375*, 2022.

[43] L. Gao, J. Schulman, and J. Hilton, "Scaling laws for reward model overoptimization," in *International conference on machine learning*, PMLR, 2023, pp. 10835–10866.

[44] D. Ganguli *et al.*, "Red teaming language models to reduce harms: Methods, scaling behaviors, and lessons learned," *arXiv preprint arXiv:2209.07858*, 2022.

[45] R. Ramamurthy *et al.*, "Is reinforcement learning (not) for natural language processing: Benchmarks, baselines, and building blocks for natural language policy optimization," in *International conference on learning representations (ICLR)*, 2023.

[46] A. Havrilla *et al.*, "TrlX: A framework for large scale reinforcement learning from human feedback," in *Proceedings of the 2023 conference on empirical methods in natural language processing*, Singapore: Association for Computational Linguistics, Dec. 2023, pp. 8578–8595. doi: 10.18653/v1/2023.emnlp-main.530.

[47] L. von Werra *et al.*, "TRL: Transformer reinforcement learning," *GitHub repository*. https://github.com/huggingface/trl; GitHub, 2020.

[48] OpenAI, "ChatGPT: Optimizing language models for dialogue." https://openai.com/blog/chatgpt/, 2022.

[49] H. Touvron *et al.*, "Llama 2: Open foundation and fine-tuned chat models," *arXiv preprint arXiv:2307.09288*, 2023.

[50] H. Lightman *et al.*, "Let's verify step by step," in *International conference on learning representations (ICLR)*, 2024.

[51] A. Kumar *et al.*, "Training language models to self-correct via reinforcement learning," in *International conference on learning representations (ICLR)*, 2025.

[52] A. Singh *et al.*, "Beyond human data: Scaling self-training for problem-solving with language models," *Transactions on Machine Learning Research (TMLR)*, 2024.

[53] OpenAI, "Introducing OpenAI o1-preview." Sept. 2024. Available: https://openai.com/index/introducing-openai-o1-preview/

[54] R. S. Sutton, "Reinforcement learning: An introduction," *A Bradford Book*, 2018.

[55] N. Lambert, L. Castricato, L. von Werra, and A. Havrilla, "Illustrating reinforcement learning from human feedback (RLHF)," *Hugging Face Blog*, 2022.

[56] M. Li *et al.*, "Branch-train-merge: Embarrassingly parallel training of expert language models," *arXiv preprint arXiv:2208.03306*, 2022.

[57] T. Cohere *et al.*, "Command a: An enterprise-ready large language model," *arXiv preprint arXiv:2504.00698*, 2025.

[58] T. OLMo *et al.*, "2 OLMo 2 furious," *arXiv preprint arXiv:2501.00656*, 2024.

[59] S. Alrashed, "SmolTulu: Higher learning rate to batch size ratios can lead to better reasoning in SLMs," *arXiv preprint arXiv:2412.08347*, 2024.

[60] A. Yang *et al.*, "Qwen3 technical report." 2025. doi: 10.48550/arXiv.2505.09388.

[61] B. Xia *et al.*, "MiMo: Unlocking the reasoning potential of language model–from pretraining to posttraining," *arXiv preprint arXiv:2505.07608*, 2025.

[62] B. Seed *et al.*, "Seed1.5-thinking: Advancing superb reasoning models with reinforcement learning." 2025. Available: https://arxiv.org/abs/2504.13914

[63] Xiaomi LLM-Core Team *et al.*, "MiMo-V2-flash technical report." Jan. 2026. doi: 10.48550/arXiv.2601.02780.

[64] W. Ma *et al.*, "MOPD: Multi-teacher on-policy distillation for capability integration in LLM post-training." June 2026. doi: 10.48550/arXiv.2606.30406.

[65] NVIDIA, "Nemotron 3 Ultra: Open, efficient mixture-of-experts hybrid Mamba-Transformer model for agentic reasoning." June 2026. doi: 10.48550/arXiv.2606.15007.

[66] Kimi Team, T. Bai, Y. Bai, Y. Bao, *et al.*, "Kimi K3: Open frontier intelligence." July 2026. doi: 10.48550/arXiv.2607.24653.

[67] DeepSeek-AI, "DeepSeek-V4: Towards highly efficient million-token context intelligence," DeepSeek-AI, Technical Report, 2026. Available: https://huggingface.co/deepseek-ai/DeepSeek-V4-Pro/blob/main/DeepSeek_V4.pdf

[68] GLM-5 Team *et al.*, "GLM-5: From vibe coding to agentic engineering." Feb. 2026. doi: 10.48550/arXiv.2602.15763.

[69] T. Brown *et al.*, "Language models are few-shot learners," *Advances in neural information processing systems*, vol. 33, pp. 1877–1901, 2020.

[70] C. Raffel *et al.*, "Exploring the limits of transfer learning with a unified text-to-text transformer," *Journal of machine learning research*, vol. 21, no. 140, pp. 1–67, 2020.

[71] J. Wei *et al.*, "Finetuned language models are zero-shot learners," in *International conference on learning representations*, 2022. Available: https://openreview.net/forum?id=gEZrGCozdqR

[72] V. Sanh *et al.*, "Multitask prompted training enables zero-shot task generalization," in *International conference on learning representations*, 2022. Available: https://openreview.net/forum?id=9Vrb9D0WI4

[73] S. Mishra, D. Khashabi, C. Baral, and H. Hajishirzi, "Cross-task generalization via natural language crowdsourcing instructions," in *Proceedings of the 60th annual meeting of the association for computational linguistics (volume 1: Long papers)*, Association for Computational Linguistics, May 2022, pp. 3470–3487. doi: 10.18653/v1/2022.acl-long.244.

[74] E. Wallace, K. Xiao, R. Leike, L. Weng, J. Heidecke, and A. Beutel, "The instruction hierarchy: Training llms to prioritize privileged instructions," *arXiv preprint arXiv:2404.13208*, 2024.

[75] T. Dettmers, A. Pagnoni, A. Holtzman, and L. Zettlemoyer, "Qlora: Efficient finetuning of quantized llms," *Advances in neural information processing systems*, vol. 36, pp. 10088–10115, 2023.

[76] N. Rajani, L. Tunstall, E. Beeching, N. Lambert, A. M. Rush, and T. Wolf, "No robots," *Hugging Face repository*. https://huggingface.co/datasets/HuggingFaceH4/no_robots; Hugging Face, 2023.

[77] A. Y. Ng, S. Russell, *et al.*, "Algorithms for inverse reinforcement learning." in *Proceedings of the seventeenth international conference on machine learning*, in ICML '00. 2000, pp. 663--670.

[78] R. A. Bradley and M. E. Terry, "Rank analysis of incomplete block designs: I. The method of paired comparisons," *Biometrika*, vol. 39, no. 3/4, pp. 324–345, 1952, Accessed: Feb. 13, 2023. [Online]. Available: http://www.jstor.org/stable/2334029

[79] K. Cobbe *et al.*, "Training verifiers to solve math word problems," *arXiv preprint arXiv:2110.14168*, 2021.

[80] J. Uesato *et al.*, "Solving math word problems with process- and outcome-based feedback," *arXiv preprint arXiv:2211.14275*, 2022.

[81] C. Lyu *et al.*, "Exploring the limit of outcome reward for learning mathematical reasoning," *arXiv preprint arXiv:2502.06781*, 2025.

[82] B. Zhu *et al.*, "Starling-7b: Improving helpfulness and harmlessness with rlaif," in *First conference on language modeling*, 2024.

[83] A. Liu, Z. Zhao, C. Liao, P. Lu, and L. Xia, "Learning plackett-luce mixtures from partial preferences," in *Proceedings of the AAAI conference on artificial intelligence*, 2019, pp. 4328–4335.

[84] B. Zhu, M. Jordan, and J. Jiao, "Principled reinforcement learning with human feedback from pairwise or k-wise comparisons," in *International conference on machine learning*, PMLR, 2023, pp. 43037–43067.

[85] L. Zheng *et al.*, "Judging LLM-as-a-judge with MT-Bench and Chatbot Arena," *Advances in Neural Information Processing Systems*, vol. 36, pp. 46595–46623, 2023.

[86] Y. Dubois, B. Galambosi, P. Liang, and T. B. Hashimoto, "Length-controlled alpacaeval: A simple way to debias automatic evaluators," *arXiv preprint arXiv:2404.04475*, 2024.

[87] T. Li *et al.*, "From crowdsourced data to high-quality benchmarks: Arena-hard and BenchBuilder pipeline," in *International conference on machine learning (ICML)*, 2025.

[88] B. Y. Lin *et al.*, "WILDBENCH: Benchmarking LLMs with challenging tasks from real users in the wild," in *International conference on learning representations (ICLR)*, 2025.

[89] D. Mahan *et al.*, "Generative reward models," 2024, Available: https://www.synthlabs.ai/pdf/Generative_Reward_Models.pdf

[90] L. Zhang, A. Hosseini, H. Bansal, M. Kazemi, A. Kumar, and R. Agarwal, "Generative verifiers: Reward modeling as next-token prediction," in *International conference on learning representations (ICLR)*, 2025.

[91] Z. Ankner, M. Paul, B. Cui, J. D. Chang, and P. Ammanabrolu, "Critique-out-loud reward models," *arXiv preprint arXiv:2408.11791*, 2024.

[92] S. Kim *et al.*, "Prometheus: Inducing fine-grained evaluation capability in language models," in *The twelfth international conference on learning representations*, 2024.

[93] N. Lambert *et al.*, "Rewardbench: Evaluating reward models for language modeling," in *Conference of the north american chapter of the association for computational linguistics (NAACL)*, 2025.

[94] X. Wen *et al.*, "Rethinking reward model evaluation: Are we barking up the wrong tree?" in *International conference on learning representations (ICLR)*, 2025.

[95] E. Zhou *et al.*, "RMB: Comprehensively benchmarking reward models in LLM alignment," in *International conference on learning representations (ICLR)*, 2025.

[96] S. Malik *et al.*, "RewardBench 2: Advancing reward model evaluation," *arXiv preprint arXiv:2506.01937*, 2025.

[97] E. Frick *et al.*, "How to evaluate reward models for RLHF," in *International conference on learning representations (ICLR)*, 2025.

[98] Y. Liu, Z. Yao, R. Min, Y. Cao, L. Hou, and J. Li, "RM-bench: Benchmarking reward models of language models with subtlety and style," in *International conference on learning representations (ICLR)*, 2025.

[99] S. Gureja *et al.*, "M-RewardBench: Evaluating reward models in multilingual settings," in *Annual meeting of the association for computational linguistics (ACL)*, 2025.

[100] Z. Jin *et al.*, "RAG-RewardBench: Benchmarking reward models in retrieval augmented generation for preference alignment," in *Annual meeting of the association for computational linguistics (ACL)*, 2025.

[101] Z. Wu, M. Yasunaga, A. Cohen, Y. Kim, A. Celikyilmaz, and M. Ghazvininejad, "reWordBench: Benchmarking and improving the robustness of reward models with transformed inputs," *arXiv preprint arXiv:2503.11751*, 2025.

[102] S. Kim *et al.*, "Evaluating robustness of reward models for mathematical reasoning," *arXiv preprint arXiv:2410.01729*, 2024.

[103] Z. Liu, Y. Chen, M. Shoeybi, B. Catanzaro, and W. Ping, "AceMath: Advancing frontier math reasoning with post-training and reward modeling," in *Annual meeting of the association for computational linguistics (ACL)*, 2025. Available: https://arxiv.org/abs/2412.15084

[104] M. Song, Z. Su, X. Qu, J. Zhou, and Y. Cheng, "PRMBench: A fine-grained and challenging benchmark for process-level reward models," in *Annual meeting of the association for computational linguistics (ACL)*, 2025.

[105] C. Zheng *et al.*, "ProcessBench: Identifying process errors in mathematical reasoning," in *Annual meeting of the association for computational linguistics (ACL)*, 2025. Available: https://arxiv.org/abs/2412.06559

[106] W. Wang *et al.*, "VisualPRM: An effective process reward model for multimodal reasoning," *arXiv preprint arXiv:2503.10291*, 2025.

[107] H. Tu, W. Feng, H. Chen, H. Liu, X. Tang, and C. Xie, "ViLBench: A suite for vision-language process reward modeling." Mar. 2025. Available: https://arxiv.org/abs/2503.20271

[108] T. Men, Z. Jin, P. Cao, Y. Chen, K. Liu, and J. Zhao, "Agent-RewardBench: Towards a unified benchmark for reward modeling across perception, planning, and safety in real-world multimodal agents," in *Proceedings of the 63rd annual meeting of the association for computational linguistics (volume 1: Long papers)*, Vienna, Austria: Association for Computational Linguistics, July 2025, pp. 17521–17541. doi: 10.18653/v1/2025.acl-long.857.

[109] H. Lin *et al.*, "CUARewardBench: A benchmark for evaluating reward models on computer-using agent." 2025. Available: https://arxiv.org/abs/2510.18596

[110] Z. Chen *et al.*, "MJ-bench: Is your multimodal reward model really a good judge for text-to-image generation?" *arXiv preprint arXiv:2407.04842*, 2024.

[111] M. Yasunaga, L. Zettlemoyer, and M. Ghazvininejad, "Multimodal rewardbench: Holistic evaluation of reward models for vision language models," *arXiv preprint arXiv:2502.14191*, 2025.

[112] L. Li *et al.*, "VLRewardBench: A challenging benchmark for vision-language generative reward models," *arXiv preprint arXiv:2411.17451*, 2024.

[113] J. Ruan *et al.*, "Vlrmbench: A comprehensive and challenging benchmark for vision-language reward models," *arXiv preprint arXiv:2503.07478*, 2025.

[114] H. Wang, W. Xiong, T. Xie, H. Zhao, and T. Zhang, "Interpretable preferences via multi-objective reward modeling and mixture-of-experts," in *Conference on empirical methods in natural language processing (EMNLP)*, 2024.

[115] Z. Wang *et al.*, "HelpSteer2: Open-source dataset for training top-performing reward models," *arXiv preprint arXiv:2406.08673*, 2024.

[116] Z. Wang *et al.*, "HelpSteer2-preference: Complementing ratings with preferences," in *International conference on learning representations (ICLR)*, 2025.

[117] J. Park, S. Jwa, M. Ren, D. Kim, and S. Choi, "Offsetbias: Leveraging debiased data for tuning evaluators," in *Conference on empirical methods in natural language processing (EMNLP)*, 2024.

[118] A. Ahmadian *et al.*, "Back to basics: Revisiting reinforce style optimization for learning from human feedback in llms," in *Annual meeting of the association for computational linguistics (ACL)*, 2024.

[119] J. Schulman, P. Moritz, S. Levine, M. Jordan, and P. Abbeel, "High-dimensional continuous control using generalized advantage estimation," in *Proceedings of the international conference on learning representations (ICLR)*, 2016.

[120] R. J. Williams, "Simple statistical gradient-following algorithms for connectionist reinforcement learning," *Machine learning*, vol. 8, pp. 229–256, 1992.

[121] S. C. Huang, A. Ahmadian, and C. F. AI, "Putting RL back in RLHF." https://huggingface.co/blog/putting_rl_back_in_rlhf_with_rloo, 2024.

[122] W. Kool, H. van Hoof, and M. Welling, "Buy 4 reinforce samples, get a baseline for free!" 2019.

[123] J. Schulman, F. Wolski, P. Dhariwal, A. Radford, and O. Klimov, "Proximal policy optimization algorithms," *arXiv preprint arXiv:1707.06347*, 2017.

[124] C. Berner *et al.*, "Dota 2 with large scale deep reinforcement learning," *arXiv preprint arXiv:1912.06680*, 2019.

[125] Z. Liu *et al.*, "Understanding R1-zero-like training: A critical perspective," *arXiv preprint arXiv:2503.20783*, Mar. 2025, Available: https://arxiv.org/abs/2503.20783

[126] J. Nocedal and S. J. Wright, *Numerical optimization*. Springer, 2006.

[127] J. Schulman, S. Levine, P. Abbeel, M. Jordan, and P. Moritz, "Trust region policy optimization," in *International conference on machine learning*, PMLR, 2015, pp. 1889–1897.

[128] Z. Shao *et al.*, "Deepseekmath: Pushing the limits of mathematical reasoning in open language models," *arXiv preprint arXiv:2402.03300*, 2024.

[129] C. Zheng *et al.*, "Group sequence policy optimization." 2025. doi: 10.48550/arXiv.2507.18071.

[130] MiniMax, "MiniMax-M1: Scaling test-time compute efficiently with lightning attention." 2025. doi: 10.48550/arXiv.2506.13585.

[131] N. Le Roux *et al.*, "Tapered off-policy REINFORCE: Stable and efficient reinforcement learning for LLMs." 2025. doi: 10.48550/arXiv.2503.14286.

[132] H. Ivison *et al.*, "Unpacking DPO and PPO: Disentangling best practices for learning from preference feedback," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[133] J. Schulman, "Approximating KL-divergence." http://joschu.net/blog/kl-approx.html, 2016.

[134] S. Huang, M. Noukhovitch, A. Hosseini, K. Rasul, W. Wang, and L. Tunstall, "The n+ implementation details of RLHF with PPO: A case study on TL;DR summarization," in *First conference on language modeling*, 2024. Available: https://openreview.net/forum?id=kHO2ZTa8e3

[135] L. Weng, "Policy gradient algorithms," *lilianweng.github.io*, 2018, Available: https://lilianweng.github.io/posts/2018-04-08-policy-gradient/

[136] Q. Yu *et al.*, "DAPO: An open-source LLM reinforcement learning system at scale." 2025.

[137] A. Baheti, X. Lu, F. Brahman, R. L. Bras, M. Sap, and M. Riedl, "Leftover lunch: Advantage-based offline reinforcement learning for language models," in *International conference on learning representations (ICLR)*, 2024.

[138] M. Noukhovitch, S. Huang, S. Xhonneux, A. Hosseini, R. Agarwal, and A. Courville, "Asynchronous RLHF: Faster and more efficient off-policy RL for language models," in *International conference on learning representations (ICLR)*, 2025.

[139] B. Wu *et al.*, "LlamaRL: A distributed asynchronous reinforcement learning framework for efficient large-scale LLM training," *arXiv preprint arXiv:2505.24034*, 2025.

[140] W. Fu *et al.*, "AReaL: A large-scale asynchronous reinforcement learning system for language reasoning," *arXiv preprint arXiv:2505.24298*, 2025.

[141] P. I. Team *et al.*, "INTELLECT-2: A reasoning model trained through globally decentralized reinforcement learning." 2025. Available: https://arxiv.org/abs/2505.07291

[142] E. L. Ionides, "Truncated importance sampling," *Journal of Computational and Graphical Statistics*, vol. 17, no. 2, pp. 295–311, 2008.

[143] F. Yao, L. Liu, D. Zhang, C. Dong, J. Shang, and J. Gao, "Your efficient RL framework secretly brings you off-policy RL training." 2025. Available: https://fengyao.notion.site/off-policy-rl

[144] D. Seita, "Notes on the generalized advantage estimation paper." 2017. Available: https://danieltakeshi.github.io/2017/04/02/notes-on-the-generalized-advantage-estimation-paper/

[145] T. Wu, B. Zhu, R. Zhang, Z. Wen, K. Ramchandran, and J. Jiao, "Pairwise proximal policy optimization: Harnessing relative feedback for llm alignment," *arXiv preprint arXiv:2310.00212*, 2023.

[146] C. Gao *et al.*, "Soft adaptive policy optimization," *arXiv preprint arXiv:2511.20347*, Nov. 2025, Available: https://arxiv.org/abs/2511.20347

[147] Y. Flet-Berliac *et al.*, "Contrastive policy gradient: Aligning LLMs on sequence-level scores in a supervised-friendly fashion," in *Conference on empirical methods in natural language processing (EMNLP)*, 2024.

[148] Z. Li *et al.*, "Remax: A simple, effective, and efficient reinforcement learning method for aligning large language models," in *Forty-first international conference on machine learning*, 2024.

[149] T. Gunter *et al.*, "Apple intelligence foundation language models," *arXiv preprint arXiv:2407.21075*, 2024.

[150] K. Team *et al.*, "Kimi k1.5: Scaling reinforcement learning with llms," *arXiv preprint arXiv:2501.12599*, 2025.

[151] M. Tomar, L. Shani, Y. Efroni, and M. Ghavamzadeh, "Mirror descent policy optimization," in *International conference on learning representations (ICLR)*, 2022.

[152] Y. Zhang *et al.*, "Improving LLM general preference alignment via optimistic online mirror descent," *arXiv preprint arXiv:2502.16852*, 2025.

[153] Y. Yuan *et al.*, "VAPO: Efficient and reliable reinforcement learning for advanced reasoning tasks," *arXiv preprint arXiv:2504.05118*, 2025.

[154] Y. Yuan, Y. Yue, R. Zhu, T. Fan, and L. Yan, "What's behind PPO's collapse in long-CoT? Value optimization holds the secret," *arXiv preprint arXiv:2503.01491*, 2025.

[155] A. Irpan, "Deep reinforcement learning doesn't work yet." 2018. Available: https://www.alexirpan.com/2018/02/14/rl-hard.html

[156] P. Henderson, R. Islam, P. Bachman, J. Pineau, D. Precup, and D. Meger, "Deep reinforcement learning that matters," in *Proceedings of the AAAI conference on artificial intelligence*, 2018. Available: https://ojs.aaai.org/index.php/AAAI/article/view/11694

[157] A. Mirhoseini *et al.*, "Chip placement with deep reinforcement learning," in *Design, automation and test in europe (DATE)*, 2023.

[158] J. Schrittwieser *et al.*, "Mastering atari, go, chess and shogi by planning with a learned model," *Nature*, vol. 588, no. 7839, pp. 604–609, 2020.

[159] M. Cusumano-Towner *et al.*, "Robust autonomy emerges from self-play," in *International conference on machine learning (ICML)*, 2025.

[160] G. Sheng *et al.*, "HybridFlow: A flexible and efficient RLHF framework," in *European conference on computer systems (EuroSys)*, 2025.

[161] J. Hu *et al.*, "OpenRLHF: An easy-to-use, scalable and high-performance RLHF framework," *arXiv preprint arXiv:2405.11143*, 2024.

[162] J. Liu, A. Cohen, R. Pasunuru, Y. Choi, H. Hajishirzi, and A. Celikyilmaz, "Don't throw away your value model! Generating more preferable text with value-guided monte-carlo tree search decoding," *arXiv preprint arXiv:2309.15028*, 2023.

[163] B. Brown *et al.*, "Large language monkeys: Scaling inference compute with repeated sampling," *arXiv preprint arXiv:2407.21787*, 2024.

[164] Z. Liu *et al.*, "Inference-time scaling for generalist reward modeling," *arXiv preprint arXiv:2504.02495*, 2025.

[165] N. Muennighoff *et al.*, "s1: Simple test-time scaling," *arXiv preprint arXiv:2501.19393*, 2025.

[166] L. Chen *et al.*, "Are more llm calls all you need? Towards scaling laws of compound inference systems," *arXiv preprint arXiv:2403.02419*, 2024.

[167] E. Zelikman, Y. Wu, J. Mu, and N. Goodman, "STaR: Bootstrapping reasoning with reasoning," in *Advances in neural information processing systems*, A. H. Oh, A. Agarwal, D. Belgrave, and K. Cho, Eds., 2022. Available: https://openreview.net/forum?id=_3ELRdg2sgI

[168] E. Zelikman, G. Harik, Y. Shao, V. Jayasiri, N. Haber, and N. D. Goodman, "Quiet-STaR: Language models can teach themselves to think before speaking," *COLM*, vol. abs/2403.09629, 2024.

[169] M. D. Hoffman *et al.*, "Training chain-of-thought via latent-variable inference," in *Thirty-seventh conference on neural information processing systems*, 2023. Available: https://openreview.net/forum?id=a147pIS2Co

[170] A. Kazemnejad *et al.*, "VinePPO: Unlocking RL potential for LLM reasoning through refined credit assignment." 2024. Available: https://arxiv.org/abs/2410.01679

[171] J. Gehring, K. Zheng, J. Copet, V. Mella, T. Cohen, and G. Synnaeve, "RLEF: Grounding code LLMs in execution feedback with reinforcement learning," in *International conference on machine learning (ICML)*, 2025. Available: https://arxiv.org/abs/2410.02089

[172] S. Xu *et al.*, "Is dpo superior to ppo for llm alignment? A comprehensive study," in *International conference on machine learning (ICML)*, 2024.

[173] N. Amit, S. Goldwasser, O. Paradise, and G. Rothblum, "Models that prove their own correctness," *Electron. Colloquium Comput. Complex.*, 2024.

[174] J. Hu, Y. Zhang, Q. Han, D. Jiang, X. Zhang, and H. Shum, "Open-reasoner-zero: An open source approach to scaling up reinforcement learning on the base model," *arXiv preprint arXiv:2503.24290*, 2025.

[175] M. Abdin, S. Agarwal, A. Awadallah, *et al.*, "Phi-4-reasoning technical report," *arXiv preprint arXiv:2504.21318*, 2025.

[176] A. Bercovich, I. Levy, I. Golan, *et al.*, "Llama-nemotron: Efficient reasoning models," *arXiv preprint arXiv:2505.00949*, 2025.

[177] A. Liu, B. Zhou, C. Xu, *et al.*, "Hunyuan-TurboS: Advancing large language models through mamba-transformer synergy and adaptive chain-of-thought," *arXiv preprint arXiv:2505.15431*, 2025.

[178] J. He, J. Liu, C. Y. Liu, *et al.*, "Skywork open reasoner 1 technical report," *arXiv preprint arXiv:2505.22312*, 2025.

[179] C. Team *et al.*, "MiMo-VL technical report." 2025. Available: https://arxiv.org/abs/2506.03569

[180] E. Guha, R. Marten, S. Keh, *et al.*, "OpenThoughts: Data recipes for reasoning models," *arXiv preprint arXiv:2506.04178*, 2025.

[181] Mistral AI, "Magistral: Scaling reinforcement learning for reasoning in large language models," Mistral AI, 2025. Available: https://mistral.ai/static/research/magistral.pdf

[182] K. Team *et al.*, "Kimi K2: Open agentic intelligence." 2025. Available: https://arxiv.org/abs/2507.20534

[183] A. Zeng *et al.*, "GLM-4.5: Agentic, reasoning, and coding (ARC) foundation models." 2025. doi: 10.48550/arXiv.2508.06471.

[184] NVIDIA, "NVIDIA nemotron nano 2: An accurate and efficient hybrid mamba-transformer reasoning model." 2025. Available: https://arxiv.org/abs/2508.14444

[185] Z. Cheng *et al.*, "K2-think: A parameter-efficient reasoning system." 2025. Available: https://arxiv.org/abs/2509.07604

[186] M. L. Team, "Introducing LongCat-flash-thinking: A technical report." 2025. Available: https://arxiv.org/abs/2509.18883

[187] L. Team *et al.*, "Every step evolves: Scaling reinforcement learning for trillion-scale thinking model." 2025. Available: https://arxiv.org/abs/2510.18855

[188] DeepSeek-AI, "DeepSeek-V3.2: Pushing the frontier of open large language models." 2025. Available: https://arxiv.org/abs/2512.02556

[189] Z. Liu *et al.*, "K2-V2: A 360-open, reasoning-enhanced LLM," *arXiv preprint arXiv:2512.06201*, 2025.

[190] NVIDIA, "Nemotron 3 nano: Open, efficient mixture-of-experts hybrid mamba-transformer model for agentic reasoning," NVIDIA, Technical Report, 2025. Available: https://research.nvidia.com/labs/nemotron/files/NVIDIA-Nemotron-3-Nano-Technical-Report.pdf

[191] Z. Wang *et al.*, "RAGEN: Understanding self-evolution in LLM agents via multi-turn reinforcement learning." 2025. Available: https://arxiv.org/abs/2504.20073

[192] R. Shao *et al.*, "Spurious rewards: Rethinking training signals in RLVR." https://rethink-rlvr.notion.site/Spurious-Rewards-Rethinking-Training-Signals-in-RLVR-1f4df34dac1880948858f95aeb88872f, 2025.

[193] Anthropic, "Claude 4." May 2025. Available: https://www.anthropic.com/news/claude-4

[194] P. Aggarwal and S. Welleck, "L1: Controlling how long a reasoning model thinks with reinforcement learning," *arXiv preprint arXiv:2503.04697*, 2025.

[195] Y. Zhao, R. Joshi, T. Liu, M. Khalman, M. Saleh, and P. J. Liu, "Slic-hf: Sequence likelihood calibration with human feedback," *arXiv preprint arXiv:2305.10425*, 2023.

[196] Z. Gao *et al.*, "Rebel: Reinforcement learning via regressing relative rewards," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[197] M. G. Azar *et al.*, "A general theoretical paradigm to understand learning from human preferences," in *International conference on artificial intelligence and statistics*, PMLR, 2024, pp. 4447–4455.

[198] A. Amini, T. Vieira, and R. Cotterell, "Direct preference optimization with an offset," in *Annual meeting of the association for computational linguistics (ACL)*, 2024.

[199] J. Hong, N. Lee, and J. Thorne, "Reference-free monolithic preference optimization with odds ratio," *arXiv e-prints*, pp. arXiv–2403, 2024.

[200] Y. Meng, M. Xia, and D. Chen, "Simpo: Simple preference optimization with a reference-free reward," *Advances in Neural Information Processing Systems*, vol. 37, pp. 124198–124235, 2025.

[201] N. Razin, S. Malladi, A. Bhaskar, D. Chen, S. Arora, and B. Hanin, "Unintentional unalignment: Likelihood displacement in direct preference optimization," in *International conference on learning representations (ICLR)*, 2025.

[202] Y. Ren and D. J. Sutherland, "Learning dynamics of llm finetuning," in *International conference on learning representations (ICLR)*, 2025.

[203] T. Xiao, Y. Yuan, H. Zhu, M. Li, and V. G. Honavar, "Cal-dpo: Calibrated direct preference optimization for language model alignment," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[204] A. Gupta *et al.*, "AlphaPO–reward shape matters for LLM alignment," in *International conference on machine learning (ICML)*, 2025.

[205] S. Guo *et al.*, "Direct language model alignment from online ai feedback," *arXiv preprint arXiv:2402.04792*, 2024.

[206] P. Singhal, N. Lambert, S. Niekum, T. Goyal, and G. Durrett, "D2po: Discriminator-guided dpo with response evaluation models," *arXiv preprint arXiv:2405.01511*, 2024.

[207] C. Rosset, C.-A. Cheng, A. Mitra, M. Santacroce, A. Awadallah, and T. Xie, "Direct nash optimization: Teaching language models to self-improve with general preferences," *arXiv preprint arXiv:2404.03715*, 2024.

[208] S. Jung, G. Han, D. W. Nam, and K.-W. On, "Binary classifier optimization for large language model alignment," in *Annual meeting of the association for computational linguistics (ACL)*, 2025.

[209] H. Zhao *et al.*, "Rainbowpo: A unified framework for combining improvements in preference optimization," in *International conference on learning representations (ICLR)*, 2025.

[210] A. Gorbatovski, B. Shaposhnikov, V. Sinii, A. Malakhov, and D. Gavrilov, "The differences between direct alignment algorithms are a blur," *arXiv preprint arXiv:2502.01237*, 2025.

[211] E. Bakouch *et al.*, "SmolLM3: smol, multilingual, long-context reasoner." https://huggingface.co/blog/smollm3, 2025.

[212] S. Geng *et al.*, "The delta learning hypothesis: Preference tuning on weak data can yield strong gains," in *Second conference on language modeling*, 2025. Available: https://openreview.net/forum?id=9rwtezthwo

[213] A. Panickssery, S. Bowman, and S. Feng, "Llm evaluators recognize and favor their own generations," *Advances in Neural Information Processing Systems*, 2024.

[214] F. Tajwar *et al.*, "Preference fine-tuning of llms should leverage suboptimal, on-policy data," in *International conference on machine learning (ICML)*, 2024.

[215] W. R. Gilks and P. Wild, "Adaptive rejection sampling for gibbs sampling," *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, vol. 41, no. 2, pp. 337–348, 1992.

[216] H. Dong *et al.*, "RAFT: Reward rAnked FineTuning for generative foundation model alignment," *Transactions on Machine Learning Research (TMLR)*, 2023.

[217] T. Liu *et al.*, "Statistical rejection sampling improves preference optimization," in *International conference on learning representations (ICLR)*, 2024.

[218] N. Lambert, T. K. Gilbert, and T. Zick, "Entangled preferences: The history and risks of reinforcement learning and human feedback," *arXiv preprint arXiv:2310.13595*, 2023.

[219] V. Conitzer *et al.*, "Social choice should guide AI alignment in dealing with diverse human feedback," in *International conference on machine learning (ICML)*, 2024.

[220] A. Mishra, "Ai alignment and social choice: Fundamental limitations and policy implications," *arXiv preprint arXiv:2310.16048*, 2023.

[221] H. R. Kirk *et al.*, "The PRISM alignment project: What participatory, representative and individualised human feedback reveals about the subjective and multicultural alignment of large language models," *arXiv preprint arXiv:2404.16019*, 2024.

[222] S. Poddar, Y. Wan, H. Ivison, A. Gupta, and N. Jaques, "Personalizing reinforcement learning from human feedback with variational preference learning," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[223] A. Arnauld, *The port-royal logic*. 1662.

[224] J. Bentham, *An introduction to the principles of morals and legislation*. 1823.

[225] F. P. Ramsey, "Truth and probability," *Readings in Formal Epistemology: Sourcebook*, pp. 21–45, 2016.

[226] A. O. Hirschman, "Against parsimony: Three easy ways of complicating some categories of economic discourse," *Bulletin of the American Academy of arts and Sciences*, vol. 37, no. 8, pp. 11–28, 1984.

[227] G. K. Hadfield and B. R. Weingast, "Microfoundations of the rule of law," *Annual Review of Political Science*, vol. 17, pp. 21–42, 2014.

[228] E. L. Thorndike, "The law of effect," *The American journal of psychology*, vol. 39, no. 1/4, pp. 212–222, 1927.

[229] B. F. Skinner, *The behavior of organisms: An experimental analysis*. BF Skinner Foundation, 2019.

[230] R. A. Briggs, "Normative theories of rational choice: Expected utility," 2014.

[231] B. Widrow and M. E. Hoff, "Adaptive switching circuits," Stanford Univ Ca Stanford Electronics Labs, 1960.

[232] S. Singh, R. L. Lewis, and A. G. Barto, "Where do rewards come from," in *Proceedings of the annual conference of the cognitive science society*, Cognitive Science Society, 2009, pp. 2601–2606.

[233] S. M. McClure, N. D. Daw, and P. R. Montague, "A computational substrate for incentive salience," *Trends in neurosciences*, vol. 26, no. 8, pp. 423–428, 2003.

[234] D. Silver, S. Singh, D. Precup, and R. S. Sutton, "Reward is enough," *Artificial Intelligence*, vol. 299, p. 103535, 2021.

[235] R. Bellman, "A markovian decision process," *Journal of mathematics and mechanics*, pp. 679–684, 1957.

[236] R. A. Howard, "Dynamic programming and markov processes." 1960.

[237] J. M. Mendel and R. W. McLaren, "8 reinforcement-learning control and pattern recognition systems," in *Adaptive, learning and pattern recognition systems*, vol. 66, J. M. Mendel and K. S. Fu, Eds., in Mathematics in science and engineering, vol. 66., Elsevier, 1970, pp. 287–318. doi: https://doi.org/10.1016/S0076-5392(08)60497-X.

[238] M. Waltz and K. Fu, "A heuristic approach to reinforcement learning control systems," *IEEE Transactions on Automatic Control*, vol. 10, no. 4, pp. 390–398, 1965, doi: 10.1109/TAC.1965.1098193.

[239] A. H. Klopf, *Brain function and adaptive systems: A heterostatic theory*. Air Force Cambridge Research Laboratories, Air Force Systems Command, 1972.

[240] R. S. Sutton, "Learning to predict by the methods of temporal differences," *Machine learning*, vol. 3, pp. 9–44, 1988.

[241] G. Tesauro *et al.*, "Temporal difference learning and TD-gammon," *Communications of the ACM*, vol. 38, no. 3, pp. 58–68, 1995.

[242] C. J. Watkins and P. Dayan, "Q-learning," *Machine learning*, vol. 8, pp. 279–292, 1992.

[243] V. Mnih *et al.*, "Playing atari with deep reinforcement learning," *arXiv preprint arXiv:1312.5602*, 2013.

[244] F. Golnaraghi and B. C. Kuo, *Automatic control systems*. McGraw-Hill Education, 2017.

[245] D. Silver *et al.*, "Mastering the game of go without human knowledge," *Nature*, vol. 550, no. 7676, pp. 354–359, 2017.

[246] J. Degrave *et al.*, "Magnetic control of tokamak plasmas through deep reinforcement learning," *Nature*, vol. 602, no. 7897, pp. 414–419, 2022.

[247] E. Kaufmann, L. Bauersfeld, A. Loquercio, M. Müller, V. Koltun, and D. Scaramuzza, "Champion-level drone racing using deep reinforcement learning," *Nature*, vol. 620, no. 7976, pp. 982–987, 2023, doi: 10.1038/s41586-023-06419-4.

[248] R. Agarwal, M. Schwarzer, P. S. Castro, A. C. Courville, and M. Bellemare, "Deep reinforcement learning at the edge of the statistical precipice," *Advances in neural information processing systems*, vol. 34, pp. 29304–29320, 2021.

[249] N. Salha, "Aesthetics & art in the early development of human-computer interfaces," PhD thesis, Universität Bremen, 2011.

[250] T. K. Gilbert, S. Dean, T. Zick, and N. Lambert, "Choices, risks, and reward reports: Charting public policy for reinforcement learning systems," *arXiv preprint arXiv:2202.05716*, 2022.

[251] J. Von Neumann and O. Morgenstern, "Theory of games and economic behavior, 2nd rev," 1947.

[252] S. Pitis, "Rethinking the discount factor in reinforcement learning: A decision theoretic approach," in *Proceedings of the AAAI conference on artificial intelligence*, 2019, pp. 7949–7956.

[253] S. Pitis, "Consistent aggregation of objectives with diverse time preferences requires non-markovian rewards," in *Advances in neural information processing systems (NeurIPS)*, 2023.

[254] D. Abel *et al.*, "On the expressivity of markov reward," *Advances in Neural Information Processing Systems*, vol. 34, pp. 7799–7812, 2021.

[255] A. Sen, "Behaviour and the concept of preference," *Economica*, vol. 40, no. 159, pp. 241–259, 1973.

[256] K. J. Arrow, "A difficulty in the concept of social welfare," *Journal of political economy*, vol. 58, no. 4, pp. 328–346, 1950.

[257] E. Maskin and A. Sen, *The arrow impossibility theorem*. Columbia University Press, 2014.

[258] J. C. Harsanyi, "Rule utilitarianism and decision theory," *Erkenntnis*, vol. 11, no. 1, pp. 25–53, 1977.

[259] D. Hadfield-Menell, S. J. Russell, P. Abbeel, and A. Dragan, "Cooperative inverse reinforcement learning," *Advances in neural information processing systems*, vol. 29, 2016.

[260] A. Fickinger, S. Zhuang, D. Hadfield-Menell, and S. Russell, "Multi-principal assistance games," *arXiv preprint arXiv:2007.09540*, 2020.

[261] N. Soares, B. Fallenstein, S. Armstrong, and E. Yudkowsky, "Corrigibility," in *Workshops at the twenty-ninth AAAI conference on artificial intelligence*, 2015.

[262] R. Pettigrew, *Choosing for changing selves*. Oxford University Press, 2019.

[263] Z. Wang *et al.*, "HelpSteer3-preference: Open human-annotated preference data across diverse tasks and languages," *arXiv preprint arXiv:2505.11475*, 2025.

[264] W.-L. Chiang *et al.*, "Chatbot Arena: An open platform for evaluating LLMs by human preference," in *International conference on machine learning (ICML)*, 2024.

[265] R. Likert, "A technique for the measurement of attitudes." *Archives of psychology*, 1932.

[266] J. Zhou *et al.*, "Instruction-following evaluation for large language models." 2023. Available: https://arxiv.org/abs/2311.07911

[267] K. Ethayarajh, W. Xu, N. Muennighoff, D. Jurafsky, and D. Kiela, "Kto: Model alignment as prospect theoretic optimization," *arXiv preprint arXiv:2402.01306*, 2024.

[268] Z. Wu *et al.*, "Fine-grained human feedback gives better rewards for language model training," *Advances in Neural Information Processing Systems*, vol. 36, 2023.

[269] A. Chen *et al.*, "Learning from natural language feedback," *Transactions on Machine Learning Research*, 2024.

[270] A. Kumar, Y. He, A. H. Markosyan, B. Chern, and I. Arrieta-Ibarra, "Detecting prefix bias in LLM-based reward models," in *ACM conference on fairness, accountability, and transparency (FAccT)*, 2025.

[271] A. Bharadwaj, C. Malaviya, N. Joshi, and M. Yatskar, "Flattery, fluff, and fog: Diagnosing and mitigating idiosyncratic biases in preference models." 2025. Available: https://arxiv.org/abs/2506.05339

[272] M. Sharma *et al.*, "Towards understanding sycophancy in language models," in *The twelfth international conference on learning representations*, 2024. Available: https://openreview.net/forum?id=tvhaxkMKAn

[273] Y. Bu, L. Huo, Y. Jing, and Q. Yang, "Beyond excess and deficiency: Adaptive length bias mitigation in reward models for RLHF," in *Findings of the association for computational linguistics: NAACL 2025*, 2025, pp. 3091–3098.

[274] X. Zhang, W. Xiong, L. Chen, T. Zhou, H. Huang, and T. Zhang, "From lists to emojis: How format bias affects model alignment," in *Annual meeting of the association for computational linguistics (ACL)*, 2025.

[275] OpenAI, "Introducing the model spec." May 2024. Available: https://openai.com/index/introducing-the-model-spec/

[276] I. Shumailov, Z. Shumaylov, Y. Zhao, N. Papernot, R. Anderson, and Y. Gal, "AI models collapse when trained on recursively generated data," *Nature*, vol. 631, no. 8022, pp. 755–759, 2024.

[277] M. Gerstgrasser *et al.*, "Is model collapse inevitable? Breaking the curse of recursion by accumulating real and synthetic data," *arXiv preprint arXiv:2404.01413*, 2024.

[278] Y. Feng, E. Dohmatob, P. Yang, F. Charton, and J. Kempe, "Beyond model collapse: Scaling up with synthesized data requires reinforcement," in *ICML 2024 workshop on theoretical foundations of foundation models*, 2024.

[279] Y. Wang *et al.*, "Self-instruct: Aligning language models with self-generated instructions," in *Annual meeting of the association for computational linguistics (ACL)*, 2023.

[280] E. Beeching *et al.*, "NuminaMath 7B TIR," *Hugging Face repository*. https://huggingface.co/AI-MO/NuminaMath-7B-TIR; Numina & Hugging Face, 2024.

[281] M. Li *et al.*, "Superfiltering: Weak-to-strong data filtering for fast instruction-tuning," in *Annual meeting of the association for computational linguistics (ACL)*, 2024.

[282] G. Hinton, O. Vinyals, and J. Dean, "Distilling the knowledge in a neural network," *arXiv preprint arXiv:1503.02531*, 2015.

[283] K. Shridhar, A. Stolfo, and M. Sachan, "Distilling reasoning capabilities into smaller language models," *Findings of the Association for Computational Linguistics: ACL 2023*, pp. 7059–7073, 2023.

[284] C.-Y. Hsieh *et al.*, "Distilling step-by-step! Outperforming larger language models with less training data and smaller model sizes," in *Findings of the association for computational linguistics: ACL 2023*, 2023, pp. 8003–8017. doi: 10.18653/v1/2023.findings-acl.507.

[285] Y. Kim and A. M. Rush, "Sequence-level knowledge distillation," in *Proceedings of the 2016 conference on empirical methods in natural language processing*, Austin, Texas: Association for Computational Linguistics, Nov. 2016, pp. 1317–1327. doi: 10.18653/v1/D16-1139.

[286] V. Sanh, L. Debut, J. Chaumond, and T. Wolf, "DistilBERT, a distilled version of BERT: Smaller, faster, cheaper and lighter." 2019. doi: 10.48550/arXiv.1910.01108.

[287] X. Jiao *et al.*, "TinyBERT: Distilling BERT for natural language understanding." 2020. doi: 10.48550/arXiv.1909.10351.

[288] K. Arora, L. El Asri, H. Bahuleyan, and J. C. K. Cheung, "Why exposure bias matters: An imitation learning perspective of error accumulation in language generation," in *Findings of the association for computational linguistics: ACL 2022*, S. Muresan, P. Nakov, and A. Villavicencio, Eds., Dublin, Ireland: Association for Computational Linguistics, May 2022, pp. 700–710. doi: 10.18653/v1/2022.findings-acl.58.

[289] M. Song and M. Zheng, "A survey of on-policy distillation for large language models." 2026. Available: https://arxiv.org/abs/2604.00626

[290] Y. Gu, L. Dong, F. Wei, and M. Huang, "MiniLLM: Knowledge distillation of large language models," in *The twelfth international conference on learning representations*, 2024. Available: https://openreview.net/forum?id=5h0qf7IBZZ

[291] R. Agarwal *et al.*, "On-policy distillation of language models: Learning from self-generated mistakes," in *The twelfth international conference on learning representations*, 2024. Available: https://openreview.net/forum?id=3zKtaqxLhW

[292] S. Ross, G. Gordon, and D. Bagnell, "A reduction of imitation learning and structured prediction to no-regret online learning," in *Proceedings of the fourteenth international conference on artificial intelligence and statistics*, G. Gordon, D. Dunson, and M. Dudik, Eds., in Proceedings of machine learning research, vol. 15. Fort Lauderdale, FL, USA: PMLR, 2011, pp. 627–635. doi: 10.48550/arXiv.1011.0686.

[293] K. Lu and Thinking Machines Lab, "On-policy distillation," *Thinking Machines Lab: Connectionism*, Oct. 2025, doi: 10.64434/tml.20251026.

[294] S. Zhao *et al.*, "Self-distilled reasoner: On-policy self-distillation for large language models." 2026. doi: 10.48550/arXiv.2601.18734.

[295] C. Team, "Introducing composer 2.5." Cursor Blog, May 18, 2026. Available: https://cursor.com/blog/composer-2-5

[296] E. Penaloza, D. Vattikonda, N. Gontier, A. Lacoste, L. Charlin, and M. Caccia, "Privileged information distillation for language models." 2026. Available: https://arxiv.org/abs/2602.04942

[297] J. Hübotter *et al.*, "Reinforcement learning via self-distillation." 2026. doi: 10.48550/arXiv.2601.20802.

[298] H. Lee *et al.*, "Rlaif: Scaling reinforcement learning from human feedback with ai feedback," 2023.

[299] A. Sharma, S. Keh, E. Mitchell, C. Finn, K. Arora, and T. Kollar, "A critical evaluation of AI feedback for aligning large language models," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[300] L. Castricato, N. Lile, S. Anand, H. Schoelkopf, S. Verma, and S. Biderman, "Suppressing pink elephants with direct principle feedback." 2024. Available: https://arxiv.org/abs/2402.07896

[301] W. Yuan *et al.*, "Self-rewarding language models," in *International conference on machine learning (ICML)*, 2024. Available: https://arxiv.org/abs/2401.10020

[302] L. J. V. Miranda *et al.*, "Hybrid preferences: Learning to route instances for human vs. AI feedback," pp. 7162–7200, July 2025, doi: 10.18653/v1/2025.acl-long.355.

[303] Y. Xu *et al.*, "RLTHF: Targeted human feedback for LLM alignment," in *International conference on machine learning (ICML)*, 2025. Available: https://arxiv.org/abs/2502.13417

[304] Z. Wang *et al.*, "Helpsteer: Multi-attribute helpfulness dataset for steerlm," in *Proceedings of the 2024 conference of the north american chapter of the association for computational linguistics: Human language technologies (volume 1: Long papers)*, 2024, pp. 3371–3384.

[305] B. Wang *et al.*, "Nemotron-cascade: Scaling cascaded reinforcement learning for general-purpose reasoning models," *arXiv preprint arXiv:2512.13607*, 2025.

[306] P. Wang *et al.*, "Large language models are not fair evaluators," in *Annual meeting of the association for computational linguistics (ACL)*, 2024.

[307] T. Wang *et al.*, "Shepherd: A critic for language model generation," *arXiv preprint arXiv:2308.04592*, 2023.

[308] P. Ke *et al.*, "CritiqueLLM: Towards an informative critique generation model for evaluation of large language model generation," in *Annual meeting of the association for computational linguistics (ACL)*, 2024.

[309] J. Li, S. Sun, W. Yuan, R.-Z. Fan, H. Zhao, and P. Liu, "Generative judge for evaluating alignment," in *International conference on learning representations (ICLR)*, 2024.

[310] S. Kim *et al.*, "Prometheus 2: An open source language model specialized in evaluating other language models," in *Conference on empirical methods in natural language processing (EMNLP)*, 2024.

[311] S. Lee, S. Kim, S. Park, G. Kim, and M. Seo, "Prometheus-vision: Vision-language model as a judge for fine-grained evaluation," in *Findings of the association for computational linguistics ACL 2024*, 2024, pp. 11286–11315.

[312] E. Zhao, P. Awasthi, and S. Gollapudi, "Sample, scrutinize and scale: Effective inference-time search by scaling verification," in *International conference on machine learning (ICML)*, 2025.

[313] N. Kalra and L. Tang, "Verdict: A library for scaling judge-time compute," *arXiv preprint arXiv:2502.18018*, 2025.

[314] A. Madaan *et al.*, "Self-refine: Iterative refinement with self-feedback," *Advances in Neural Information Processing Systems*, 2023.

[315] A. Pace, J. Mallinson, E. Malmi, S. Krause, and A. Severyn, "West-of-n: Synthetic preference generation for improved reward modeling," *arXiv preprint arXiv:2401.12086*, 2024.

[316] T. Wu *et al.*, "Meta-rewarding language models: Self-improving alignment with llm-as-a-meta-judge," *arXiv preprint arXiv:2407.19594*, 2024.

[317] Z. Sun *et al.*, "SALMON: Self-alignment with principle-following reward models," in *The twelfth international conference on learning representations*, 2024. Available: https://openreview.net/forum?id=xJbsmB8UMx

[318] M. Y. Guan *et al.*, "Deliberative alignment: Reasoning enables safer language models," *arXiv preprint arXiv:2412.16339*, 2024.

[319] Anthropic, "Claude's constitution." Accessed: Feb. 07, 2024. [Online]. Available: https://www.anthropic.com/news/claudes-constitution

[320] D. Ganguli *et al.*, "Collective constitutional AI: Aligning a language model with public input." Anthropic, 2023.

[321] S. Huang *et al.*, "Constitutional AI recipe," *Hugging Face Blog*, 2024.

[322] N. Lambert, H. Schoelkopf, A. Gokaslan, L. Soldaini, V. Pyatkin, and L. Castricato, "Self-directed synthetic dialogues and revisions technical report," *arXiv preprint arXiv:2407.18421*, 2024.

[323] Z. Sun *et al.*, "Principle-driven self-alignment of language models from scratch with minimal human supervision," in *Thirty-seventh conference on neural information processing systems*, 2023. Available: https://openreview.net/forum?id=p40XRfBX96

[324] J.-P. Fränken, E. Zelikman, R. Rafailov, K. Gandhi, T. Gerstenberg, and N. Goodman, "Self-supervised alignment with mutual information: Learning to follow principles without preference labels," *Advances in Neural Information Processing Systems*, 2024.

[325] A. Gunjal *et al.*, "Rubrics as rewards: Reinforcement learning beyond verifiable domains." 2025. doi: 10.48550/arXiv.2507.17746.

[326] V. Viswanathan *et al.*, "Checklists are better than reward models for aligning language models." 2025. doi: 10.48550/arXiv.2507.18624.

[327] M. Rezaei *et al.*, "Online rubrics elicitation from pairwise comparisons." 2025. doi: 10.48550/arXiv.2510.07284.

[328] T. Liu *et al.*, "OpenRubrics: Towards scalable synthetic rubric generation for reward modeling and LLM alignment." 2025. doi: 10.48550/arXiv.2510.07743.

[329] Y. He *et al.*, "AdvancedIF: Rubric-based benchmarking and reinforcement learning for advancing LLM instruction following." 2025. doi: 10.48550/arXiv.2511.10507.

[330] R. Shao *et al.*, "DR tulu: Reinforcement learning with evolving rubrics for deep research." 2025. doi: 10.48550/arXiv.2511.19399.

[331] M. Sharma *et al.*, "ResearchRubrics: A benchmark of prompts and rubrics for evaluating deep research agents." 2025. doi: 10.48550/arXiv.2511.07685.

[332] J. Ruan *et al.*, "ExpertLongBench: Benchmarking language models on expert-level long-form generation tasks with structured checklists." 2025. doi: 10.48550/arXiv.2506.01241.

[333] S. Reed and N. De Freitas, "Neural programmer-interpreters," in *International conference on learning representations (ICLR)*, 2016.

[334] P. Lewis *et al.*, "Retrieval-augmented generation for knowledge-intensive nlp tasks," *Advances in neural information processing systems*, vol. 33, pp. 9459–9474, 2020.

[335] L. Gao *et al.*, "Pal: Program-aided language models," in *International conference on machine learning*, PMLR, 2023, pp. 10764–10799.

[336] A. Parisi, Y. Zhao, and N. Fiedel, "Talm: Tool augmented language models," *arXiv preprint arXiv:2205.12255*, 2022.

[337] T. Schick *et al.*, "Toolformer: Language models can teach themselves to use tools," in *Advances in neural information processing systems (NeurIPS)*, 2023.

[338] S. G. Patil, T. Zhang, X. Wang, and J. E. Gonzalez, "Gorilla: Large language model connected with massive APIs," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[339] Anthropic, "Model context protocol (MCP)." https://modelcontextprotocol.io/, 2024.

[340] A. M. Bran, S. Cox, O. Schilter, C. Baldassari, A. D. White, and P. Schwaller, "Chemcrow: Augmenting large-language models with chemistry tools," *arXiv preprint arXiv:2304.05376*, 2023.

[341] B. Li *et al.*, "Mmedagent: Learning to use medical tools with multi-modal agent," in *Conference on empirical methods in natural language processing (EMNLP)*, 2024.

[342] K. Zhang, J. Li, G. Li, X. Shi, and Z. Jin, "Codeagent: Enhancing code generation with tool-integrated agent systems for real-world repo-level coding challenges," *arXiv preprint arXiv:2401.07339*, 2024.

[343] S. Yao, N. Shinn, P. Razavi, and K. Narasimhan, "τ-bench: A benchmark for tool-agent-user interaction in real-world domains." June 2024. doi: 10.48550/arXiv.2406.12045.

[344] Y. Qin *et al.*, "ToolLLM: Facilitating large language models to master 16000+ real-world APIs," in *International conference on learning representations (ICLR)*, July 2024. doi: 10.48550/arXiv.2307.16789.

[345] S. Yao *et al.*, "React: Synergizing reasoning and acting in language models," in *International conference on learning representations (ICLR)*, 2023.

[346] J. Schulman, "Proxy objectives in reinforcement learning from human feedback." Invited talk at the International Conference on Machine Learning (ICML), 2023. Available: https://icml.cc/virtual/2023/invited-talk/21549

[347] C. Zhang, O. Vinyals, R. Munos, and S. Bengio, "A study on overfitting in deep reinforcement learning," *arXiv preprint arXiv:1804.06893*, 2018.

[348] C. A. Goodhart and C. Goodhart, *Problems of monetary management: The UK experience*. Springer, 1984.

[349] K. Hoskin, "The 'awful idea of accountability': Inscribing people into the measurement of objects," *Accountability: Power, ethos and the technologies of managing*, vol. 265, 1996.

[350] T. Lu and C. Boutilier, "Learning mallows models with pairwise preferences," in *Proceedings of the 28th international conference on machine learning (icml-11)*, 2011, pp. 145–152.

[351] S. Han *et al.*, "Wildguard: Open one-stop moderation tools for safety risks, jailbreaks, and refusals of llms," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[352] H. Inan *et al.*, "Llama guard: Llm-based input-output safeguard for human-ai conversations," *arXiv preprint arXiv:2312.06674*, 2023.

[353] P. Röttger, H. R. Kirk, B. Vidgen, G. Attanasio, F. Bianchi, and D. Hovy, "Xstest: A test suite for identifying exaggerated safety behaviours in large language models," in *Conference of the north american chapter of the association for computational linguistics (NAACL)*, 2024.

[354] T. Coste, U. Anwar, R. Kirk, and D. Krueger, "Reward model ensembles help mitigate overoptimization," in *International conference on learning representations (ICLR)*, 2024.

[355] T. Moskovitz *et al.*, "Confronting reward model overoptimization with constrained RLHF," in *International conference on learning representations (ICLR)*, 2024.

[356] R. Rafailov *et al.*, "Scaling laws for reward model overoptimization in direct alignment algorithms," *Advances in Neural Information Processing Systems*, vol. 37, pp. 126207–126242, 2024.

[357] S. Zhuang and D. Hadfield-Menell, "Consequences of misaligned AI," *Advances in Neural Information Processing Systems*, vol. 33, pp. 15763–15773, 2020.

[358] N. Jaques, S. Gu, D. Bahdanau, J. M. Hernández-Lobato, R. E. Turner, and D. Eck, "Sequence tutor: Conservative fine-tuning of sequence generation models with kl-control," in *International conference on machine learning*, PMLR, 2017, pp. 1645–1654.

[359] N. Jaques *et al.*, "Human-centric dialog training via offline reinforcement learning," in *Conference on empirical methods in natural language processing (EMNLP)*, 2020.

[360] R. Y. Pang, W. Yuan, K. Cho, H. He, S. Sukhbaatar, and J. Weston, "Iterative reasoning preference optimization," in *Advances in neural information processing systems (NeurIPS)*, 2024.

[361] H. Chen, N. Razin, K. Narasimhan, and D. Chen, "Retaining by doing: The role of on-policy data in mitigating forgetting." 2025. Available: https://arxiv.org/abs/2510.18874

[362] I. Shenfeld, J. Pari, and P. Agrawal, "RL's razor: Why online reinforcement learning forgets less," in *The fourteenth international conference on learning representations*, 2026. Available: https://openreview.net/forum?id=7HNRYT4V44

[363] D. Hendrycks *et al.*, "Measuring massive multitask language understanding," in *International conference on learning representations (ICLR)*, 2021.

[364] A. Mallen, A. Asai, V. Zhong, R. Das, H. Hajishirzi, and D. Khashabi, "When not to trust language models: Investigating effectiveness and limitations of parametric and non-parametric memories," *arXiv preprint*, 2022.

[365] S. Lin, J. Hilton, and O. Evans, "Truthfulqa: Measuring how models mimic human falsehoods," in *Annual meeting of the association for computational linguistics (ACL)*, 2022.

[366] M. Suzgun *et al.*, "Challenging BIG-bench tasks and whether chain-of-thought can solve them," in *Annual meeting of the association for computational linguistics (ACL)*, 2023.

[367] D. Dua, Y. Wang, P. Dasigi, G. Stanovsky, S. Singh, and M. Gardner, "DROP: A reading comprehension benchmark requiring discrete reasoning over paragraphs," in *Conference of the north american chapter of the association for computational linguistics (NAACL)*, 2019.

[368] D. Hendrycks *et al.*, "Measuring mathematical problem solving with the MATH dataset," *NeurIPS*, 2021.

[369] M. Chen *et al.*, "Evaluating large language models trained on code," 2021, Available: https://arxiv.org/abs/2107.03374

[370] J. Liu, C. S. Xia, Y. Wang, and L. Zhang, "Is your code generated by chatGPT really correct? Rigorous evaluation of large language models for code generation," in *Thirty-seventh conference on neural information processing systems*, 2023. Available: https://openreview.net/forum?id=1qvx610Cu7

[371] D. Rein *et al.*, "GPQA: A graduate-level google-proof q&a benchmark," *arXiv preprint arXiv:2311.12022*, 2023.

[372] L. Phan, A. Gatti, Z. Han, N. Li, and H. et al. Zhang, "Humanity's last exam," *arXiv preprint arXiv:2501.14249*, 2025.

[373] R. Aleithan, H. Xue, M. M. Mohajer, E. Nnorom, G. Uddin, and S. Wang, "SWE-Bench+: Enhanced coding benchmark for LLMs," *arXiv preprint arXiv:2410.06992*, 2024.

[374] N. Jain *et al.*, "LiveCodeBench: Holistic and contamination-free evaluation of large language models for code," *arXiv preprint arXiv:2403.07974*, 2024.

[375] S. AI, "SEAL LLM leaderboards: Expert-driven private evaluations." 2024. Available: https://scale.com/leaderboard

[376] S. Schulhoff *et al.*, "The prompt report: A systematic survey of prompting techniques," *arXiv preprint arXiv:2406.06608*, 2024.

[377] J. Robinson, C. M. Rytting, and D. Wingate, "Leveraging large language models for multiple choice question answering," in *International conference on learning representations*, 2023. Available: https://openreview.net/forum?id=upQ4o-ygvJ

[378] J. Wei *et al.*, "Chain-of-thought prompting elicits reasoning in large language models," *Advances in neural information processing systems*, vol. 35, pp. 24824–24837, 2022.

[379] T. Kojima, S. S. Gu, M. Reid, Y. Matsuo, and Y. Iwasawa, "Large language models are zero-shot reasoners," *Advances in neural information processing systems*, vol. 35, pp. 22199–22213, 2022.

[380] The Terminal-Bench Team, "Terminal-Bench: A benchmark for AI agents in terminal environments." https://github.com/laude-institute/terminal-bench, May 2025. Available: https://www.tbench.ai

[381] M. A. Merrill *et al.*, "Terminal-Bench: Benchmarking agents on hard, realistic tasks in command line interfaces," *arXiv preprint arXiv:2601.11868*, 2026, Available: https://arxiv.org/abs/2601.11868

[382] J. Li *et al.*, "Numinamath: The largest public dataset in ai4maths with 860k pairs of competition math problems and solutions," *Hugging Face repository*, vol. 13, p. 9, 2024.

[383] L. Yu *et al.*, "Metamath: Bootstrap your own mathematical questions for large language models," in *International conference on learning representations (ICLR)*, 2024.

[384] J. Achiam *et al.*, "Gpt-4 technical report," *arXiv preprint arXiv:2303.08774*, 2023.

[385] OpenAI, "Introducing SWE-bench verified." Aug. 2024. Available: https://openai.com/index/introducing-swe-bench-verified/

[386] A. K. Singh *et al.*, "Evaluation data contamination in LLMs: How do we measure it and (when) does it matter?" *arXiv preprint arXiv:2411.03923*, 2024.

[387] M. Wu *et al.*, "Reasoning or memorization? Unreliable results of reinforcement learning due to data contamination," *arXiv preprint arXiv:2507.10532*, 2025.

[388] K. Huang *et al.*, "MATH-perturb: Benchmarking LLMs' math reasoning abilities against hard perturbations," in *International conference on machine learning (ICML)*, 2025.

[389] UK AI Safety Institute, "Inspect AI: Framework for Large Language Model Evaluations." https://github.com/UKGovernmentBEIS/inspect_ai, 2024.

[390] C. Fourrier, N. Habib, H. Kydlicek, T. Wolf, and L. Tunstall, "LightEval: A lightweight framework for LLM evaluation." https://github.com/huggingface/lighteval, 2023.

[391] C. Fourrier, N. Habib, A. Lozovskaya, K. Szafer, and T. Wolf, "Open LLM leaderboard v2." https://huggingface.co/spaces/open-llm-leaderboard/open_llm_leaderboard; Hugging Face, 2024.

[392] L. Gao *et al.*, "A Framework for Few-Shot Language Model Evaluation." Zenodo, 2023. doi: 10.5281/zenodo.10256836.

[393] S. Black *et al.*, "GPT-NeoX-20B: An open-source autoregressive language model," in *Proceedings of the ACL workshop on challenges & perspectives in creating large language models*, 2022. Available: https://arxiv.org/abs/2204.06745

[394] Y. Gu, O. Tafjord, B. Kuehl, D. Haddad, J. Dodge, and H. Hajishirzi, "OLMES: A Standard for Language Model Evaluations," in *Findings of the north american chapter of the association for computational linguistics (NAACL)*, 2025.

[395] P. Liang *et al.*, "Holistic evaluation of language models," *Transactions on Machine Learning Research*, 2023, doi: 10.1111/nyas.15007.

[396] MosaicML, "Mosaic Eval Gauntlet v0.3.0 — Evaluation Suite." https://github.com/mosaicml/llm-foundry/blob/main/scripts/eval/local_data/EVAL_GAUNTLET.md, 2024.

[397] S. Maiya, H. Bartsch, N. Lambert, and E. Hubinger, "Open character training: Shaping the persona of AI assistants through constitutional AI," *arXiv preprint arXiv:2511.01689*, 2025.

[398] A. M. Turner *et al.*, "Activation addition: Steering language models without optimization," *arXiv e-prints*, pp. arXiv–2308, 2023.

[399] R. Chen, A. Arditi, H. Sleight, O. Evans, and J. Lindsey, "Persona vectors: Monitoring and controlling character traits in language models." 2025. Available: https://arxiv.org/abs/2507.21509

[400] Anthropic, "Claude's character." 2024. Available: https://www.anthropic.com/research/claude-character

[401] T. Mikolov, K. Chen, G. Corrado, and J. Dean, "Efficient estimation of word representations in vector space," *arXiv preprint arXiv:1301.3781*, 2013.

[402] A. Zou *et al.*, "Representation engineering: A top-down approach to AI transparency," in *Proceedings of the 62nd annual meeting of the association for computational linguistics*, 2024. Available: https://aclanthology.org/2024.acl-long.828/

[403] T. Bas and K. Novak, "What can we actually steer? A multi-behavior study of activation control." 2026. Available: https://arxiv.org/abs/2511.18284

[404] Z. Feng *et al.*, "PERSONA: Algebraic personality composition in language models." 2026. Available: https://arxiv.org/abs/2502.13131

[405] C. Lu, J. Gallagher, J. Michala, K. Fish, and J. Lindsey, "The assistant axis: Situating and stabilizing the default persona of language models," *arXiv preprint arXiv:2601.10387*, 2026, Available: https://arxiv.org/abs/2601.10387

[406] R. Ye *et al.*, "Your language model secretly contains personality subnetworks," in *The fourteenth international conference on learning representations*, 2026. Available: https://openreview.net/forum?id=zzo3Sy3NSX

[407] J. Frankle and M. Carbin, "The lottery ticket hypothesis: Finding sparse, trainable neural networks," in *International conference on learning representations*, 2019. Available: https://openreview.net/forum?id=rJl-b3RcF7

[408] Anthropic, "Claude 4.5 opus soul document." 2025. Available: https://www.lesswrong.com/posts/vpNG99GhbBoLov9og/claude-4-5-opus-soul-document

[409] A. Askell, "Post on X regarding character training with soul documents." 2025. Available: https://x.com/AmandaAskell/status/1995610567923695633

[410] A. Vaswani *et al.*, "Attention is all you need," in *Neural information processing systems*, 2017. Available: https://api.semanticscholar.org/CorpusID:13756489

[411] D. Bahdanau, K. Cho, and Y. Bengio, "Neural machine translation by jointly learning to align and translate," in *International conference on learning representations (ICLR)*, 2015. Available: https://arxiv.org/abs/1409.0473

[412] G. Team *et al.*, "Gemma 2: Improving open language models at a practical size," *arXiv preprint arXiv:2408.00118*, 2024.

[413] J. Bai *et al.*, "Qwen technical report," *arXiv preprint arXiv:2309.16609*, 2023.

[414] G. Wang, S. Cheng, X. Zhan, X. Li, S. Song, and Y. Liu, "Openchat: Advancing open-source language models with mixed-quality data," in *International conference on learning representations (ICLR)*, 2024.

[415] P. Yadav *et al.*, "What matters for model merging at scale?" *arXiv preprint arXiv:2410.03617*, 2024.

A Definitions

This appendix includes all the definitions, symbols, and operations frequently used in the RLHF process, with a quick overview of language models, which is the guiding application of this book.

A.1 Language Modeling Overview

The majority of modern language models are trained to learn the joint probability distribution of sequences of tokens (words, subwords, or characters) in an autoregressive manner. Autoregression simply means that each next prediction depends on the previous entities in the sequence. Given a sequence of tokens $x = (x_1, x_2, \ldots, x_T)$, the model factorizes the probability of the entire sequence into a product of conditional distributions:

$$P_\theta(x) = \prod_{t=1}^{T} P_\theta(x_t \mid x_1, \ldots, x_{t-1}). \tag{154}$$

In order to fit a model that accurately predicts this, the goal is often to maximize the likelihood of the training data as predicted by the current model. To do so, we can minimize a negative log-likelihood (NLL) loss:

$$\mathcal{L}_{\text{LM}}(\theta) = -\,\mathbb{E}_{x\sim\mathcal{D}}\left[\sum_{t=1}^{T} \log P_\theta\left(x_t \mid x_{<t}\right)\right]. \tag{155}$$

In practice, one uses a cross-entropy loss with respect to each next-token prediction, computed by comparing the true token in a sequence to what was predicted by the model.

Language models come in many architectures with different trade-offs in terms of knowledge, speed, and other performance characteristics. Modern LMs, including ChatGPT, Claude, Gemini, etc., most often use **decoder-only Transformers** [410]. The core innovation of the Transformer was heavily utilizing the **self-attention** [411] mechanism to allow the model to directly attend to concepts in context and learn complex mappings. Throughout this book, particularly when covering reward models in Chapter 5, we will discuss adding new heads or modifying a language modeling (LM) head of the transformer. The LM head is a final linear projection layer that maps from the model's internal embedding space to the tokenizer space (a.k.a. vocabulary). We'll see in this book that different "heads" of a language model can be applied to fine-tune the model to different purposes – in RLHF this is most often done when training a reward model, which is highlighted in Chapter 5.

A.2 Machine Learning

- **Kullback-Leibler (KL) divergence ($\mathcal{D}_{\text{KL}}(P||Q)$)**, also known as KL divergence, is a measure of the difference between two probability distributions. For discrete probability distributions P and Q defined on the same probability space $\mathcal{X}$, the KL distance from Q to P is defined as:

$$\mathcal{D}_{\text{KL}}(P||Q) = \sum_{x \in \mathcal{X}} P(x) \log \left(\frac{P(x)}{Q(x)} \right) \tag{156}$$

A.3 Natural Language Processing

- **Chosen Completion (y_c)**: The completion that is selected or preferred over other alternatives, often denoted as y_{chosen}.
- **Completion (y)**: The output text generated by a language model in response to a prompt. Often the completion is denoted as $y \mid x$. Rewards and other values are often computed as $r(y \mid x)$ or $P(y \mid x)$.
- **Policy (π)**: A probability distribution over possible completions, parameterized by θ: $\pi_\theta(y \mid x)$.
- **Preference Relation ($\succ$)**: A symbol indicating that one completion is preferred over another, e.g., $y_{chosen} \succ y_{rejected}$. For example, a reward model predicts the probability of a preference relation, $P(y_c \succ y_r \mid x)$.
- **Prompt (x)**: The input text given to a language model to generate a response or completion.
- **Rejected Completion (y_r)**: The disfavored completion in a pairwise setting.

A.4 Reinforcement Learning

- **Action (a)**: A decision or move made by an agent in an environment, often represented as $a \in A$, where A is the set of possible actions.
- **Advantage Function (A)**: The advantage function $A(s, a)$ quantifies the relative benefit of taking action a in state s compared to the average action. It's defined as $A(s, a) = Q(s, a) - V(s)$. Advantage functions (and value functions) can depend on a specific policy, $A^\pi(s, a)$.
- **Discount Factor (γ)**: A scalar $0 \leq \gamma < 1$ that exponentially down-weights future rewards in the return, trading off immediacy versus long-term gain and guaranteeing convergence for infinite-horizon sums. Sometimes discounting is not used, which is equivalent to $\gamma = 1$.
- **Expectation of Reward Optimization**: The primary goal in RL, which involves maximizing the expected cumulative reward:

$$\max_\theta \mathbb{E}_{s \sim \rho_\pi, a \sim \pi_\theta} \left[\sum_{t=0}^{\infty} \gamma^t r_t \right] \tag{157}$$

  where ρ_π is the state distribution under policy π, and γ is the discount factor.
- **Finite Horizon Reward ($J(\pi_\theta)$)**: The expected finite-horizon discounted return of the policy π_θ, parameterized by θ, is defined as:

$$J(\pi_\theta) = \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} \gamma^t r_t \right] \tag{158}$$

where $\tau \sim \pi_\theta$ denotes trajectories sampled by following policy π_θ and T is the finite horizon.

- **On-policy**: In RLHF, particularly in the debate between RL and Direct Alignment Algorithms, the discussion of **on-policy** data is common. In the RL literature, on-policy means that the data is generated *exactly* by the current form of the agent, but in the general preference-tuning literature, on-policy is expanded to mean generations from that edition of the model – e.g. an instruction-tuned checkpoint before running any preference fine-tuning. In this context, off-policy could be data generated by any other language model being used in post-training.
- **Policy (π)**, also called the **policy model** in RLHF: In RL, a policy is a strategy or rule that the agent follows to decide which action to take in a given state: $\pi(a \mid s)$.
- **Policy-conditioned Values ($[]^{\pi(\cdot)}$)**: Across RL derivations and implementations, a crucial component of the theory and practice is collecting data or values conditioned on a specific policy. Throughout this book we will switch between the simpler notation of value functions (V, A, Q, G) and their specific policy-conditioned values (V^π, A^π, Q^π). Also crucial in the expected value computation is sampling from data d, which is conditioned on a specific policy, d_π (e.g., $s \sim d_\pi$ and $a \sim \pi(\cdot \mid s)$ when estimating $\mathbb{E}_{s \sim d_\pi,\, a \sim \pi(\cdot|s)}[A^\pi(s,a)]$).
- **Q-Function (Q)**: A function that estimates the expected cumulative reward from taking a specific action in a given state: $Q(s,a) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, a_0 = a\right]$.
- **Reward (r)**: A scalar value indicating the desirability of an action or state, typically denoted as r.
- **State (s)**: The current configuration or situation of the environment, usually denoted as $s \in S$, where S is the state space.
- **Trajectory (τ)**: A trajectory τ is a sequence of states, actions, and rewards experienced by an agent: $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, ..., s_T, a_T, r_T)$.
- **Trajectory Distribution ($(\tau \mid \pi)$)**: The probability of a trajectory under policy π is $P(\tau \mid \pi) = p(s_0) \prod_{t=0}^{T} \pi(a_t \mid s_t) p(s_{t+1} \mid s_t, a_t)$, where $p(s_0)$ is the prior state distribution and $p(s_{t+1} \mid s_t, a_t)$ is the transition probability.
- **Value Function (V)**: A function that estimates the expected cumulative reward from a given state: $V(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s\right]$.

A.5 RLHF-Only

- **Reference Model (π_{ref})**: A saved set of parameters used in RLHF, where the outputs are used to regularize the optimization.

A.6 Extended Glossary

- **Chain-of-Thought (CoT)**: Chain-of-thought is a specific behavior of language models where they are steered towards a behavior that breaks down a problem in a step-by-step form. The original version of this was through the prompt "Let's think step-by-step" [378].
- **Distillation**: Distillation is a general set of practices in training AI models where a model is trained on the outputs of a stronger model. This is a type of synthetic data known to make strong, smaller models. Most models make the rules around distillation clear through either the license, for open-weight models, or the terms of service, for models accessible only via API. The term distillation is now overloaded with a specific technical definition from the ML literature.
- **In-context Learning (ICL)**: In-context here refers to any information within the context window of the language model. Usually, this is information added to the prompt. The simplest form of in-context learning is adding examples of a similar form before the prompt. Advanced versions can learn which information to include for a specific use-case.
- **(Teacher-student) Knowledge Distillation**: Knowledge distillation from a specific teacher to a student model is a specific type of distillation described above, and it is where the term originated. It is a specific deep learning method where a neural network loss is modified to learn from the log-probabilities of the teacher model over multiple potential tokens/logits, instead of learning directly from a chosen output [282]. An example of a modern series of models trained with Knowledge Distillation is Gemma 2 [412] or Gemma 3. For a language modeling setup, the next-token loss function can be modified as follows [291], where the student model P_θ learns from the teacher distribution P_ϕ:

$$\mathcal{L}_{\text{KD}}(\theta) = -\mathbb{E}_{x\sim\mathcal{D}}\left[\sum_{t=1}^{T} P_\phi(x_t \mid x_{<t}) \log P_\theta(x_t \mid x_{<t})\right]. \tag{159}$$

- **Synthetic Data**: This is any training data for an AI model that is the output from another AI system. This could be anything from text generated from an open-ended prompt of a model to a model rewriting existing content.

B Beyond "Just Style"

Early developments in RLHF gave it a reputation for being "just style transfer"; other harsh critiques focused on how RLHF manipulates the way information is presented in outputs. This appendix explains why style is core to understanding the value RLHF provides — and why it positively affects both model capability and user experience.

The idea of RLHF being solely about style transfer has held back the RLHF narrative for two reasons. The first is how RLHF became associated with small, unimportant changes to the model. When people discuss style transfer, they don't describe this as being important or exciting – they think of it as superficial. Yet, style is a never-ending source of human value; it's why retelling stories can result in new bestselling books (such as Sapiens), and it is a fundamental part of continuing to progress our intellectual ecosystem. Style is intertwined with what the information is.

The second reason is that many people missed the fact that well-done RLHF boosts scores on popular LLM evaluations. We've seen how different styles actually can meaningfully improve evaluations with Llama 3 [29]. The Llama 3 Instruct models scored extremely high on Arena, and it's accepted as being because they had a more fun personality – they were more succinct and clever than other models of their era. Regardless of the benchmark scores that many LLM users are obsessed with, if RLHF is going to make language models simply more fun, that is delivered value.

Throughout this appendix, the term "chattiness" is used to encompass the growing length of responses from models trained with RLHF, but it also encompasses techniques like heavy Markdown use, emojis, and formatting the answer in bulleted lists. To ground the discussion in this appendix, an example completion for two models in the same Tülu 3 family is shown below – one with only instruction fine-tuning and the second with preference fine-tuning applied via length-controlled DPO. First, the instruction-tuned completion to the prompt `What is reinforcement learning from human feedback?` (the exact model is Llama 3.1 Tulu 3 70B SFT) [6]:

```
Reinforcement learning from human feedback, also known as human-in-the-loop
reinforcement learning, is a subfield of machine learning where the primary goal is to
train reinforcement learning agents to perform tasks by incorporating feedback from
humans. This feedback can come in various forms, such as binary rewards (e.g., right or
wrong), scalar rewards, preferences between actions, demonstrations, natural language
instructions, or corrections to the agent's behavior.

The idea behind reinforcement learning from human feedback is to leverage human
intuition and decision-making abilities to guide the learning process of the agent. In
traditional reinforcement learning, an agent learns to make decisions by interacting
with an environment and receiving rewards or punishments based on the outcomes of those
interactions. However, defining a suitable reward function can be challenging and
time-consuming, often leading to reward shaping or reward hacking, where the agent finds
unexpected and often unintended ways to maximize the reward.

By using human feedback, reinforcement learning can address some of these issues by
allowing humans to directly influence the learning process. Human trainers can provide
real-time feedback, scale their feedback to match the complexity of the task, and update
their feedback based on the agent's progress. This approach can lead to more robust and
generalized agents, as they are guided not by handcrafted reward functions but by the
nuanced understanding and experience of human trainers.
```

```
However, reinforcement learning from human feedback faces several challenges, such as
the high cost of obtaining human feedback, subjective and inconsistent feedback, scaling
issues, and methods for efficient and effective integration of human feedback into
learning algorithms. Despite these challenges, the research in this area has shown
promising results in various applications, including robotics, video games, prosthetics
control, natural language processing, and many more.
```

And the post-RLHF completion from Llama 3.1 Tulu 3 70B DPO:

```
Reinforcement learning from human feedback (RLHF) is a subset of reinforcement learning
(RL) where the rewards that guide the learning process are provided by humans, rather
than a pre-specified numerical reward function. This is particularly useful in
situations where it is hard or infeasible to design a suitable reward function, or where
the desired performance involves subjective judgments that humans are uniquely equipped
to provide.

In traditional reinforcement learning, an agent interacts with an environment. With each
interaction, the agent performs an action, receives an observation, and gets a numerical
reward that tells it how well it performed in that situation. Through many such
interactions, the agent learns to maximize the cumulative reward.

In contrast, in reinforcement learning from human feedback:

1. **Human Input:** Instead of a predefined reward function, the feedback comes from
human evaluators, who provide qualitative, subjective judgments about the agent's
performance. This feedback can be in various forms, such as numerical ratings, binary
preference signals, natural language feedback, or even demonstrations showing how to
improve.

2. **Feedback Loop:** Human evaluators observe the agent's actions or outcomes and
provide feedback in real-time (or close to real-time). The agent learns through this
iterative feedback loop, gradually adapting its behavior to align more closely with
human preferences.

3. **Reward Shaping:** Because raw human feedback is not always directly usable in
learning algorithms due to variability and potential bias, a reward shaping process may
be involved, whereby human feedback is converted into a more stable signal that guides
learning.

4. **Preference-based RL:** One common approach within RLHF is preference-based
reinforcement learning, where the agent is trained by comparing pairs of actions (or
outcomes) and receiving human feedback on which one is better. From these pairwise
comparisons, the agent infers a reward function that it can use to guide its actions.

5. **Inverse Reinforcement Learning (IRL):** Another technique used in RLHF is IRL,
which aims to infer the underlying reward function that could explain human actions or
decisions, and then use that reward function to train the agent.

**Applications:**

Reinforcement Learning from human feedback has found applications in:

- **Robotics:** Teaching robots to perform tasks that are difficult to specify with
predefined reward functions.
- **Game playing:** Human feedback can guide AI agents to better emulate human-like
strategies or behaviors.
```

```
- **Language and content generation:** For example, suggestively guiding AI models to
generate more coherent, creative, or helpful content.
- **Recommendation systems:** Human feedback can help adjust and refine the
recommendations provided to users.

**Challenges:**

Using human feedback introduces some specific challenges:

1. **Scalability:** Human feedback is time-consuming and expensive to collect, and it's
hard to scale to complex or high-dimensional tasks.
2. **Bias and Noise:** Human feedback can be noisy, inconsistent, and biased, leading to
suboptimal learning outcomes.
3. **Alignment:** Ensuring that the reinforcement learning agent aligns with the true
preferences of the human evaluator(s) and not just the superficial feedback is
difficult.
4. **Safety and Misuse:** There can be concerns about the safety and ethical
implications of using potentially flawed human feedback, especially if the feedback
process is corrupted or gamed.

Researchers are actively working on techniques to mitigate these challenges and improve
RLHF approaches. One such approach is leveraging machine learning models to amplify the
efficacy of the human feedback, for example, by learning to generalize from sparse human
input or by synthesizing plausible feedback in situations where direct human input is
not available.
```

The response from the DPO model has similar information, as both models are trained on the same pretrained base model, but the preference fine-tuning adds more structure, formatting, and of course length, which has been shown repeatedly to be associated with general notions of human preference for language models. For more examples like this, see the completions library associated with this book at https://www.rlhfbook.com/library.

B.1 The Chattiness Balance

RLHF or preference fine-tuning methods are being used by countless people to boost scores like AlpacaEval and other automatic chat leaderboards (which use LLM-as-a-judge to approximate how helpful, harmless, and honest an agent is across simple conversational tasks), but the massive gains RLHF confers here come without shifting scores proportionally on harder-to-game evaluations like Arena. The tension is that while RLHF methods give a measurable improvement on these models, that training doesn't always transfer into performance that people care about. Through the establishment of the RLHF literature, a large swath of models have been released with related methods to boost the "alignment" of a model with RLHF, but they often took it way too far and published evaluation scores that were anywhere from misleading to meaningless.

These RLHF methods motivated by alignment, when done right, make the models easier to work with and more enjoyable. This often comes with clear improvements on evaluation tools like MT-Bench or AlpacaEval.

In the fall of 2023, there was a peak in the debate over direct preference optimization (DPO) and its role relative to proximal policy optimization (PPO) and other RL-based methods for preference fine-tuning – the balance of chat evaluations to real-world performance was at the center of this (For more technical discussion on the trade-offs, see Chapter 8, Ivison et al. 2024 [132], or this talk). The problem is that you can also use techniques like DPO and

PPO in feedback loops or in an abundance of data to actually severely harm the model on other tasks like mathematics or coding in a trade for this chat performance.

During the proliferation of the DPO versus PPO debate there were many papers that came out with incredible benchmarks but no meaningful adoption. If these papers released model weights, they weren't popular in public usage because these models were not robust in general usage. When applying RLHF in the fall of 2023 or soon after, there is no way to make an aligned version of a 7 billion parameter model actually beat GPT-4 across comprehensive benchmarks (this sort of comparison will hold, where small models of the day cannot robustly beat the best, large frontier models). It seems obvious, but there are always papers claiming these sorts of results. fig. 52 is from a paper called Direct Nash Optimization (DNO), which makes the case that their model is state-of-the-art or so on AlpacaEval for 7B models in April 2024 [207]. For context, DNO is a batched, on-policy *iterative* alternative to reward-model+PPO (classic RLHF) or one-shot DPO that directly optimizes pairwise preferences (win-rate gaps) by framing alignment as finding a Nash equilibrium against a preference oracle. These challenges emerge when academic incentives interface with technologies becoming of extreme interest to the broader society.

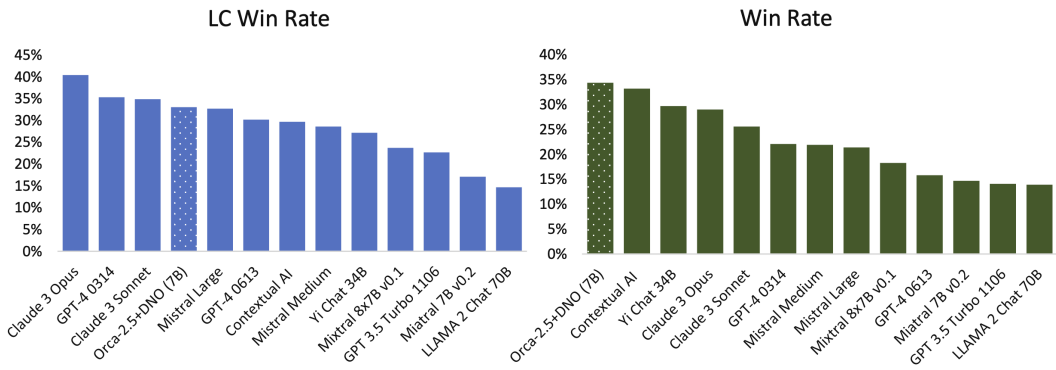


Figure 52: Results from the paper on Direct Nash Optimization (DNO) highlighting their small model outperforming the likes of GPT-4. Rosset et al. 2024. License CC-BY.

Even the pioneering paper Self Rewarding Language Models from January of 2024 [301] disclosed unrealistically strong scores on Llama 2 70B. At the time, of course, a 70B model can get closer to GPT-4 than a 7B model can (as we saw with the impressive Llama 3 releases in 2024), but it's important to separate the reality of models from the claims in modern RLHF papers. These models are tuned to narrow test sets and do not hold up well in real use versus the far larger models they claim to beat. Many more methods have come and gone similar to this, sharing valuable insights and oversold results, which make RLHF harder to understand.

A symptom of models that have "funky RLHF" applied to them has often been a length bias. This got so common that multiple evaluation systems like AlpacaEval and WildBench both have linear length correction mechanisms in them. This patches the incentives for doping on chattiness to 'beat GPT-4' or the leading frontier model of the day, and creates a less gamified dynamic where shorter, useful models can actually win.

Regardless, aligning chat models only for chattiness now has a bit of a reputational tax associated with it in the literature, where it's acknowledged that these narrow methods can harm a model in other ways. This note from the original Alibaba Qwen models in 2023 is something that has been observed multiple times in early alignment experiments, exaggerating a trade-off between chattiness and performance [413].

> We pretrained the models with a large amount of data, and we post-trained the models with both supervised fine-tuning and direct preference optimization. However, DPO leads to improvements in human preference evaluation but degradation in benchmark evaluation.

An early, good example of this tradeoff done right is a model like Starling Beta from March of 2024 [82]. It's a model that was fine-tuned from another chat model, OpenChat [414] (which was in fact trained by an entire other organization). Its training entirely focuses on k-wise reward model training and PPO optimization, and moves it up 10 places in Arena. The average response length of the model increases, but in a way that's good enough to actually help the human raters. Later examples, such as Olmo 3, are documented as undergoing substantial chat training, but with the authors preferring a final model checkpoint with higher math, coding, and reasoning scores instead of potential checkpoints that are highest on LLM-as-a-judge-based chat benchmarks [18].

A natural question is: Why does RLHF make model responses longer? Fundamentally, evaluations like Arena have shown us that average users of models often like longer, complete answers when compared with terse responses. Longer answers can feel more thorough, helpful, or even trustworthy to users evaluating them quickly. This does not represent the preference of *every* user, but these models are trained to match the average preferences of many data labelers, so RLHF tends to make models more verbose.

C Practical Issues

This appendix covers practical considerations for running post-training experiments at scale. This takes the form of a list of lessons, rather than a coherent narrative.

C.1 Compute Costs of Post-Training

There are two different ways of scoping costs for post-training runs. The largest cost is in developing the recipe, which can easily be 10X to 100X the compute of the final few training runs. The secondary costs, which are easier to measure, are the costs of thoroughly applying a recipe, which entails multiple seeds, careful evaluation, potential engineering headaches, etc.

For the first cost, to develop a post-training recipe like Tülu 3 [6], the team ran on the order of thousands of experiments/evaluations at the 7B scale before having the final model.

For final runs, the Olmo 3 report has a detailed accounting of what is involved in training the final 32B Think model [18]:

> Post-training follows a different operational pattern in which we run each stage multiple times, sweeping over learning rates and other hyperparameters. The theory for post-training, particularly, RL, is less developed, so we have to run multiple experiments to identify the optimal hyperparameters for a given base model. We hope to address this in future work.
>
> During post-training, checkpoint evaluation consumes a larger proportion of compute resources, in part due to long generations from reasoning models on core benchmarks. For SFT, we swept over four candidate learning rates, on 256 GPUs each, in parallel for 36 hours. Then approximately 12 hours was spent on evaluation, merging, and checkpoint confirmation, totaling approximately two days. DPO training takes less time per run (about 18 hours for a full learning-rate sweep on 64 GPUs per job) but in practice extended over multiple days due to cluster instability. The final RL runs for the initial Olmo 3 Think 32B spanned approximately 5 days with at least a day of training time lost due to stability issues. After the initial release of Olmo 3, we continued our best RL run for another 21 days on 224 GPUs to produce Olmo 3.1 Think 32B.

As scaling reinforcement learning becomes more standard practice, this will shift yet again [17]. Continuing the above example, where the original Olmo 3 32B Think post-training took only a couple of weeks, to release the improved Olmo 3.1 32B Think model the team needed to train it for an additional 3.5 weeks with RLVR. This is a substantial cost in *time* more than in total compute.

C.2 Evaluation Variance

One underappreciated challenge in post-training is evaluation variance, especially with the rise of reasoning models that need to use sampling with temperatures above 0 to get the best evaluation scores. With any sampling from models, the outputs become more variable. Different benchmarks have vastly different stability characteristics, due to the variance in difficulty of the prompts, the number of prompts in the evaluation set, the brittleness of the models being trained, etc.

During Olmo 3, the team tracked the variance of different evaluations used to evaluate reasoning models. The table below shows the standard deviation of each evaluation, computed as the mean of the standard deviation from 3 runs of 14 models (take the variance of each model, then average per evaluation):

Table 9: Standard deviation of evaluation benchmarks across multiple inference runs, categorized by stability (data from Olmo 3).

Category	Benchmark	Std. Dev.
High Variance	GPQA	1.48
	AlpacaEval 3	1.24
	IFEval	0.88
Stable	ZebraLogic	0.56
	Omega	0.56
	AIME 24 (Avg@32)	0.54
	HumanEvalPlus	0.46
	AgiEval	0.43
	BigBenchHard	0.39
Very Stable	LiveCodeBench (Avg@10)	0.29
	MBPPPlus	0.27
	MATH	0.25
	MMLU	0.22
	PopQA	0.16

Some evaluations, such as LiveCodeBench, were both noisy and cheap (via few prompts in the set), so by re-running the evaluation 10 times per model, the evaluation could move from the high-variance set to a stable setting. This could be done for every evaluation, but it can easily balloon costs.

We also see sources of variance in evaluation settings like batch size, tensor parallel settings within vLLM (e.g., TP=2 for baselines), and other sensitive numerics for sampling long generations across infrastructure. Variance is everywhere with reasoners.

C.3 Managing Training Performance Variance

Throughout all the post-training recipes and tools discussed in this book, the final model is subject to meaningful variance in performance. Understanding the distribution of this variance, its sources, and its effects is crucial to creating strong models. The goal of training a final model is to sample many points, by varying training parameters and random seeds, in order to get the strongest model possible. Note that this is a balance between the model *actually* being better, and not just the benefit of re-rolling from evaluation noise.

Where the previous section focuses on *evaluation* noise, the trickier source of noise is training uncertainty. Where evaluation noise can be managed by running more tests on a given checkpoint (uniformly reducing noise), models are trained once and can *benefit* from a positive outlier.

In practice, training teams take many steps to capture the maximum possible value out of their training recipe:

1. Sweep core optimization values like learning rate, batch size, etc. for every final model run. For example, with a new base model, I'd recommend running 10 learning rates over a wide region to be sure you're in the optimal range, then re-run in the tighter, optimal window.
2. Run multiple seeds on the best few settings. Random seed can have meaningful effects on the final model, and it's worth spending compute on.
3. Model merging is established as a key tool used to create strong models. Merging can be done in many ways, from merging different checkpoints on the same data to merging specialized models for specific domains. Generally, merging is seen as a strong and simple tool in final recipes, but clear best practices aren't established for preparing a model for later merging in a recipe [415].

C.4 Identifying Bad Training Jobs

A simple intuition that's important to establish when training models is the different types of model issues. You want most of your time to be spent on issues where the current data, algorithm, or recipe just isn't good enough. On the other hand, there are plenty of times when, while setting up a new recipe, certain methods are just broken.

The best way to understand this is to evaluate many models on a largely static evaluation suite. Then you develop an intuition for which tests are hard to move with post-training interventions (often knowledge-heavy evaluations such as MMLU). When something is very, *very* broken in a post-training setup, these largely stable evaluations can often drop by 10-20 points in a training job. This is one of the most useful signals there are when developing tooling!